Uniform Solving for ω-regular Games

Quasipolynomial Computation of Nested Fixpoints (TACAS 2021)

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Winning Regions in Games are Nested Fixpoints

- Parity game winning regions are nested fixpoints over powerset lattice
- Recent breakthrough result: solving parity games is in QP
- Idea: Adapt QP parity game solving algorithms to compute general nested fixpoints, obtain same results for more general games / logics

Main contribution:

QP algorithm for computing nested fixpoints over arbitrary finite lattices

Nested Fixpoints over Finite Lattices

Finite lattice $(L, \sqsubseteq)$ with basis B of size $n = |B|$

Nested Fixpoints over L

For monotone function $f : L^d \to L$ (w.l.o.g. d even), put

$$\mathsf{NFP}\, f := \mathsf{GFP}\, X_d.\ \mathsf{LFP}\, X_{d-1}. \ldots \mathsf{LFP}\, X_1. f(X_1, \ldots, X_d)$$

(Our results actually hold for fixpoint equation systems)

Nested Fixpoint for Parity Games

Parity game $G = (V_\exists \,\dot\cup\, V_\forall, E, \Omega)$, n nodes, d priorities

One-step game function $f_{\mathsf{PG}} : (2^n)^d \to 2^n$:

$$(X_1, \dots, X_d) \mapsto (V_\exists \cap \Diamond X_\Omega) \cup (V_\forall \cap \Box X_\Omega)$$

Theorem [Walukiewicz, 1996]

Player $\exists$ wins v if and only if $v \in \mathsf{NFP}\, f_{\mathsf{PG}}$.

Nested Fixpoint for Energy Parity Games

Energy parity game $G = (V_\exists \mathbin{\dot\cup} V_\forall, E, \Omega, w)$, $w : E \to \mathbb{Z}$

- Bound on histories $c = n \cdot d \cdot w_{\max}$ [Chatterjee, Doyen, 2012]

One-step game function $f_{\mathsf{EPG}} : (c^n)^d \to (c^n)$:

$$(X_1, \dots, X_d) \mapsto (V_\exists \sqcap \Diamond_E X_\Omega) \sqcup (V_\forall \sqcap \Box_E X_\Omega)$$

Theorem [Amram, Maoz, Pistiner, Ringert, 2020]

Player $\exists$ wins v with initial credit c_0 if and only if $(\mathsf{NFP}\ f_{\mathsf{EPG}})(v) \leq c_0$.

A Progress Measure Algorithm

- Progress measure algorithm for computing NFP f ($f : L^d \to L$)
- Progress is measured using nodes in universal tree

Main Contribution: Theorem

The progress measure algorithm computes NFP f.

Corollary [Czerwinski et al. 2018]

Nested fixpoints over finite lattices can be computed with quasipolynomially many iterations.

Conclusion

Results:

- Quasipolynomial solving of fixpoint equations by universal trees
- Highly general quasipolynomial progress measure algorithm for
  - Parity games / model checking μ-calculus
  - Energy parity games / model checking energy μ-calculus
  - Mean pay-off parity games
  - Stochastic parity games (both qualitative and quantitative)
- Typical runtime: $\mathcal{O}((hn)^{\log d})$ (notable exception: stochastic games)

Ongoing work:

- Implement algorithm to obtain generic game solver
- Does this work for all games with finite-history winning strategies?