Advanced Topics in Automata
Exercise 1

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Exercise

1. Give a family of languages $L_1, L_2, \ldots$ all over the same **small** alphabet such that L_i is accepted by an NFW (nondeterministic finite automaton on words) with $O(i)$ states but every DFW (deterministic finite automaton on words) that accepts L_i has at least $2^{O(i)}$ states.

   Bonus:

   Make the construction tight. That is, L_i accepted by an NFW with i states and least DFW with $2^i - 1$ states.

Food for thought

2. A 'strange' automaton with ϵ-moves is $A = \langle \Sigma, Q, \delta, q_0, F \rangle$ where $\delta : Q \times (\Sigma \cup \{\epsilon\}) \to Q$. Notice that the choice is restricted to ϵ-moves and advancing-moves and not more. A run of such an automaton on $w = w_0, \ldots, w_m$ is a sequence of states and locations $r = (q_0, i_0), \ldots, (q_n, i_n)$ such that forall j one of the following holds.

   - $i_{j+1} = i_j$ and $q_{j+1} = \delta(q_j, \epsilon)$.
   - $i_{j+1} = i_j + 1$ and $q_{j+1} = \delta(q_j, w_{i_j})$.

   The run is accepting if $i_n = m + 1$ and $q_n \in F$.

   Show that NFW and 'strange' automata are polynomially related.

3. Given a regular language L all the following are correct.

   - $[L]_\sim = \{vu \mid uv \in L\}$ is regular.
   - $u^{-1}L = \{v \mid uv \in L\}$ is regular.
   - $[u]_L = \{v \mid v^{-1}L = u^{-1}L\}$ is regular and $\{[u]_L\}$ is finite.
   - $L_{n^2} = \{v \mid \exists u \,.\, |u| = |v|^2 \text{ and } vu \in L\}$ is regular.