Automated Deduction - SS 2019

Problem Set 2
Due May 2, 2019, 11am

Problem 2.1. (10 points) Consider the following formula:

$$x = y \rightarrow (x = a \vee x = b),$$

where x, y are variables and a, b are constants.

Describe the class of interpretations that makes this formula valid.

Problem 2.2. (20 points) Consider two axiomatizable theories $\mathcal{T}_1$ and $\mathcal{T}_2$ such that $\Sigma_{\mathcal{T}_1} = \Sigma_{\mathcal{T}_2} = \Sigma$ and $A_{\mathcal{T}_2} \subseteq A_{\mathcal{T}_1}$. Recall that $\Sigma_{\mathcal{T}_i}$ and $A_{\mathcal{T}_i}$ denote, respectively, a signature and set of axioms definining $\mathcal{T}_i$, for $i = 1, 2$. Let F be a formula over signature Σ.

(a) If F is valid in $\mathcal{T}_1$, it is also valid in $\mathcal{T}_2$? Prove your answer or give a counterexample.

(b) If F is valid in $\mathcal{T}_2$, it is also valid in $\mathcal{T}_1$? Prove your answer or give a counterexample.

Problem 2.3. (30 points) For each formula below, decide whether it is satisfiable or not by relying on the decision procedure of the theory or arrays. If the formula is satisfiable, give an interpretation that satisfies the formula.

(a) $read(write(A, i, e), j) = e \ \wedge \ i \neq j \ \wedge read(A, j) \neq e,$
where i, e, j, are integer-valued constants and A is an array constant.

(b) $read(write(write(A, i, e), j, f), k) = read(A, j) \wedge i = k \wedge read(A, k) \neq g \wedge read(A, j) \neq g,$
where i, e, j, f, k, g, are integer-valued constants and A is an array constant.

Problem 2.4. (20 points) Consider the formula:

$$g(f(a - 2)) = a + 2 \ \wedge \ g(f(b)) = b - 2 \ \wedge \ b + 1 = a - 1,$$

where a, b are constants interpreted over integers, f, g are function symbols, and $+, -, 1, 2$ are interpreted in the standard way over the integers.

(a) Use the Nelson-Oppen decision procedure to determine the satisfiability of the above formula. Use the decision procedures for the theory of uninterpreted functions and use simple mathematical reasoning for deriving new equalities among the constants in the theory of linear integer arithmetic. If the formula is satisfiable, give an interpretation that satisfies the formula.

(b) Encode the above formula as an input to the Z3 SMTsolver and evaluate Z3 on your encoding, using `http://rise4fun.com/Z3`. Interpret the result of Z3. Provide the electronic version of your Z3 encoding together with your solution.

Problem 2.5. (20 points) Consider the formula:

$$read(write(A, a, f(b)), c + 1) = f(d) \ \wedge \ f(b) \neq f(d - 1) \ \wedge \ (b + 1 = d \ \vee \ c = a - 1),$$

where A is an array constant with integer elements, a, b, c, d are constants interpreted over integers, f is a function symbol, and $+, 1$ are interpreted in the standard way over the integers.

(a) Use the Nelson-Oppen decision procedure to determine the satisfiability of the above formula. Use the decision procedures for the theory of uninterpreted functions and the theory of arrays, and use simple mathematical reasoning for deriving new equalities among the constants in the theory of linear integer arithmetic. If the formula is satisfiable, give an interpretation that satisfies the formula.

(b) Encode the above formula as an input to the Z3 SMTsolver and evaluate Z3 on your encoding, using `http://rise4fun.com/Z3`. Interpret the result of Z3. Provide the electronic version of your Z3 encoding together with your solution.