Automated Deduction - SS 2019

Problem Set 1
Due April 2, 2019, 12 o'clock

Problem 1.1. (10 points) Prove, using splitting, that the formulas $p \leftrightarrow \neg q$ and $\neg p \leftrightarrow q$ are equivalent.

Problem 1.2. (10 points) Apply the optimized definitional clausal transformation algorithm to the formula:

$$\neg((p \leftrightarrow q) \leftrightarrow (r \leftrightarrow q))$$

Apply the DPLL algorithm to the resulting set of clauses. If the resulting set of clauses is satisfiable, give a model of the formula above.

Problem 1.3. (25 points) Consider the formula:

$$(p \rightarrow (\neg q \wedge r)) \ \wedge \ (p \vee (q \rightarrow (r \vee p)))$$

(a) For each atom in the above formula, decide whether it is monotonic, anti-monotonic, or neither.

(b) Compute a clausal normal form C of the above formula by applying the optimized definitional clausal transformation algorithm.

(c) Decide the satisfiability of the computed CNF formula C by applying the DPLL method on C. If C is satisfiable, give an interpretation which satisfies it.

Problem 1.4. (10 points) A formula A is in disjunctive normal form, or simply DNF, if it is either $\top$, or $\bot$, or it is a disjunction of conjunction of literals:

$$A = \bigvee_i \bigwedge_j L_{i,j},$$

where $L_{i,j}$ are literals.

Show that satisfiability of formulas in DNF can be checked in polynomial time.

Problem 1.5. (25 points) Consider the Sudoku puzzle for placing digits from 1 to 9 into a 9x9 grid. The grid is additionally divided into 9 blocks, each block representing a 3x3 grid. In the Sudoku puzzle, each cell in the 9x9 grid is filled with one digit from 1 to 9, by satisfying the following constraints:

(S1) every cell of the 9x9 grid contains exactly one digit from 1 to 9;

(S2) every row of the 9x9 grid must contain one of each digit from 1 to 9;

(S3) every column of the 9x9 grid must contain one of each digit from 1 to 9;

(S4) every 3x3 block of the 9x9 grid must contain one of each digit from 1 to 9.

Consider an instance of the Sudoku puzzle, as given in Figure 1:

4		8	7	9				
		9	8	2		5		
	2							
9		2			6		1	7
		6	5	8	7	9		
7	8		2			4		6
							4	
		5		4	8	2		
	9			7	2	3		5

Figure 1: A Sudoku instance

Does this Sudoku instance has a solution? That is, is it possible to assign digits to the empty cells of Figure 1 by satisfying the Sudoku constraints? Provide your solution by solving the following tasks:

(a) Express the Sudoku constraints (S1)-(S4) in CNF, using propositional clauses. How many clauses does your CNF formalization use?

(b) Formalize the Sudoku instance of Figure 1 as an instance of SAT.

(c) Encode the Sudoku instance of Figure 1 as an input to the MiniSat solver and evaluate MiniSat on your encoding. Is Figure 1 satisfiable? If yes, fill-in the empty cells of Figure 1! Provide the electronic version of your MiniSat encoding together with your solution.

Problem 1.6. (20 points)
Consider the set consisting of the following clauses:

$p_0 \lor \neg p_1 \lor p_2$	$p_0 \lor \neg p_1 \lor p_2 \lor p_4$	$\neg p_0 \lor p_1 \lor \neg p_2$	$\neg p_0 \lor \neg p_1 \lor \neg p_2$
$p_0 \lor \neg p_1 \lor p_4$	$p_3 \lor p_2 \lor p_4 \lor \neg p_0$	$\neg p_2 \lor \neg p_2 \lor p_4 \lor p_3$	$\neg p_2 \lor \neg p_0 \lor p_4 \lor p_4$
$p_0 \lor p_3 \lor \neg p_4$	$p_0 \lor \neg p_1 \lor \neg p_2 \lor \neg p_3$	$\neg p_1 \lor \neg p_2 \lor \neg p_3$	$p_1 \lor \neg p_2 \lor \neg p_3 \lor \neg p_4$
$p_1 \lor p_2$	$p_2 \lor p_3 \lor \neg p_4 \lor p_3$	$\neg p_0 \lor \neg p_2 \lor \neg p_3 \lor \neg p_4$	$p_0 \lor p_2 \lor p_4$

(a) For each of the variables p_0, p_1, p_2, p_3, p_4 find the probability that WSAT will choose this variable for flipping when the current interpretation is:

$$\{p_0 \mapsto 1, p_1 \mapsto 1, p_2 \mapsto 1, p_3 \mapsto 1, p_4 \mapsto 1\}.$$

(b) Answer the same question as above, but using GSAT instead of WSAT.