Advanced Topics in Automata
Exercise 3

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Exercise

1. A nondeterministic generalized Büchi automaton is $A = \langle \Sigma, Q, Q_0, \delta, \alpha \rangle$ where $\alpha = \{F_1, \ldots, F_n\}$. A run r of A is accepting if for **every** set $F \in \alpha$ we have $inf(r) \cap F \neq \emptyset$.

   For every generalized Büchi automaton we know how to construct a Büchi automaton. The essence of the construction was that we can overlook infinitely many visits to each of the acceptance sets, as long as we make sure that we don't overlook too many. In this exercise we show a more efficient construction. We show that we can ignore many more visits to each acceptance set and still stay with enough.

   Consider a nondeterministic generalized Büchi automaton $A = \langle \Sigma, Q, Q_0, \delta, \alpha \rangle$ where $\alpha = \{F_1, \ldots, F_n\}$. Let $[n]$ denote the set $\{1, \ldots, n\}$. Let $A' = \langle \Sigma, Q \times [n], Q_0 \times \{1\}, \delta', F_1 \times \{1\} \rangle$ where δ' is defined for every state $(q, i) \in Q \times [n]$ and letter $a \in \Sigma$ as follows.

   $$\delta'((q,i),a) = \begin{cases} \delta(q,a) \times \{i\} & q \notin F_i \\ \delta(q,a) \times \{(i \ mod \ n) + 1\} & q \in F_i \end{cases}$$

   The automaton A' has $[n]$ copies of the automaton A. Copy i is waiting for a visit to the set F_i. As long as F_i is not visited, A' stays in copy i. Once F_i is visited A' moves to copy $(i \ mod \ n) + 1$ (This is just a fancy way of saying if $i = n$ go to 1 otherwise, increase i by 1).

   Prove that $L(A) = L(A')$.

2. A Müller automaton is $A = \langle \Sigma, Q, Q_0, \delta, \alpha \rangle$ where $\alpha = \{F_1, \ldots, F_n\}$. A run r of A on an infinite word $w = w_0 w_1 \cdots$ is an infinite sequence $r = q_0 q_1 \cdots$ such that: (a) $q_0 \in Q_0$ (b) Forall $i \geq 0$ we have $q_{i+1} \in \delta(q_i, w_i)$. A run r is accepting if $inf(r) \in \alpha$.

   That is, a Müller automaton, may have many runs on a given word. It accepts this word if for one of these runs $inf(r)$ is **one** of the sets in the acceptance condition. Note that $inf(r)$ defines both the states that are visited infinitely often and the states that are visited finitely often.

   Show that for every Müller automaton there exists a nondeterministic Büchi automaton that accepts the same langauge.

   Hint. Use the closure of Büchi automata under union. Use the construction showing how to complement a deterministic Büchi automaton and generalize it. Use the conversion of generalized Büchi to Büchi.