Fibrancy of $\mathcal{U}$ derived "on the right". We write Equiv for universe of equivalences and $p_0, p_1 \colon \mathsf{Equiv} \to \mathcal{U}$ for its endpoint projections. We abbreviate $I = \Delta[1]$ and denote its endpoint inclusions $\delta_0 \colon \{0\} \to I$ and $\delta_1 \colon \{1\} \to I$.

From the equivalence extension property, we have that p_0 and p_1 are trivial fibrations. We want to show that $\mathcal{U}$ is fibrant. This means to show that $[\delta_0, \mathcal{U}], [\delta_1, \mathcal{U}] \colon [I, \mathcal{U}] \to \mathcal{U}$ are trivial fibrations. We only deal with $[\delta_0, \mathcal{U}]$, the other case is dual.

Recall that δ_0 is a strong homotopy equivalence. This means that the map of arrows $\theta_0 \mathbin{\widehat{\times}} \delta_0 \colon \delta_0 \to \delta_0 \mathbin{\widehat{\times}} \delta_0$ is split mono where $\theta_0 = (!, \delta_1) \colon !_{0 \to 1} \to \delta_0$. Applying the functor $[-, \mathcal{U}]$, we get that the map

$$\begin{array}{ccc} [I, [I, \mathcal{U}]] & \xrightarrow{[I, [\delta_1, \mathcal{U}]]} & [I, \mathcal{U}] \\ {\scriptstyle \langle [\delta_0 [I, \mathcal{U}]], [I, [\delta_0, \mathcal{U}]] \rangle} \downarrow & & \downarrow {\scriptstyle [\delta_0, \mathcal{U}]} \\ [I, \mathcal{U}] \times_{\mathcal{U}} [I, \mathcal{U}] & \xrightarrow{[\delta_1, \mathcal{U}] \circ \pi_0} & \mathcal{U} \end{array} \tag{0.1}$$

of arrows from the left to the right is split epi.

We have a map

$$\begin{array}{ccc} [I, \mathcal{U}] & \xrightarrow{\quad c \quad} & \mathsf{Equiv} \\ & {\scriptstyle \langle [\delta_0, \mathcal{U}], [\delta_1, \mathcal{U}] \rangle} \searrow \quad \swarrow {\scriptstyle \langle p_0, p_1 \rangle} & \\ & \mathcal{U} \times \mathcal{U} & \end{array}$$

that converts equalities into equivalences between types. We use it to split the map of arrows (0.1) into two factors:

$$\begin{array}{ccccc} [I, [I, \mathcal{U}]] & \xrightarrow{[I, c]} & [I, \mathsf{Equiv}] & \xrightarrow{[I, p_1]} & [I, \mathcal{U}] \\ {\scriptstyle \langle [\delta_0 [I, \mathcal{U}]], [I, [\delta_0, \mathcal{U}]] \rangle} \downarrow & & \downarrow {\scriptstyle \widehat{\exp}(\delta_0, p_0)} & & \downarrow {\scriptstyle [\delta_0, \mathcal{U}]} \\ [I, \mathcal{U}] \times_{\mathcal{U}} [I, \mathcal{U}] & \xrightarrow{c \times_{\mathcal{U}} [I, \mathcal{U}]} & \mathsf{Equiv} \times_{\mathcal{U}} [I, \mathcal{U}] & \xrightarrow{p_1 \circ \pi_0} & \mathcal{U} \end{array}$$

Since the composite morphism is split epi, so is its right factor (depicted by the right square). Since p_0 is a trivial fibration, so is $\widehat{\exp}(\delta_0, p_0)$. Since $[\delta_0, \mathcal{U}]$ is its retract, it is also a trivial fibration.