Problem 3.1. (10 points) Apply the unification algorithm and show the most general unifier of the following formulas:

(a) $p(f(x), f(y), y)$ and $p(y, z, f(x))$;

(b) $p(g(x, y), g(y, g(z, b)))$ and $p(z, g(a, x))$.

Note: x, y, z denote variables, f, g are function symbols, p is a predicate symbol and a, b are constants.

Problem 3.2. (10 points) Consider an ordering $\succ$ on ground non-equality atoms that is total and well-founded. We denote the literal ordering induced by $\succ$ also by $\succ$. Let C and D be ground clauses without equality literals. Let A and B respectively denote the maximal atoms of C and D wrt $\succ$.
Assume that A and B are syntactically the same atoms. Assume also that A occurs negatively in C but only positively in D. Show that $C \succ_{bag} D$.

Problem 3.3. (10 points) Let Σ be a signature containing only function symbols such that Σ contains at least one constant. Let $\gg$ be a precedence relation on Σ and $w : \Sigma \to \mathbb{N}$ be a weight function compatible with $\gg$. Consider the (ground) Knuth-Bendix order $\succ_{KB}$ induced by $\gg$ and w on the set of ground terms of Σ. Describe the set of ground terms that have the minimal weight wrt $\succ_{KB}$.

Problem 3.3. (20 points) Consider the following set S of clauses:

$$\neg p(z, a) \vee \neg p(z, x) \vee \neg p(x, z)$$
$$p(y, a) \vee p(y, f(y))$$
$$p(w, a) \vee p(f(w), w)$$

where p is a predicate symbol, f is a function symbol, x, y, z, w are variables and a is a constant.
Give a refutation proof of S by using the non-ground binary resolution inference system $\mathbb{BR}$. For each newly derived clause, label the clauses from which it was derived by which inference rule and indicate most general unifiers.

Problem 3.4. (20 points) Consider the KBO ordering $\succ$ generated by the precedence $f \gg a \gg b \gg c$ and the weight function that assigns weight 1 to each symbol from $\{f, a, b, c\}$. Let σ be a well-behaved selection function w.r.t. $\succ$. Consider the set S of ground formulas:

$$a = b \vee a = c$$
$$f(a) \neq f(b)$$
$$b = c$$

Apply saturation on S using an inference process based on the ground superposition calculus $\text{Sup}_{\succ,\sigma}$ (including the inference rules of ground binary resolution with selection).
Show that S is unsatisfiable by finding a refutation of S such that during saturation only 4 new clauses are generated. Give details on what literals are selected and which terms are maximal.

Problem 3.5. (30 points) Consider the group theory axiomatisation used in the lecture. Prove that the group's left identity element e is also a right identity.

- Formalize the problem in `TPTP` and solve it using `Vampire`, by running `Vampire` with the additional option specification `--avatar off` . Provide your TPTP encoding and `Vampire` output.
- Explain the superposition reasoning part of the `Vampire` proof by detailing the superposition inferences, generated clauses and mgu (if any) in the `Vampire` poof.