An heuristic for verifying safety properties of infinite-state systems

GERARDO SCHNEIDER

UPPSALA UNIVERSITY

DEPARTMENT OF INFORMATION TECHNOLOGY

UPPSALA, SWEDEN

Joint work with **Michael Baldamus** and **Richard Mayr**

Motivation

- How to build **correct** complex systems?

Motivation

- How to build **correct** complex systems?
- Synthesis (from the specification)

Motivation

- How to build **correct** complex systems?
- Synthesis (from the specification)
- Build them and then
  - Test
  - Simulate

Motivation

- How to build **correct** complex systems?
- Synthesis (from the specification)
- Build them and then
  - Test
  - Simulate
- Alternative: Formal verification

What is Verification?

- Instance:
  - P: Program (Hw circuit, communication protocol, distributed system, C program, Real-time system, etc)
  - ϕ: Specification
- Question:
  - Does P satisfies ϕ?

Formal Verification

- It is a very active field for theoretical research and practical development
- Deductive vs Algorithmic approach

Formal Verification

- Model Checking (Algorithmic)
  - By now, a quite well-established theory (80's)
  - Exhaustive exploration of the state-space
  - Fully automatic
  - Practical applications:
    - Hardware controllers
    - Circuit design
    - Many communication protocols

Formal Verification

- Limitations of Model Checking:
  - Finite-state systems
  - State explosion problem

Formal Verification

- Limitations of Model Checking:
  - Finite-state systems
  - State explosion problem
- Infinite-state systems: More general but more difficult to analyse!

Verification of Infinite-State Systems

- Key aspects to take into account
  - Non-bounded variables and/or data structures (e.g. counters, clocks, queues)
  - Parameterised systems (e.g. nets of unbounded number of id. processes)
  - Mobility
  - Security

Verification of Infinite-State Systems

- Examples of infinite-state systems
  - Timed and hybrid automata
  - Process rewrite systems
  - Push-down automata
  - Communicating FSA (e.g. Lossy channel systems)
  - Petri nets
  - Parameterised systems (mutual exclusion protocols, broadcast protocols, etc)

Verification of Infinite-State Systems

- Techniques:
  - Abstraction
  - Symbolic analysis
  - Well-quasi-ordering (WQO)

The Problem

- Our Dream: Verify the π-calculus!

The Problem

- Our Dream: Verify the π-calculus!
- Not yet there! We start with something simpler: CCS-like Calculus

The Problem

- Our Dream: Verify the π-calculus!
- Not yet there! We start with something simpler: CCS-like Calculus
- Which kind of properties?
  - Safety properties (Reachability)

The Problem

- Our Dream: Verify the π-calculus!
- Not yet there! We start with something simpler: CCS-like Calculus
- Which kind of properties?
  - Safety properties (Reachability)
- Problems?
  - Verifying safety properties is undecidable in CCS
  - Termination

Our Solution

Algorithm:

- Give a Petri net semantics to CCS-like Agents
  Agent: A, Petri net: N_A
- Obtain an over-approximation Petri net $W(N_A)$
- Prove that $W(N_A)$ is a Well-Structured System
- Reachability is decidable in $W(N_A)$

Our Solution

- Our algorithm is partial:
  - If it says (NO) YES: the property is (not) satisfied
  - Sometimes it says UNKNOWN

Agenda

- Preliminaries
  - Well-Structured Systems
  - An Agent Language (CCS-like)
  - Petri Nets
- Petri Nets Semantics of the Agent Lang.
- Safety Properties Verification
- Concluding Remarks

Well-Structured Systems: Preliminaries

Let $< S, \rightarrow >$ (where $S = Q \times D$ is a set of *states*) be a **labelled transition system** (LTS) and $\preceq$ a **preorder** (reflexive and transitive)

Well-Structured Systems: Preliminaries

Let $< S, \rightarrow >$ (where $S = Q \times D$ is a set of *states*) be a **labelled transition system** (LTS) and $\preceq$ a **preorder** (reflexive and transitive)

- $\preceq$ is a **WQO** if there is no infinite sequence $a_0, a_1, \ldots$, so that $a_i \not\preceq a_j$ for any $i \leq j$

UPPSALA UNIVERSITET

Well-Structured Systems: Preliminaries

Let $< S, \rightarrow >$ (where $S = Q \times D$ is a set of *states*) be a **labelled transition system** (LTS) and $\preceq$ a **preorder** (reflexive and transitive)

- Let D be a set. A subset $U \subseteq D$ is **upward closed** if whenever $a \in U, b \in D$ and $a \preceq b$, then $b \in U$. The *upward closure* of a set

  $A \subseteq D$ is

$$\mathcal{C}(A) := \{b \in D \mid \exists a \in A.\, a \preceq b\}$$

Well-Structured Systems: Preliminaries

Let $< S, \rightarrow >$ (where $S = Q \times D$ is a set of *states*) be a **labelled transition system** (LTS) and $\preceq$ a **preorder** (reflexive and transitive)

- A LTS $< S, \rightarrow >$ is **monotonic** if, whenever $s \preceq t$ and $s \stackrel{\alpha}{\rightarrow} s'$, then $t \stackrel{\alpha}{\rightarrow} t'$ for some t' so that $s' \preceq t'$

UPPSALA UNIVERSITET

Well-Structured Systems: Definition

A trans. system $\mathcal{L} =< S, \rightarrow >$ (with $\preceq$ on data values) is **well-structured** if

- $\preceq$ is a well–quasi–ordering, and
- $< S, \rightarrow >$ is monotonic with respect to $\preceq$, and
- for all $s \in S$ and $\alpha \in L$, the set $\mathtt{min}(\mathtt{pre}_\alpha(\mathcal{C}(\{s\})))$ is computable

WSS: Some Nice Properties

Theorem:

- Let $< S, \rightarrow >$ be a WSS, $< q, d >$ a state and U an upward–closed subset of the set of data values
- Then it is decidable whether it is possible to reach, from $< q, d >$, any state $< q', d' >$ with $d' \in U$

An Agent Language (CCS-like)

- Given:
  - A set of *names*, $\mathcal{N}$ ($a, b, x, y \ldots$)
  - A set of *co-names*, $\overline{\mathcal{N}} = \{\overline{a} \mid a \in \mathcal{N}\}$. The set of *visible actions*: $Act = \mathcal{N} \cup \overline{\mathcal{N}}$
  - We denote by $Act_\tau = \mathcal{N} \cup \overline{\mathcal{N}} \cup \{\tau\}$ (α)

UPPSALA UNIVERSITET

An Agent Language (CCS-like)

- Given:
  - A set of *names*, $\mathcal{N}$ $(a, b, x, y \ldots)$
  - A set of *co-names*, $\overline{\mathcal{N}} = \{\overline{a} \mid a \in \mathcal{N}\}$. The set of *visible actions*: $Act = \mathcal{N} \cup \overline{\mathcal{N}}$
  - We denote by $Act_\tau = \mathcal{N} \cup \overline{\mathcal{N}} \cup \{\tau\}$ (α)
- The syntax is given by:

$$P ::= \mathbf{0} \mid \alpha.P \mid P + Q \mid P \backslash c \mid P \parallel P \mid A$$

Where $A \stackrel{\text{def}}{=} P$

UPPSALA UNIVERSITET

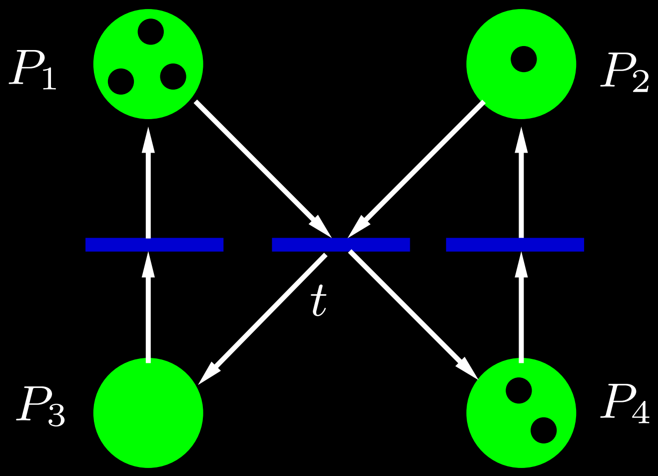
Petri Nets

- A Petri net is a tuple $N = (P, A, T, M_0)$:
  - P is a finite set of *places*
  - A is a finite set of *actions* (or *labels*)
  - $T \subseteq \mathcal{M}(P) \times A \times \mathcal{M}(P)$ is a finite set of *transitions*
  - M_0 is the *initial marking*

where $\mathcal{M}(P)$ is a collection of multisets (bags) over P

UPPSALA UNIVERSITET

Petri Nets: Graphical representation



Marking

M : is a mapping from places to the set of natural numbers

$$M(P_1) = 3 \quad M(P_2) = 1$$
$$M(P_3) = 0 \quad M(P_4) = 2$$

Agenda

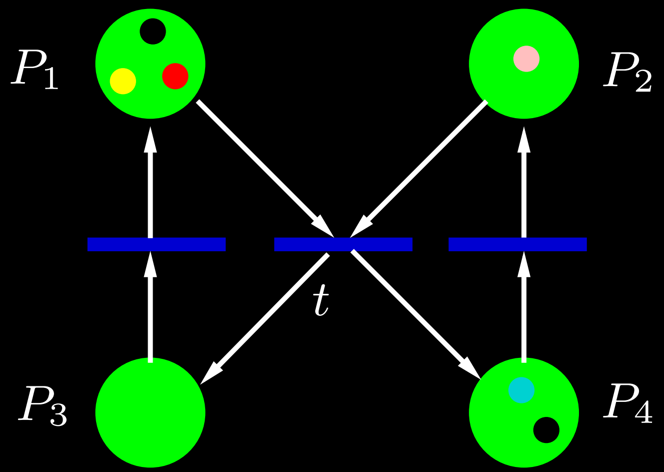
- Preliminaries
  - Well-Structured Systems
  - An Agent Language (CCS-like)
  - Petri Nets
- Petri Nets Semantics of the Agent Lang.
- Safety Properties Verification
- Concluding Remarks

Petri Nets Semantics of the Agent Lang.

- We will use Coloured Petri Nets

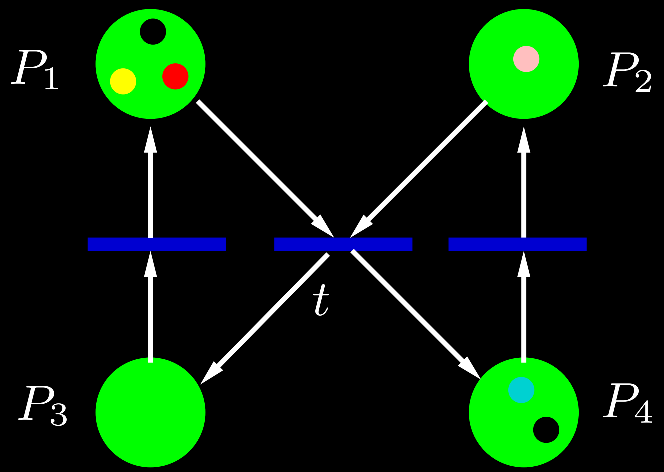
Petri Nets Semantics of the Agent Lang.

- We will use Coloured Petri Nets



Petri Nets Semantics of the Agent Lang.

- We will use Coloured Petri Nets



- In particular, we will use *strings* as colours

Petri Nets Semantics of the Agent Lang.: Formal Definition

- Places : all agent constants together with all agents and sub–agents that occur on the right–hand side of any defining equation within the environment

Petri Nets Semantics of the Agent Lang.: Formal Definition

- Places
- Transitions :

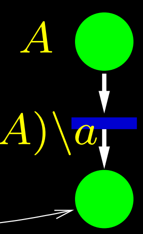
$$\begin{aligned}
\texttt{Trans}(\alpha.P) &= \left\{\left\langle\{\alpha.P\},\{P\}\right\rangle \mapsto \alpha\right\} \\
\texttt{Trans}(P+Q) &= \left\{\left\langle\{P+Q\},\{P\}\right\rangle, \left\langle\{P+Q\},\{Q\}\right\rangle\right\} \\
\texttt{Trans}(P|Q) &= \left\{\left\langle\{P|Q\},\{P \mapsto \mathrm{l}, Q \mapsto \mathrm{r}\}\right\rangle\right\} \\
\texttt{Trans}(P\backslash c) &= \left\{\left\langle\{P\backslash c\},\{P\}\right\rangle \mapsto \backslash c\right\} \\
\texttt{Trans}(A) &= \left\{\left\langle\{A\},\{P\}\right\rangle\right\}, \text{ given that } A \triangleq P
\end{aligned}$$

Petri Nets Semantics of the Agent Lang.: Example

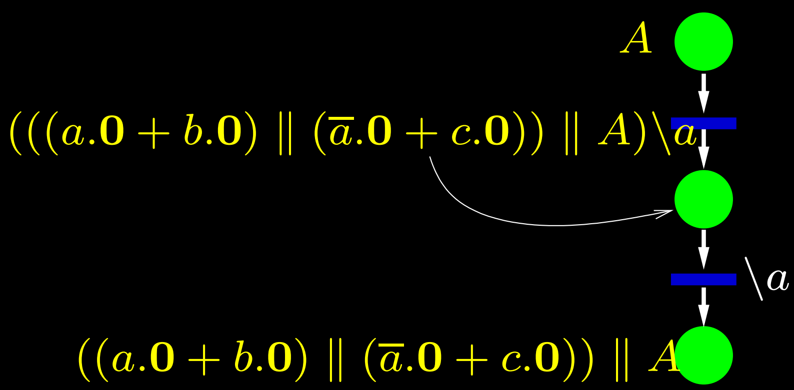
$$A \stackrel{\mathrm{def}}{=} (((a.\mathbf{0} + b.\mathbf{0}) \parallel (\overline{a}.\mathbf{0} + c.\mathbf{0})) \parallel A)\backslash a$$

Petri Nets Semantics of the Agent Lang.: Example

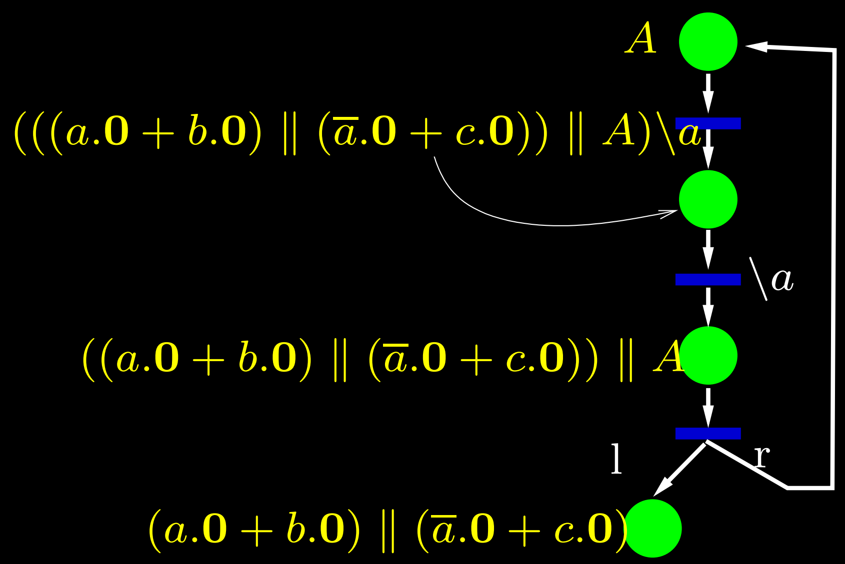
$$(((a.\mathbf{0} + b.\mathbf{0}) \parallel (\overline{a}.\mathbf{0} + c.\mathbf{0})) \parallel A) \backslash a$$



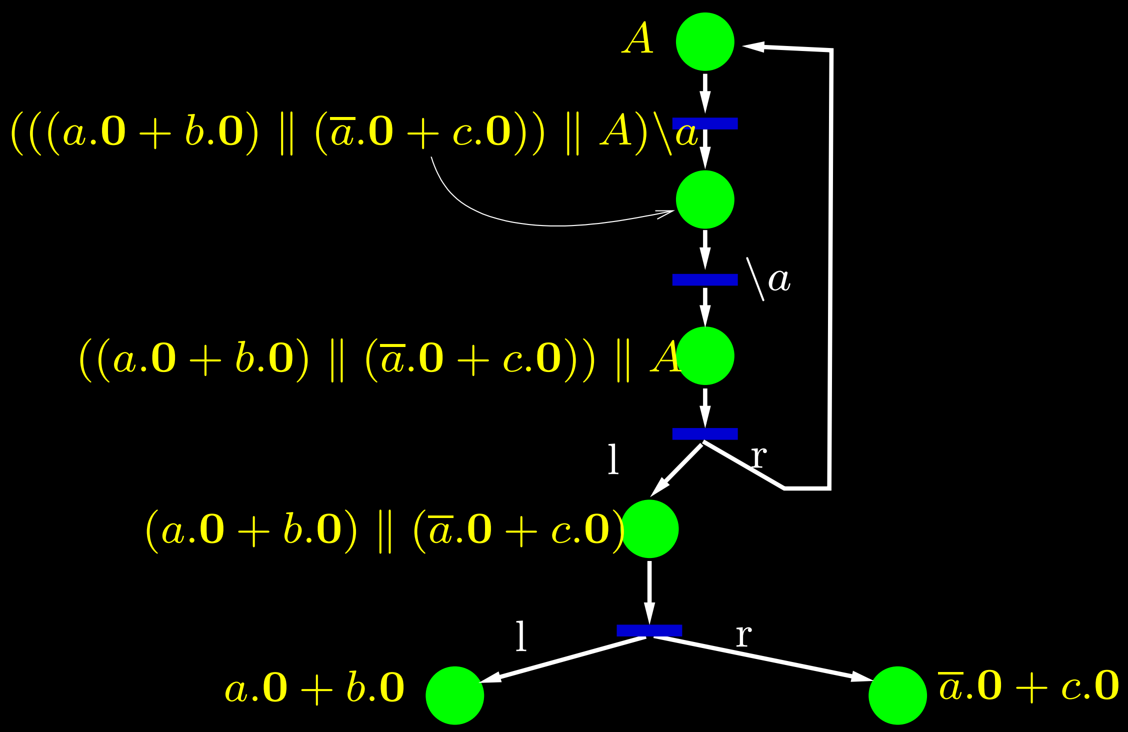
Petri Nets Semantics of the Agent Lang.: Example



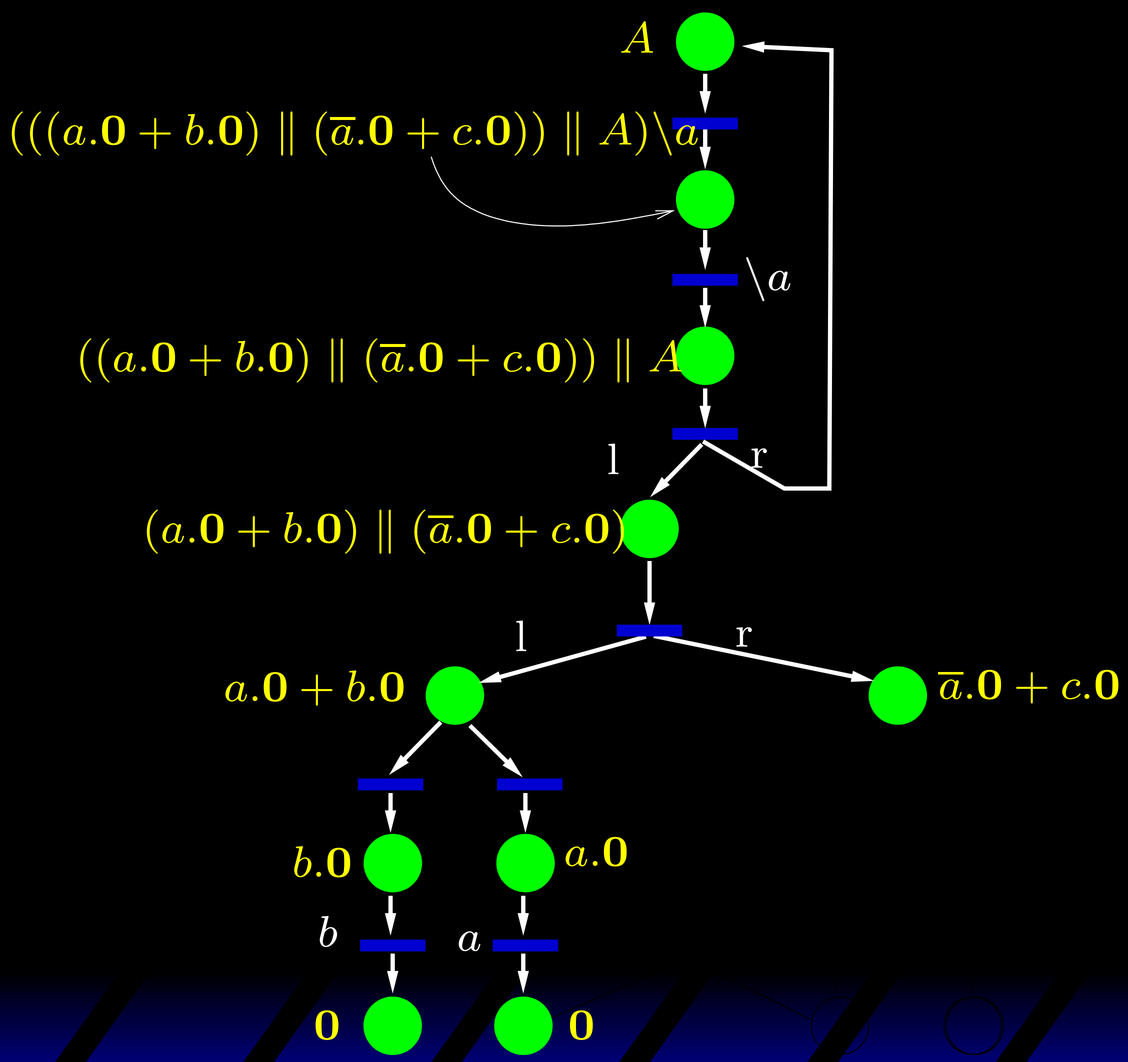
Petri Nets Semantics of the Agent Lang.: Example



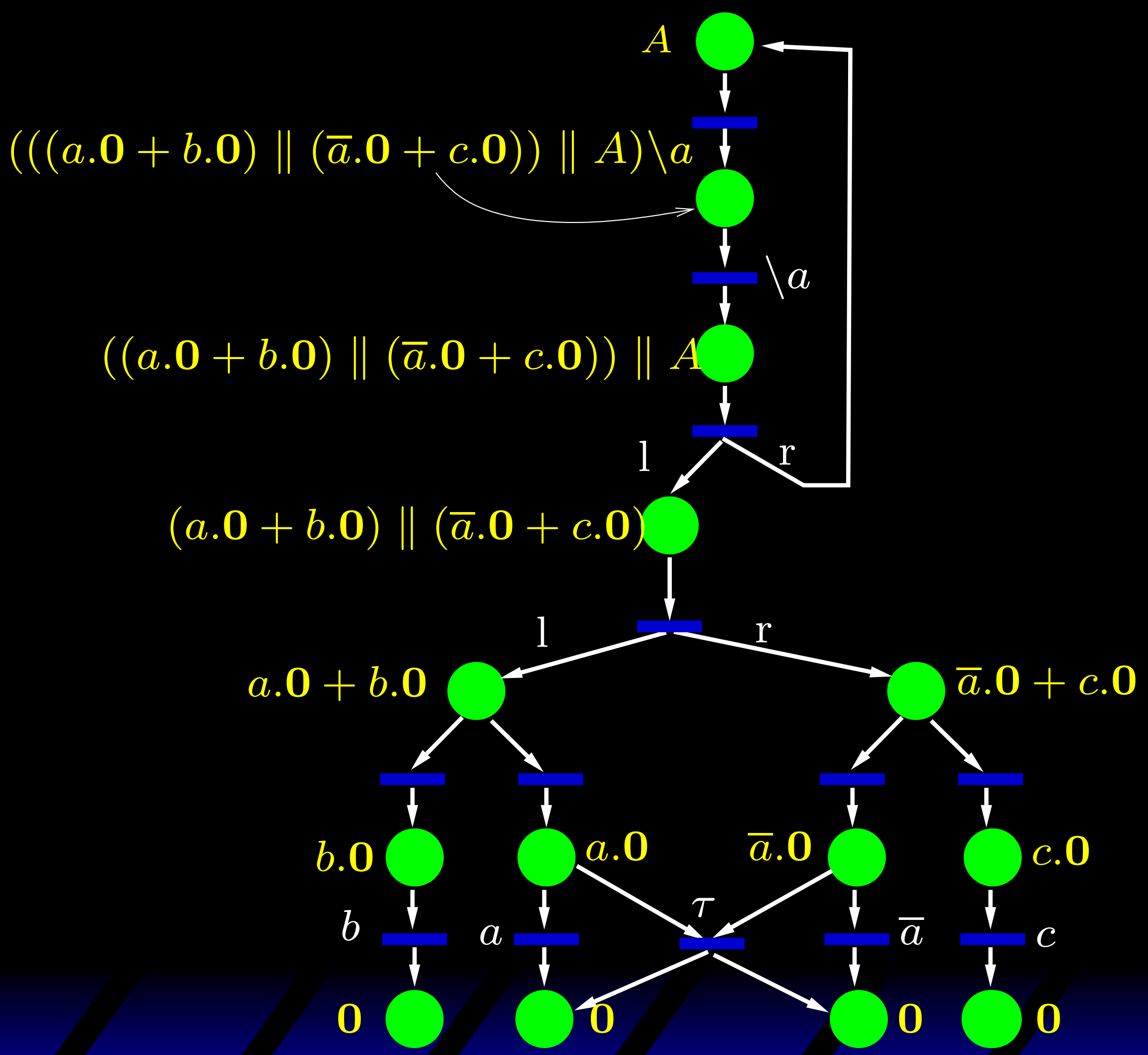
Petri Nets Semantics of the Agent Lang.: Example



Petri Nets Semantics of the Agent Lang.: Example



Petri Nets Semantics of the Agent Lang.: Example



Petri Nets Semantics of the Agent Lang.: Formal Definition

- Tokens : $(Act \cup \{\mathrm{l}, \mathrm{r}\})^*$; Empty token: ϵ. They carry history information about:
  - Concurrent threads, and
  - In which scope w.r.t. restriction they are

UPPSALA UNIVERSITET

Petri Nets Semantics of the Agent Lang.: Formal Definition

- Tokens
- Firing (Enabling of Transitions):
  - For transition t with one input place and a token θ, t is enabled if *some* of the following hold
    - t is **not** labelled with a visible action
    - t is labelled with a visible action a and θ doesn't contain a

Petri Nets Semantics of the Agent Lang.: Formal Definition

- Tokens
- Firing (Enabling of Transitions):
  - For transition t with two input places p_1 and p_2 and tokens θ_1 and θ_2, t is enabled if *both* of the following hold
    - $\texttt{pc}(pre_i(t)) \setminus Act \neq \epsilon$, $i = 1, 2$, while $\texttt{pc}(pre_1(t)) \setminus Act \neq \texttt{pc}(pre_2(t)) \setminus Act$
    - $\texttt{maxpref}_a(\texttt{pc}(pre_1(t))) = \texttt{maxpref}_a(\texttt{pc}(pre_2(t)))$

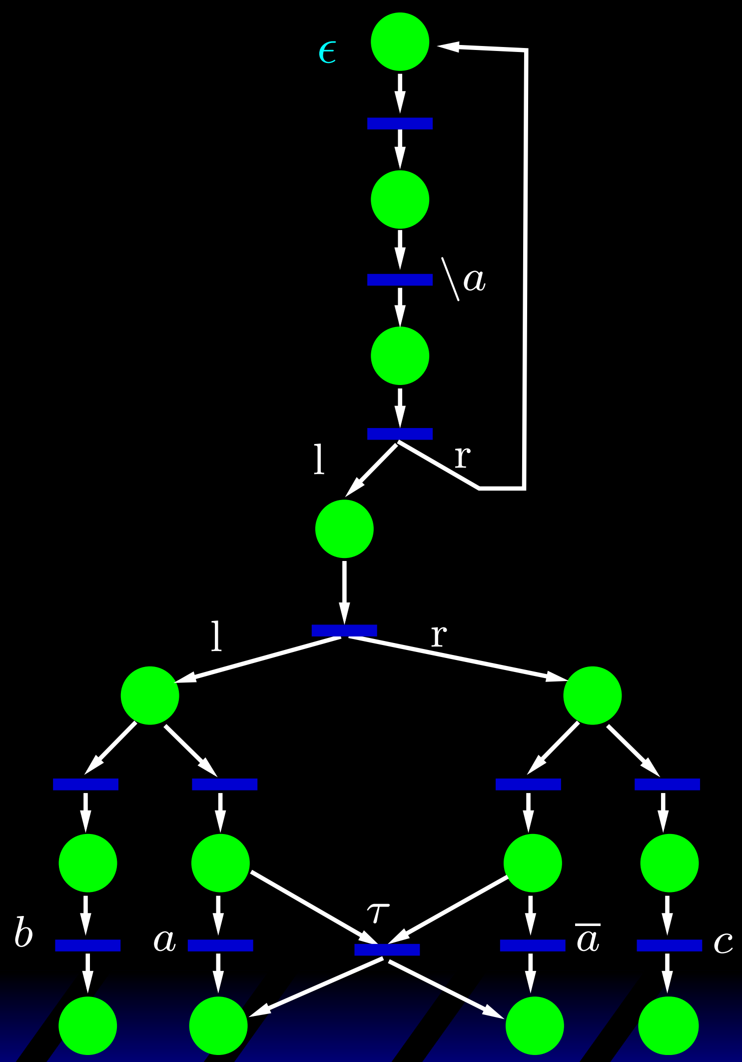
UPPSALA UNIVERSITET

Petri Nets Semantics of the Agent Lang.: Example

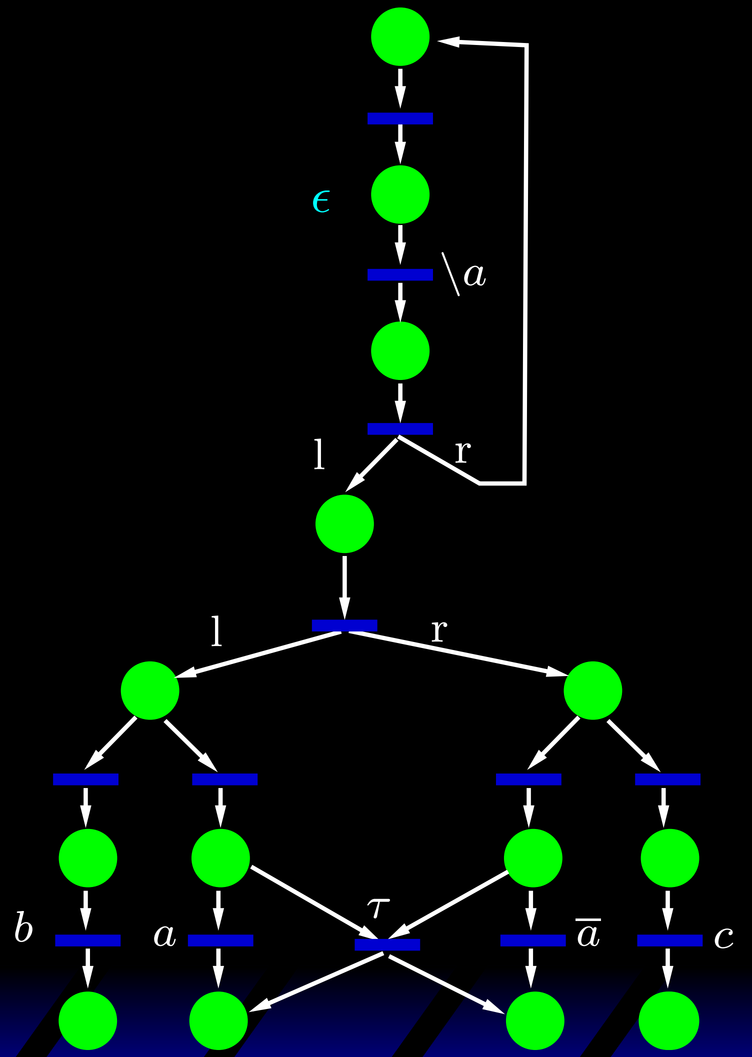
$$A \stackrel{\text{def}}{=} (((a.\mathbf{0} + b.\mathbf{0}) \parallel (\overline{a}.\mathbf{0} + c.\mathbf{0})) \parallel A)\backslash a$$

UPPSALA UNIVERSITET

Petri Nets Semantics of the Agent Lang.: Example

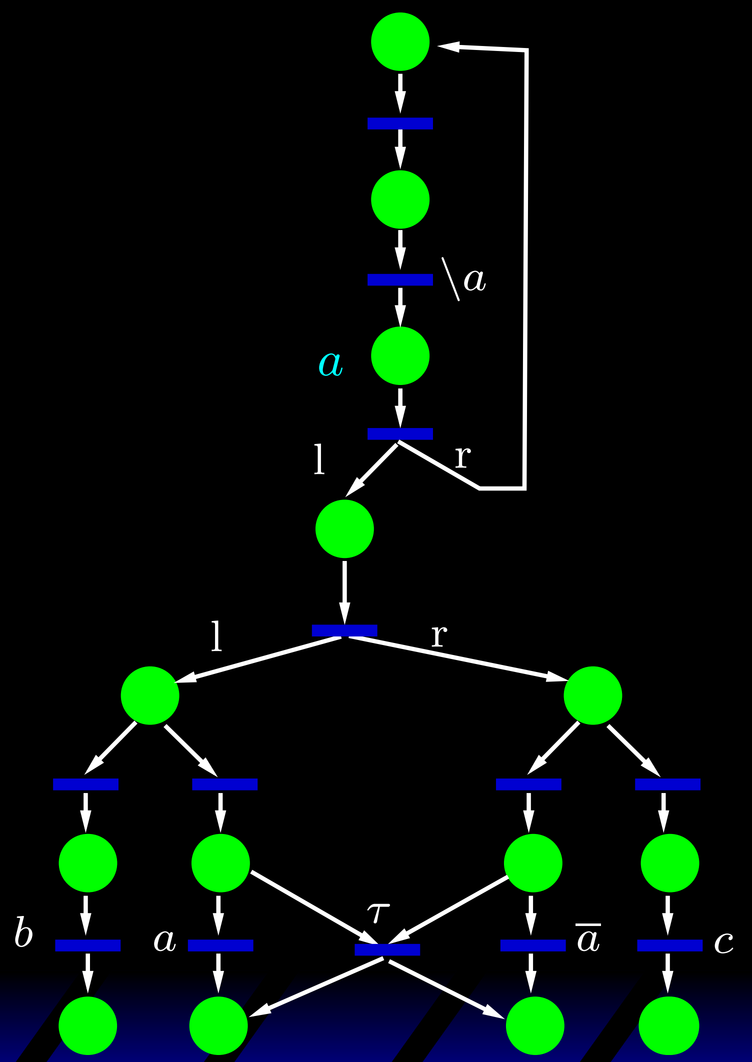


Petri Nets Semantics of the Agent Lang.: Example

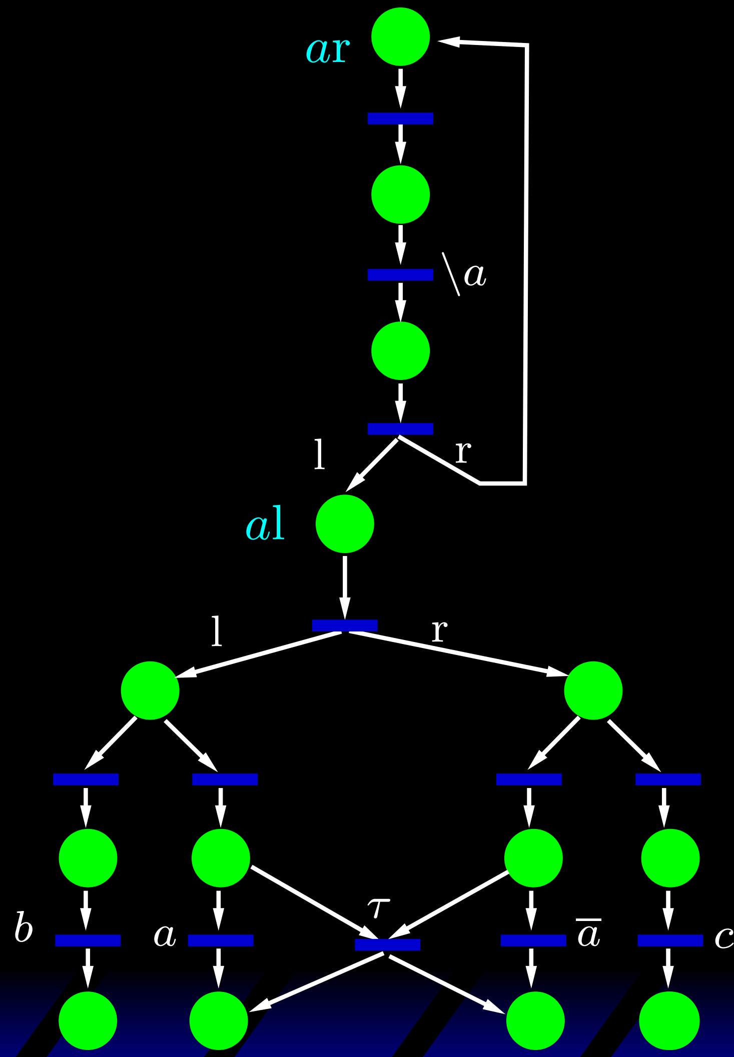


UPPSALA UNIVERSITET

Petri Nets Semantics of the Agent Lang.: Example

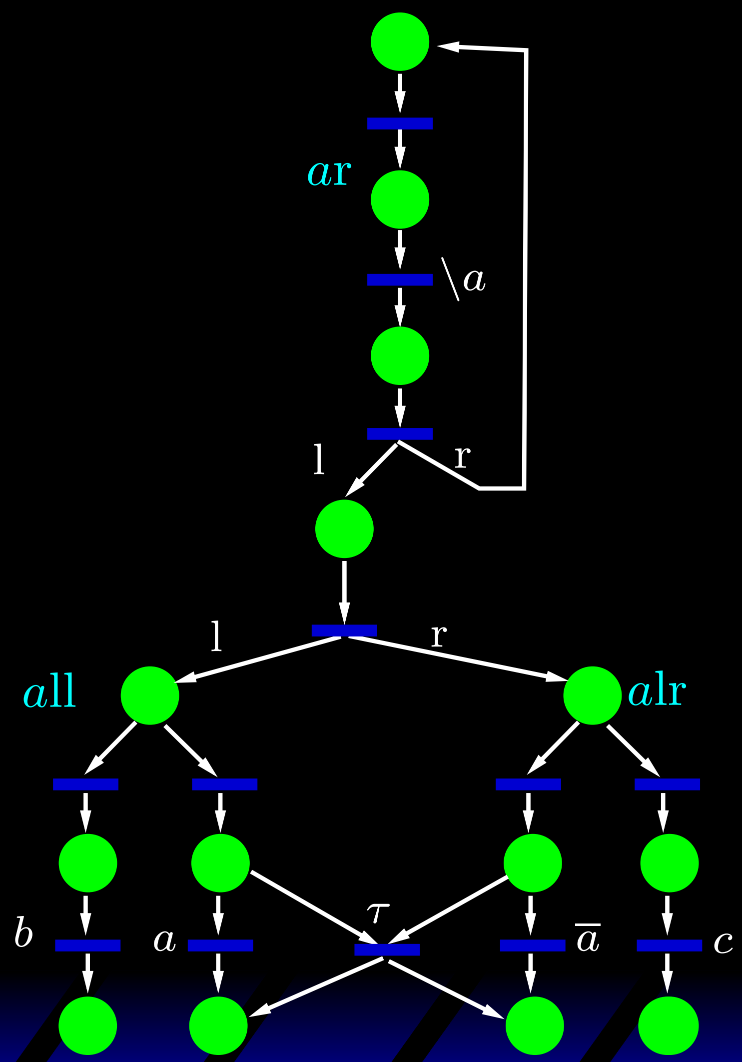


Petri Nets Semantics of the Agent Lang.: Example

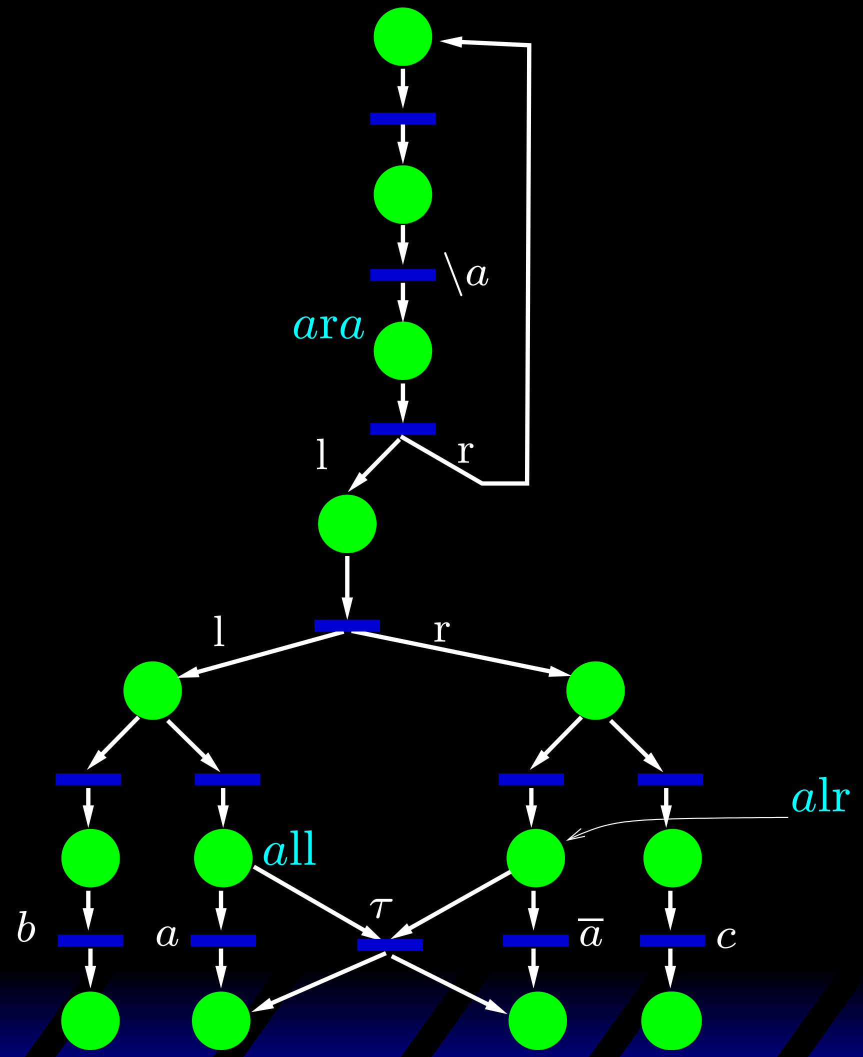


UPPSALA UNIVERSITET

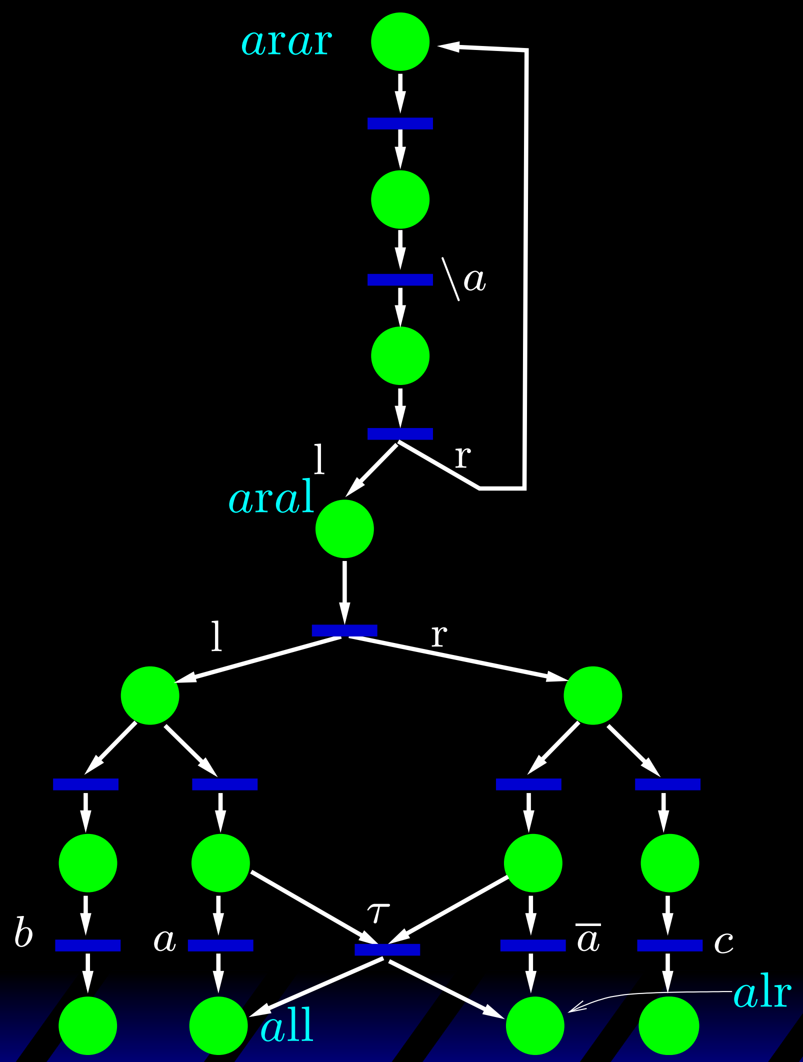
Petri Nets Semantics of the Agent Lang.: Example



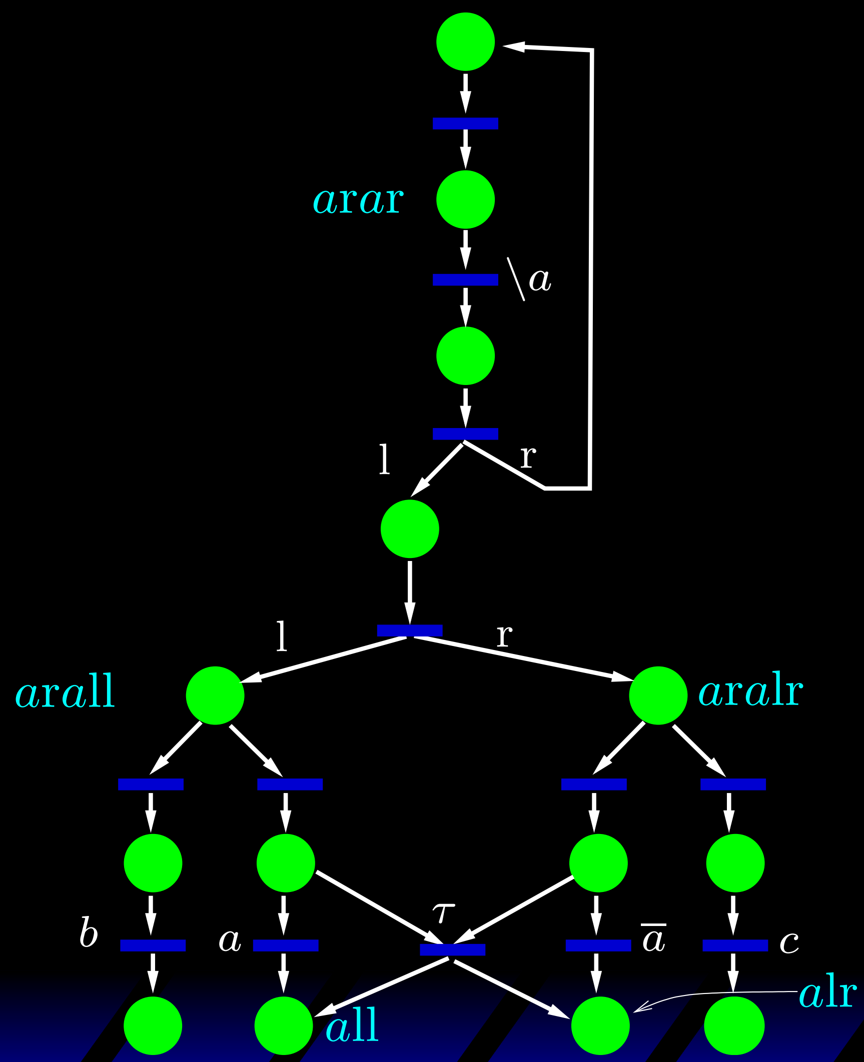
Petri Nets Semantics of the Agent Lang.: Example



Petri Nets Semantics of the Agent Lang.: Example

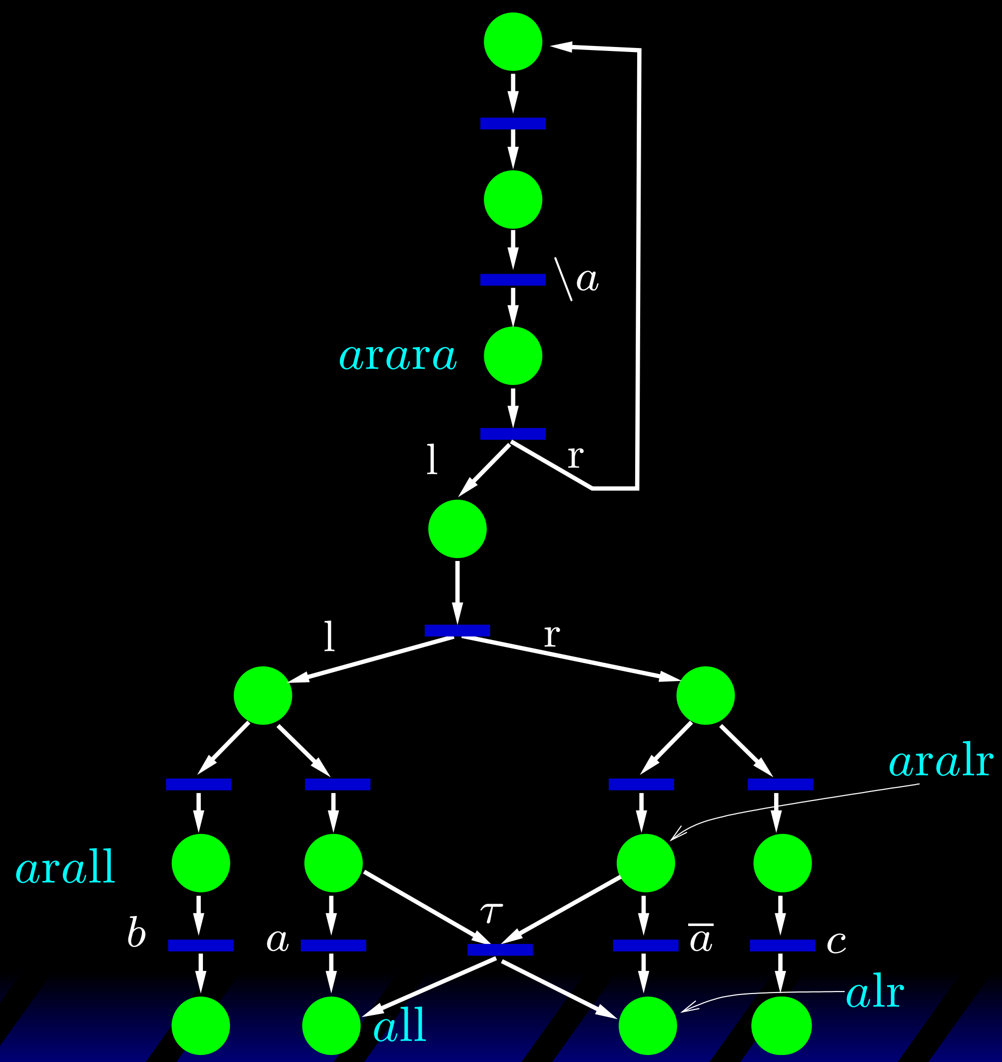


Petri Nets Semantics of the Agent Lang.: Example



UPPSALA UNIVERSITET

Petri Nets Semantics of the Agent Lang.: Example



Petri Nets Semantics of the Agent Lang.: Extra Structure

- We define a preorder between tokens:

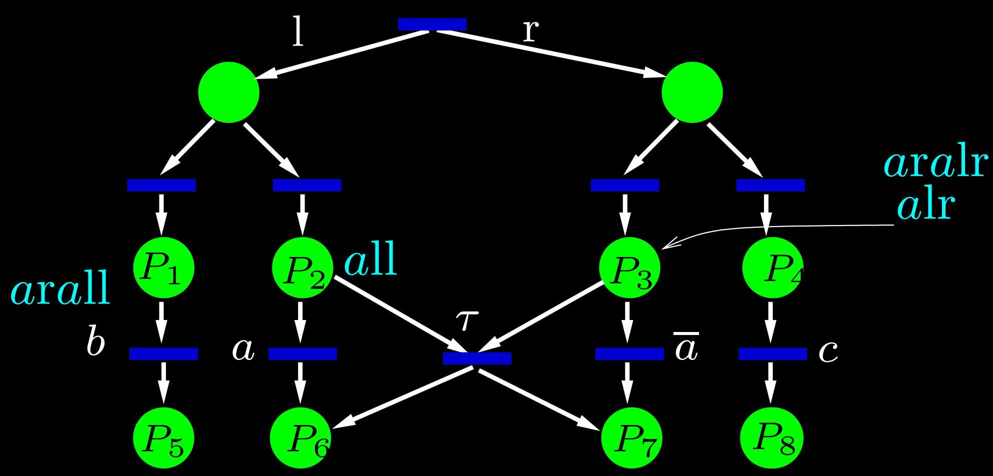
  $\eta \preceq \theta$ if η is a (not necessarily contiguous) substring of θ

  Example:

$$all \preceq ararall$$

Petri Nets Semantics of the Agent Lang.: Extra Structure

- We define an ordering between markings:
  $m_1 \sqsubseteq m_2$
  Example: m_1



Petri Nets Semantics of the Agent Lang.: Extra Structure

- We define an ordering between markings:
  $m_1 \sqsubseteq m_2$
  Example:

$$
\begin{aligned}
m_1 &= \{\ldots,(P_1,\{arall\}),(P_2,\{all\}),(P_3,\{alr,aralr\}),\\
&\qquad (P_4,\{\}),(P_5,\{\}),(P_6,\{\}),(P_7,\{\}),(P_8,\{\})\}\\
m_2 &= \{\ldots,(P_1,\{arall\}),(P_2,\{ararall\}),(P_3,\{alr,aralr\}),\\
&\qquad (P_4,\{araralr\}),(P_5,\{all\}),(P_6,\{\}),(P_7,\{\}),(P_8,\{\})\}
\end{aligned}
$$

Petri Nets Semantics of the Agent Lang.: Extra Structure

- We define an ordering between markings: $m_1 \sqsubseteq m_2$

  Intuition: $m \sqsubseteq m'$ if m' represents a (not necessarily strictly) longer firing history than m

UPPSALA UNIVERSITET

Petri Nets Semantics of the Agent Lang.: Extra Structure

- We define an ordering between markings: $m_1 \sqsubseteq m_2$
- Markings represent upward closed sets

Example:

$$\begin{aligned} m_1 \quad &= \{\ldots, (P_1, \{arall\}), (P_2, \{all\}), (P_3, \{alr, aralr\}), \\ &(P_4, \{\}), (P_5, \{\}), (P_6, \{\}), (P_7, \{\}), (P_8, \{\})\} \end{aligned}$$

UPPSALA UNIVERSITET

Our Petri Nets are not WSS

Very nice, but...

Our Petri Nets are not WSS

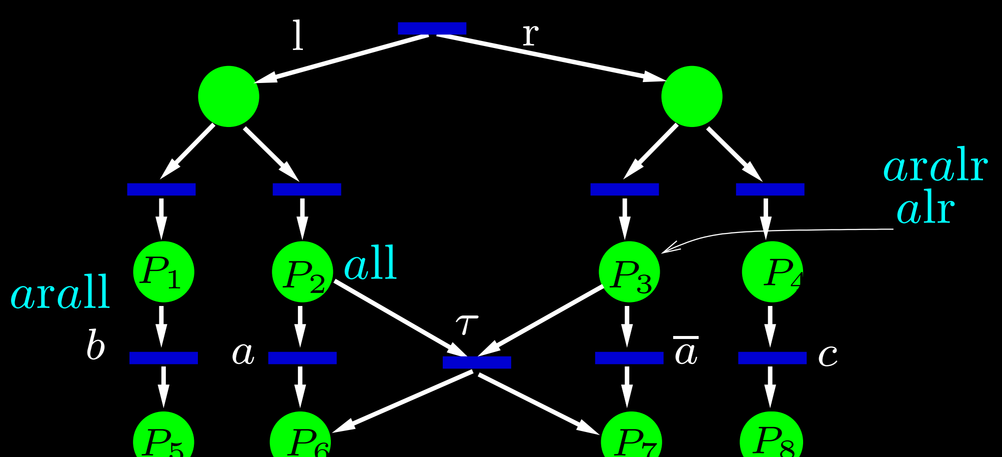
Very nice, but...

- Our Petri nets are not monotonic!

Our Petri Nets are not WSS

- Counter-example: Let

$$m_1 = \{\ldots, (P_1, \{arall\}), (P_2, \{all\}), (P_3, \{alr, aralr\}), (P_4, \{\}), (P_5, \{\}), (P_6, \{\}), (P_7, \{\}), (P_8, \{\})\}$$



Our Petri Nets are not WSS

- Counter-example: Let

$$m_1 = \{\ldots, (P_1, \{arall\}), (P_2, \{all\}), (P_3, \{alr, aralr\}), (P_4, \{\}), (P_5, \{\}), (P_6, \{\}), (P_7, \{\}), (P_8, \{\})\}$$

$$m_2 = \{\ldots, (P_1, \{arall\}), (P_2, \{ararall\}), (P_3, \{alr, aralr\}), (P_4, \{araralr\}), (P_5, \{all\}), (P_6, \{\}), (P_7, \{\}), (P_8, \{\})\}$$

- Notice that $m_1 \sqsubseteq m_2$

UPPSALA UNIVERSITET

Our Petri Nets are not WSS

- Counter-example: Let

$$m_1 = \{\ldots, (P_1, \{arall\}), (P_2, \{all\}), (P_3, \{alr, aralr\}), (P_4, \{\}), (P_5, \{\}), (P_6, \{\}), (P_7, \{\}), (P_8, \{\})\}$$

- Moreover, $m_1 \rightarrow m_1'$, where

$$m_1' = \{\ldots, (P_1, \{arall\}), (P_2, \{\}), (P_3, \{aralr\}) (P_4, \{\}), (P_5, \{\}), (P_6, \{all\}), (P_7, \{alr\}), (P_8, \{\})\}$$

Our Petri Nets are not WSS

- Counter-example: Let

$$
\begin{aligned}
m_1 \; &= \{\ldots,(P_1,\{arall\}),(P_2,\{all\}),(P_3,\{alr,aralr\}),\\
&\quad (P_4,\{\}),(P_5,\{\}),(P_6,\{\}),(P_7,\{\}),(P_8,\{\})\}
\end{aligned}
$$

But, there is **no** m'_2 such that $m'_1 \sqsubseteq m'_2$ and $m_2 \longrightarrow m'_2$

$\Longrightarrow$ It is not monotonic!

UPPSALA UNIVERSITET

Petri Nets as WSS

We make an over-approximation of the Petri net

- We change the synchronisation policy: A transition may be fired even if the tokens don't synchronise (Weak Firings)

Petri Nets as WSS

We make an over-approximation of the Petri net

- We change the synchronisation policy: A transition may be fired even if the tokens don't synchronise (Weak Firings)

Lemma: P–nets with *weak firings* are well–structured systems

Petri Nets as WSS

We make an over-approximation of the Petri net

- We change the synchronisation policy: A transition may be fired even if the tokens don't synchronise (Weak Firings)

Lemma: P–nets with *weak firings* are well–structured systems

Corollary: The control state reachability problem is decidable for p–nets with weak firings

UPPSALA UNIVERSITET

Agenda

- Preliminaries
  - Well-Structured Systems
  - An Agent Language (CCS-like)
  - Petri Nets
- Petri Nets Semantics of the Agent Lang.
- Safety Properties Verification
- Concluding Remarks

Verification of Safety Properties: The Problem

Instance: An agent A with initial state ini and an atomic action a

Question: Can agent A ever execute action a?

Verification of Safety Properties: The Algorithm

Preparatory phase:

[1] Build the p–net N associated with A

[2] For every transition t labelled with a there is a minimal marking m_t that enables t. It is given by an ϵ–token on all places in $pre(t)$. Then $M^a = \{m_t \,|\, t$ labelled by $a\}$.

Verification of Safety Properties: The Algorithm

Preparatory phase:

[1] Build the p–net N associated with A

[2] For every transition t labelled with a there is a minimal marking m_t that enables t. It is given by an ϵ–token on all places in $pre(t)$. Then $M^a = \{m_t \,|\, t$ labelled by $a\}$.

Remark: m_t is an upward closed set: "At least one token in $pre(t)$"

Verification of Safety Properties: The Algorithm

Algorithm:

function $Reachability(N, M^a, ini)$:
$(OB, s) := Search_{backward}(M^a, ini)$
if $ini \notin OB$
then $\longleftarrow$ NO
else $\longleftarrow$ $Search_{forward}(ini, M^a, OB, b(s))$

UPPSALA UNIVERSITET

Agenda

- Preliminaries
  - Well-Structured Systems
  - An Agent Language (CCS-like)
  - Petri Nets
- Petri Nets Semantics of the Agent Lang.
- Safety Properties Verification
- Concluding Remarks

Concluding Remarks

- We have given a (finite-control) Petri net semantics to a CCS-like calculus
- We have presented a general technique for reachability analysis of non-WSS
  - It combines backward and forward reachability analysis
  - It produces answers: YES, NO, UNKNOWN (YES and NO always correct)
- We have applied it to partially decide the reachability problem for a CCS-like calculus

Future Work (Research Topics)

- Use this methodology for verifying safety properties of
  - π-calculus
  - Concurrent Constraint Programming
  - Others?
- Implementation of the Algorithm

MUITO OBRIGADO!