Advanced Topics in Automata
Exercise 2

Submission: April 8, 2003

Exercise

1. Prove that the $R^k_{i,j}$ construction yields (in the worst case) an exponential blow up.

Food for thought

2. Consider the languages:

$$L_n = \left\{ \{0,1,\sharp\}^*\sharp w\sharp\{0,1,\sharp\}^*\$w \mid w \in \{0,1\}^n \right\}$$

We have shown that a DFW accepting L_n has at least 2^{2^n} states. We gave the general idea of how to construct a concurrent AFW (an E, A, C machine) of size $O(log(n))$ that accepts L_n. Formalize, these ideas.

3. Let e be a regular expression and let $E = \{e_1, \ldots, e_k\}$ be a finite set of regular expressions over a common alphabet Σ. Let Σ_E be the alphabet $\{a_1, \ldots, a_k\}$. Intuitively, Σ_E consists of names for the regular expresssions in E. Let f be a regular expression over Σ_E. the regular expression $f(E)$ over the alphabet Σ is obtained by substituting e_i for a_i in f.

We say that f is an *approximation* of e with respect to E if $L(f(E))$ is contained in $L(e)$. We say that f is a rewriting of e with respect to E if $L(f(E))$ is equal to $L(e)$.

(a) Given e and E, use the DFW for e and the NFW for the regular expressions in E (all over the alphabet Σ), to construct a DFW (over the alphabet Σ_E) that accepts some word iff there is a nonempty approximation of e with respect to E.

(b) Replace every transition reading a_i in the automaton above by an automaton for e_i. Show that the resulting automaton accepts $L(e)$ iff there is a rewriting of e with respect to E.