Homotopy type theory and constructive higher category theory

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Project description. Homotopy type theory [2] sits at the intersection between computer science and mathematics. It is a synthetic language for homotopy types, which models higher-dimensional structure as for example found in topological spaces.

Category theory is an organizing language for mathematics with applications to computer science. Higher categories generalize categories, replacing *sets* of morphisms between objects by *homotopy types*. For example, every model of homotopy type theory presents an ∞-category. Certain phenomena (such as descent/univalence) admit a satisfactory explanation only in the ∞-categorical world.

Many models for ∞-categories have been proposed, but the most mature is *quasicategories* by Joyal and Lurie [1]. In contrast to ordinary category theory, their development relies on classical logic. For example, the axiom of excluded middle is used to make every simplicial set cofibrant. This contrasts with ordinary category theory, which is largely constructive and has computational content.

This project will investigate further the connections between constructive mathematics, higher category theory, and homotopy type theory. To list just two example topics:

- Developing constructively (the basics of) higher category theory such as (co)limits, adjunctions, fibrations, presheaves, the ∞-category of ∞-categories, etc.
- Clarifying the semantics of homotopy type theory in higher categories. For example, a univalent universe should correspond to a univalent map in the presented ∞-category.

Prerequisites. You should be familiar with category theory. Familiarity and/or interest in any of the following topics is beneficial: constructive mathematics, type theory, abstract homotopy theory, models of higher categories, simplicial sets.

Starting out. The precise nature of your research will depend on your own background and interests. Most likely, you will spend the first year getting to know the state of the start. For this, you will study the relevant literature, solve and present exercises, and discuss with researchers in the field.

Environment. You will be working in the *Logic and Types* unit, a team of leading researchers in the area of dependent type theory. The group is well-known for the development of proof assistants, including the Agda system. It also conducts research in category theory, semantics of type theory, homotopy type theory, and synthetic formalization of mathematics.

Senior members of the group include Andreas Abel, Robin Adams, Jean-Philippe Bernardy, Ana Bove, Thierry Coquand, Nils Anders Danielsson, Peter Dybjer, and Christian Sattler. It currently has three postdoctoral researchers and five doctoral students.

We have an open culture based around frequent discussions and free exchange of ideas. You will be expected to participate in this culture, for example by giving informal talks.

References

[1] Jacob Lurie. *Higher topos theory*. AM-170. Princeton University Press, 2009.

[2] The Univalent Foundations Program. *Homotopy Type Theory: Univalent Foundations of Mathematics*. `https://homotopytypetheory.org/book`, IAS, 2013.