Permutation Games for the Weakly Aconjunctive μ-Calculus

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Satisfiability Games for the μ-Calculus

Constructing satisfiability games for the μ-calculus typically involves determinization of parity automata (**tracking automata**).

Key observation: For **aconjunctive** formulas, tracking of **fixpoints** already is deterministic; easier determinization of tracking automata

Results:

- Concept of limit-deterministic parity automata along with determinization procedure
- Asymptotically smaller satisfiability games for aconjunctive formulas
- Implementation of solver for these games, **coalgebraic** and **on-the-fly**

Limit-determinisitic parity automata

Parity automata (PA)

$\mathcal{A} = (V, \Sigma, \delta, q_0, \alpha)$ with transition relation $\delta \subseteq V \times \Sigma \times V$ and priority function $\alpha : \delta \to \mathbb{N}$. Priorities are assigned to **transitions** rather than states, e.g. [Schewe,Varghese 2014].

Büchi automata are PA with priorities just 1 and 2 ($\delta_1 = \overline{F}$, $\delta_2 = F$).

Limit-deterministic PA

PA $\mathcal{A}$ is **limit-deterministic** (LD) if all its accepting runs are deterministic from some point on.

Büchi automaton is LD iff for all $t \in F$, scc(t) is deterministic.

Determinizing LDBA, example

Theorem [Esparza, Kretínský, Raskin, Sickert, TACAS 2017]

LDBA $\mathcal{B}$ of size n can be determinized to DPA $\mathrm{Det}(\mathcal{B})$ of size $\mathcal{O}(n!)$ and with $\mathrm{L}(\mathcal{B}) = L(\mathrm{Det}(\mathcal{B}))$.

Safraless determinization, using **permutations** of states.

Lemma

LDPA $\mathcal{A}$ of size n with k priorities can be transformed to LDBA $\mathrm{Büchi}(\mathcal{A})$ of size $\mathcal{O}(nk)$ and with $\mathrm{L}(\mathcal{A}) = L(\mathrm{Büchi}(\mathcal{A}))$.

Corollary

LDPA $\mathcal{B}$ of size n with k priorities can be determinized to equivalent DPA $\mathrm{Det}(\mathrm{Büchi}(\mathcal{B}))$ of size $\mathcal{O}((nk)!)$.

This brings the bound down from $\mathcal{O}(((nk)!)^2)$.

The Aconjunctive μ-Calculus

The μ-**calculus** [Kozen, 88]: expressive logic, extending modal logic with fixpoint operators ($\mu X.\,\psi$, $\nu X.\,\psi$); models are standard Kripke structures.

Aconjunctive formulas: in conjunctions $\psi_1 \wedge \psi_2$, at most one ψ_i contains an **active** μ-**variable**, i.e. a variable that can be transformed to a formula containing a free least fixpoint variable by (repeatedly) replacing variables with their binding fixpoint.

E.g. $\nu X.\mu Y.(\Diamond X \wedge \Box Y)$ is aconjunctive while $\mu X.\nu Y.(\Diamond X \wedge \Box Y)$ is not.

Weak aconjunctivity [Walukiewicz, 2000] relaxes this.

Tracking automata

Tracking automaton $\mathcal{A}_\psi$ for formula ψ: NPA that tracks single formulas through potential models, accepting **bad branches**, i.e. infinite paths on which some least fixpoint is unfolded infinitely often.

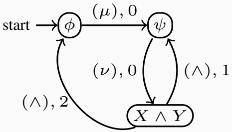
Priorities according to **alternation depth** of passed fixpoint variables.

Lemma

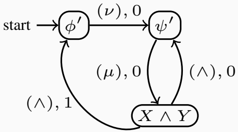
If ψ is weakly aconjunctive, then the tracking automaton $\mathcal{A}_\psi$ is limit-deterministic.

E.g.: $\phi := \mu X. \underbrace{\nu Y.(X \wedge Y)}_{=:\psi}$ $\qquad$ $\phi' := \nu X. \underbrace{\mu Y.(X \wedge Y)}_{=:\psi'}$

$\mathcal{A}_\phi$:



$\mathcal{A}_{\phi'}$:



Satisfiability games

Permutation games for the weakly aconjunctive μ-calculus

Input: Weakly aconjunctive formula ψ

1. Tracking automaton $\mathcal{A}_\psi$ is **LDPA** of size $n = |\psi|$ with $k = \text{ad}(\psi)$ priorities, recognizes **bad branches** in pre-tableaux for ψ.
2. Determinize $\mathcal{A}_\psi$ using permutation method, obtaining equivalent DPA $\mathcal{B}_\psi$ of size $\mathcal{O}((nk)!)$ and with $\mathcal{O}(nk)$ priorities.
3. Complement DPA $\mathcal{B}_\psi$, obtaining DPA $\mathcal{C}_\psi$ of same size.
4. Solve resulting satisfiability game on states of $\mathcal{C}_\psi$ in time $\mathcal{O}((nk)!^{nk})$.

Build the game step by step and solve it **on-the-fly**, using the fixpoint iteration algorithm for parity games, see e.g. [Bruse, Falk, Lange, 2014].

Implementation

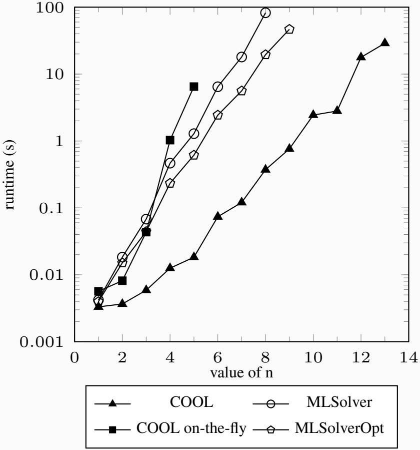
Permutation games work for the **coalgebraic** μ-calculus (covering e.g. alternating-time and probabilistic fixpoint logics, and game logic).

They have been implemented as part of the **Coalgebraic Ontology Logic Reasoner** (COOL):

`https://www8.cs.fau.de/research:software:cool`

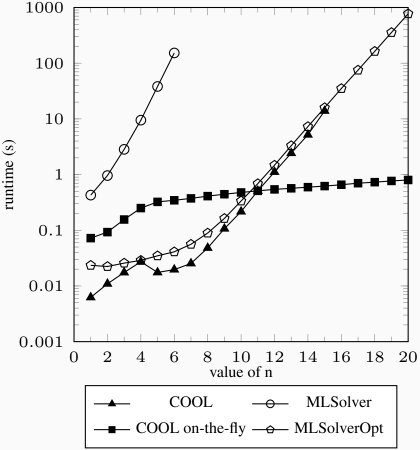
We compare COOL with MLSolver (which supports the full μ-calculus) on some series of aconjunctive formulas.

Benchmarking



"Parity automata with *n* priorities can be transformed to equivalent parity automata with 3 priorities." (valid)

Benchmarking



early-ac($n, 4, 2$) (unsatisfiable)