On Shadow Volume Silhouettes

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Abstract

In shadow volume rendering, the shadow volume silhouette *edges are used to create primitives that model the shadow volume. A common misconception is that the vertices on such silhouettes can only be connected to two silhouette edges, i.e., have degree two. Furthermore, some believe that the degree of such a vertex can have any degree. In this short note, we present a geometric proof that shows that the degree of a silhouette vertex must be even, and not necessarily two.*

1 Introduction

The shadow volume (SV) algorithm [4] has become a very popular algorithm for real-time rendering [5] of hard shadows. Recently, the SV algorithm has been extended to handle soft shadow as well [1, 2]. In all these algorithms the shadow volume silhouette (SVS) of the shadow casting objects are found. An edge of such an SVS is connected to two polygons, where one is frontfacing and the other is backfacing as seen from the light source. The *degree* of an SVS vertex is the number of SVS edges connected to it. Note that silhouettes with edges as defined here may not necessarily be true silhouettes, and should therefore rather be referred to as possible silhouettes. For example, an SVS edge may very well be in shadow of another geometric object. In order for SV algorithms to work properly, the shadow casting objects must be polygonal and closed (two-manifold) [3].

During our research on soft shadows, we realized (to our surprise) that a vertex of an SVS can be connected to more than two SVS edges. This might be obvious to some, but we have realized that many others also believed (or believe) that an SVS vertex must have degree two. This is not so, as we will show.

2 The Degree of SVS Vertices

Theorem The degree of a shadow volume silhouette vertex is always even.

In the following, when we say vertex or edge, we refer to a SVS vertex or edge.

Proof: A geometric proof by contradiction follows here. Assume that we have a vertex with degree two, which is the smallest degree of a vertex. The definition of an edge says that the geometry connected to the edge on one side must be frontfacing (FF) and on the other side it must be backfacing (BF), or vice versa. This is illustrated in Figure 1a. Now, assume that we desire to augment the vertex so that it has degree three. This is done by inserting an edge, c, that connects to the vertex. See Figure 1b. Note that the same must hold for this new edge: FF on one side and BF on the other. However, as can be seen to the right in the figure, the geometry between edge b and c now contains both FF and BF geometry, which implies that a new edge must be located there. When the new edge c was inserted, one can also swap places between the FF and BF for c, but this leads to the same inconsistency. The only difference is that another new edge appears between a and c. Induction gives that each newly inserted edge that makes the degree odd generates a new edge to maintain consistency.

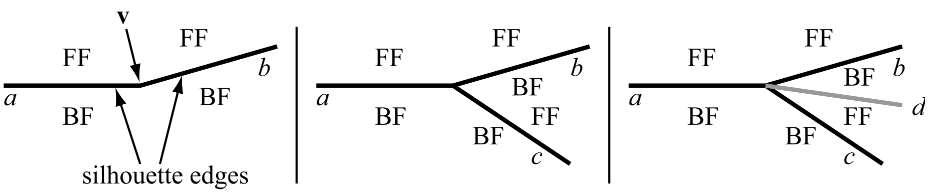

Figure 1: Left: two silhouette edges a and b meet at a vertex $\mathbf{v}$. Assume that a light source is located above the edges, and that the geometry above the edges are front facing (FF) as seen from the light source, and that the geometry below is backfacing (BF). Middle: add a new silhouette edge, c. Right: To cure the FF/BF inconsistency, a new edge d must be inserted.

At this point we have shown that the degree of an SVS vertex must be even. To prove that it can be larger than two, we give a simple example of a very simple geometrical model with vertices with degree four. See Figure 2.

Finally, we end this short note with an intuitive example that explains

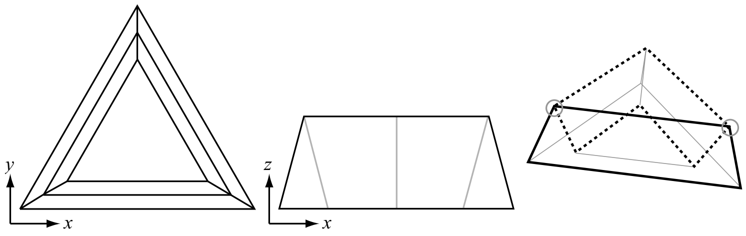

Figure 2: The left and middle parts illustrate a simple object from two orthographic views. To the right, a projection of the three-dimensional object is shown. Here, we asssume that a light source is located where the viewer is. This implies that the black continuous loop is an SVS, and the black dashed loop is another SVS. The circled vertices have degree four.

why SVSs with vertices with odd degrees would not work for shadow volume based algorithms. As can be seen in Figure 3, such scenes would easily create inconsistent results as well. In our silhouette generation software, we have seen vertices with degrees 2,4,6, and 8.

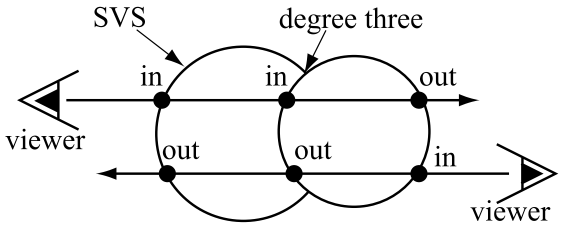

Figure 3: An example of incorrect shadow volumes. To understand the purpose of this illustration, imagine that the two view rays coincide. Normally, one would expect that the reciprocity law holds here, i.e., the left viewer experiences exactly what the right viewer experiences (only difference is direction).

References

[1] Akenine-Möller, Tomas, and Ulf Assarsson, "Approximate Soft Shadows on Arbitrary Surfaces using Penumbra Wedges," *13th Eurographics Workshop on Rendering*, pp. 309–318, June 2002. 133

[2] Assarsson, Ulf, and Tomas Akenine-Möller, "A Geometry-based Soft Shadow Volume Algorithm using Graphics Hardware," to appear in *SIGGRAPH 2003*, 2003. 133

[3] Bergeron, P., "A General Version of Crow's Shadow Volumes," *IEEE Computer Graphics and Applications*, vol. 6, no. 9., pp. 17–28, September 1986. 133

[4] Crow, Frank, "Shadow Algorithms for Computer Graphics," *Computer Graphics (Proceedings of ACM SIGGRAPH 77)*, pp. 242–248, July 1977. 133

[5] Heidmann, Tim, "Real shadows, real time," *Iris Universe*, no. 18, pp. 23–31, November 1991. 133