Introduction to Hybrid Systems

GERARDO SCHNEIDER

gerardo@irisa.fr

IRISA/INRIA

ÉQUIPE LANDE

RENNES - FRANCE

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Motivation

- Computers are everywhere
  - Electronic commerce
  - Education
  - Thermostat
  - Automated highway systems
  - Air traffic management systems
  - Automotive industry (robots)
  - Chemical plants
  - ...

Motivation

- Computers are everywhere
- Many of these systems have a *“hybrid”* nature. Systems exhibiting both:
  - Continuous evolutions
  - Discrete transitions

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Motivation

- Computers are everywhere
- Many of these systems have a *"hybrid"* nature. Systems exhibiting both:
  - Continuous evolutions
  - Discrete transitions
- Some examples:
  - Thermostat: Temperature + switch On/Off
  - Robotic systems: Distance, speed, etc + switch direction
  - Chemical plants: Chemical reactions + closing/opening valves
  - . . .

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Hybrid Systems: Why a new theory?

- Two main reasons: Academic and practical

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  Academic reason: People competent in specific domains of knowledge
  - Control theoreticians
  - Computer scientists
  - Mathematicians

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  Practical reason: Finding suitable abstract models and analysis techniques for natural phenomena

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  - Mathematicians

  Practical reason: Finding suitable abstract models and analysis techniques for natural phenomena

- Hybrid models offer clean modelling solutions for phenomena for which classical models are inadequate

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Overview of the presentation

- Continuous models
- Discrete systems
- Hybrid automata
- Verification
- Discussion

Continuous Models

Traditional formalisms for describing system dynamics are based on continuous dynamical systems

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- Initially: Conceived for predicting the behaviour of *uncontrolled* systems (e.g. solar system)
- Later: Adapted for systems with *inputs* - *controlled* systems (e.g. robots)
  - In the presence of disturbance or control signals: Need for *input* (or *control*) variables

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- Such systems are specified by differential or difference equations, describing the evolution of the state-variable

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Continuous Models: Limitations

- The dynamics of many physical components of plants cannot be modelled using purely-continuous models
  - Behaviour of valves and switches are best modelled as discrete systems
  - Continuous sensors and actuators are saturated beyond certain values

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Continuous Models: Limitations

- The dynamics of many physical components of plants cannot be modelled using purely-continuous models
- Some “intelligent” control might not be expressed in terms of continuous trajectories
  - Movement in physical space may contain “objects” and “places”: Inherently discrete involving phenomena like collision

Continuous Models: Limitations

- The dynamics of many physical components of plants cannot be modelled using purely-continuous models
- Some “intelligent” control might not be expressed in terms of continuous trajectories
- Even in the presence of continuous models, the dynamics could be highly *non-linear*
  - Many models based on a linear approximation are valid only in a certain region. When leaving such region a new linear model should be used

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- The dynamics of many physical components of plants cannot be modelled using purely-continuous models
- Some “intelligent” control might not be expressed in terms of continuous trajectories
- Even in the presence of continuous models, the dynamics could be highly *non-linear*
- Many control systems need interaction with entities other than continuous sensors: e.g. with computers or human operators
  - Such entities may activate or suspend the controller execution or force it to switch to another mode of operation

Continuous Models: Practical Solution

- Control Engineers know how to solve many of the above problems:

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  - A continuous model is given for each “mode” of operation
  - Control laws are synthesised for each of these modes and then “glue” together

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- Control Engineers know how to solve many of the above problems:
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  - Control laws are synthesised for each of these modes and then “glue” together
- However, the transition between them is not a part of the “official” dynamics of the system
  - The formal notion of dynamical system is reserved only for the continuous modes; other phenomena are treated as “extra-modelic”

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Discrete Systems

- The design of reactive systems in Computer Science has similar goals to Control Theory
  - To design systems that interact with an external environment

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- The design of reactive systems in Computer Science has similar goals to Control Theory
  - To design systems that interact with an external environment
- **Example:** A mechanism which controls the access of clients to some shared resources

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Discrete Systems

- Main difference between Control Theory and Computer Science
  - Control Theory:
    - State variables are physical magnitudes (e.g. temperature)
    - Interaction is done through measurements of physical magnitudes
  - Computer Science:
    - State variables are non-numerical values (e.g. “ready”, “waiting”)
    - Interaction via *messages* and *events* such as “request” or “release”

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Discrete Systems: How to Model?

- State-transition dynamics: For each state and input event, it defines what is the *next* state

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Discrete Systems: How to Model?

- State-transition dynamics: For each state and input event, it defines what is the *next* state
- Small state-space: It can be explicitly written in a table
- Larger systems are described using two methods:
  - *Composition*, where interacting sub-systems are described separately
  - *Implicit* (symbolic) description (e.g. using programming formalisms)

Discrete vs. Continuous Systems

The state space of a discrete system is *much smaller* than that of a continuous one

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Not Always!

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Discrete vs. Continuous Systems

The state space of a discrete system is *much smaller* than that of a continuous one

- D.S. are defined on impoverish mathematical domains: Analysis and synthesis are more difficult
- **Examples:**
  - $A\mathbf{x} = \mathbf{b}$ defined over the reals: Simple solution using division and subtraction
  - Finding whether there is a Boolean vector satisfying a formula in propositional logic, is an NP-hard problem: We need to explore all possible vectors!

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Discrete vs. Continuous Systems

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- D.S. are defined on impoverish mathematical domains: Analysis and synthesis are more difficult
- **Examples:**
  - In C.S. $\dot{\mathbf{x}} = A\mathbf{x}$: Knowledge about the trajectories by inspecting A
  - In D.S.: No "holistic" way to capture the global behaviour of the systems. Sometimes, need to explore all the possible trajectories.

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Hybrid Automata

- Hybrid automata are a good formalism for modelling
  - The continuous "modes" of operation, and
  - The discrete switches between such modes
- They are a generalisation of a well-established formalism: Timed Automata

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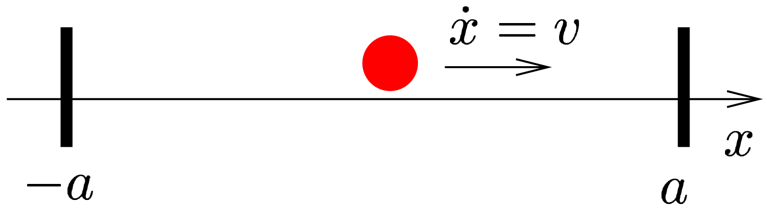
Hybrid Automata: Examples

- Frictionless movement of a particle in a bounded interval $[-a, a] \in \mathcal{R}$ subject to elastic collisions at the endpoints of the interval



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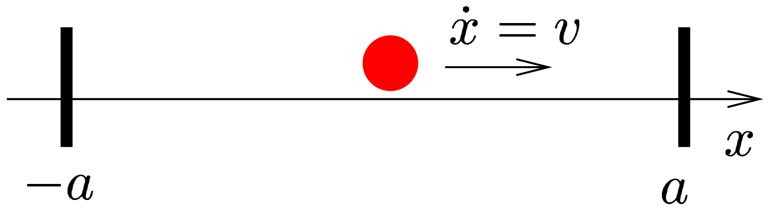
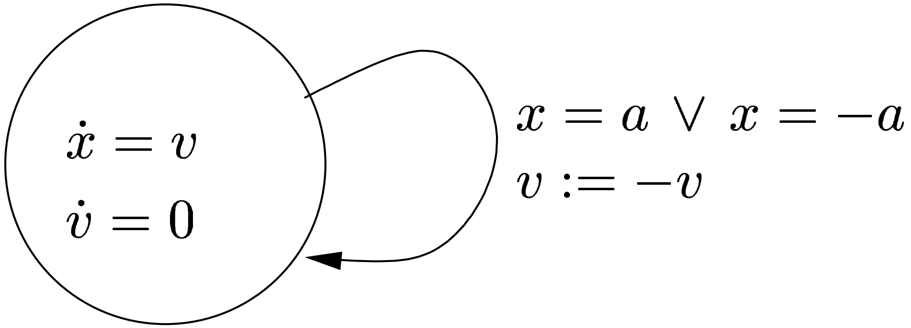
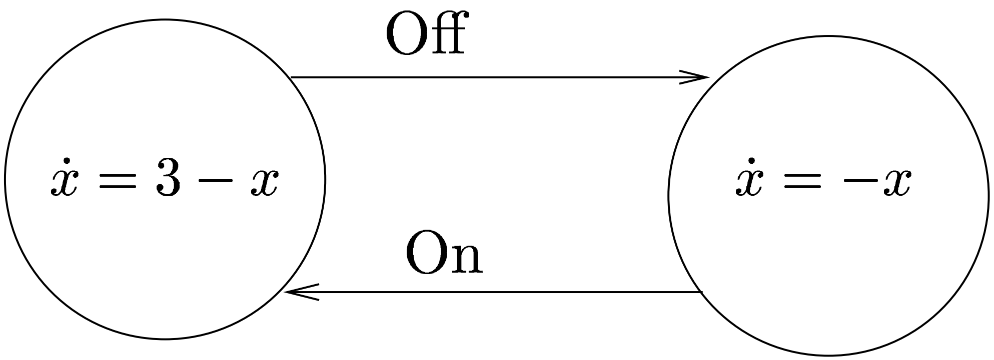




Hybrid Automata: Examples

- A heating system with external On/Off commands



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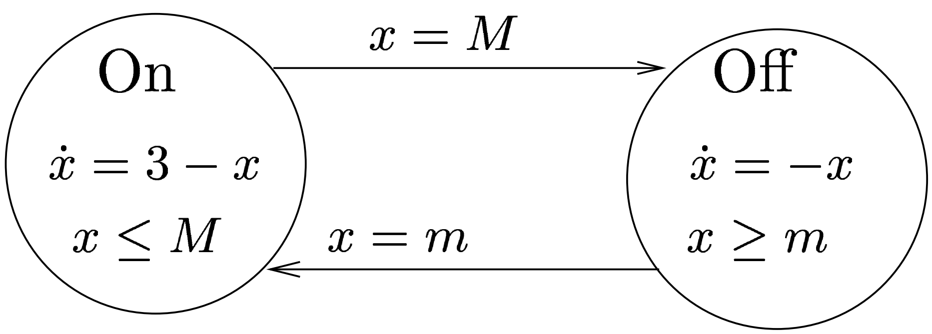
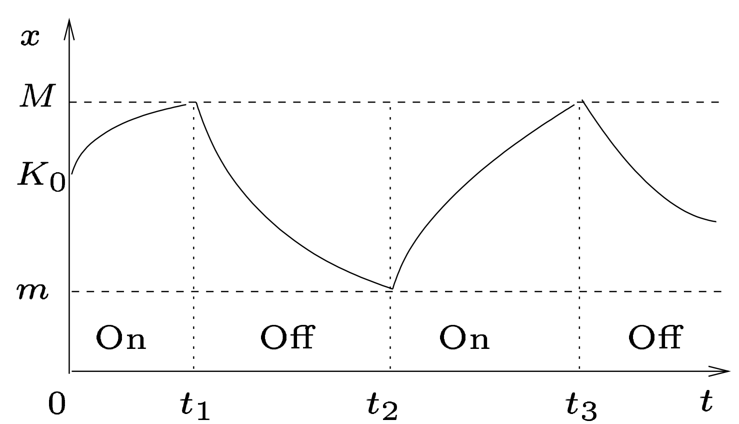
Hybrid Automata: Examples

- A heating system with a thermostat





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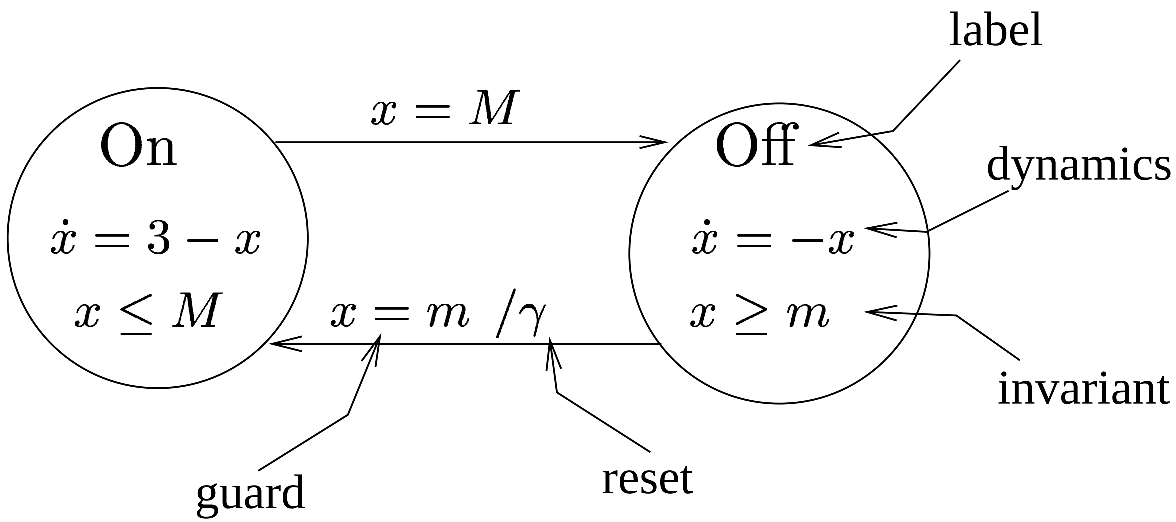

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Verification: Motivation

- How to build **correct** complex systems?

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- Synthesis (from the specification)

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- How to build **correct** complex systems?
- Synthesis (from the specification)
- Build them and then
  - Test
  - Simulate

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Verification: Motivation

- How to build **correct** complex systems?
- Synthesis (from the specification)
- Build them and then
  - Test
  - Simulate
- Alternative: Formal verification

What is Verification?

- Instance:
  - P: Program (e.g. Hw circuit, communication protocol, distributed system, C program, Real-time system, hybrid automata)
  - ϕ: Specification
- Question:
  - Does P satisfies ϕ?

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- Question:
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- **Example:**
  - P: Thermostat
  - ϕ: The temperature remains always between m and M

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Verification of Discrete Systems: Methodology

- **Modelling formalisms:** Based on interacting automata and other variants of transition systems

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- **Modelling formalisms:** Based on interacting automata and other variants of transition systems
- **Formalisms for specifying systems requirements:** Automata, regular expressions or formulae in temporal logic

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Verification of Discrete Systems: Methodology

- **Modelling formalisms:** Based on interacting automata and other variants of transition systems
- **Formalisms for specifying systems requirements:** Automata, regular expressions or formulae in temporal logic
- **Methods to verify** that a controller, composed with its environment, generates only acceptable behaviours
  - Algorithmic: Explore the paths in the transition graph
  - Deductive: Try to prove some claims about all system behaviours

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Discussion

- Many natural phenomena and industrial applications are hybrid by nature (continuous + discrete behaviours)
- Hybrid systems are studied by mathematicians, computer scientists and control theoreticians
- Hybrid automata are a good formalism for modelling hybrid systems
- A lot of work is still to be done for verifying and synthesising hybrid systems!

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MUITO OBRIGADO!

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