

# Specification and Analysis of Contracts

## Lecture 5

### Deontic Logic

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# Why Deontic Logic?

- We have propose the use of 'deontic' e-contracts in the context of Service-Oriented Computing and Components
- Such contracts are based on deontic logic, which has many applications
- Deontic logic has been identified as a good specification language for information systems in general
  - Norms play a role in knowledge-based and intelligent systems
    - Databases
    - Legal expert systems
    - Electronic contracting
    - Fault tolerant systems
  - There is a need to capture the **dynamic** aspect of evolving computer systems
  - The ideas behind deontic logic can be used in the specification of long transactions

# The Role of Deontic Logic in the Specification of Information Systems

- An **information system** (IS) is a system storing data about the real world
- A conceptual model of an IS describes the properties of the data
- Any property known to be true about the IS is an **integrity constraint**
- For **normal** (*hard*) constraints we can use different logics
  - Predicate logic: “all employees are persons”
  - Temporal logic: “the age of a person can never decrease”

[MWD96] J.-J. Meyer, R.J. Wieringa and F.P.M. Dignum. **The role of deontic logic in the specification of information systems.**

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- What about **desirable** properties that can be **violated**? —**exceptional** (*soft*) constraints
- Needs **deontic logic**

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# Deontic Logic and Violations of Constraints

- Deontic logic is good to reason about **ideal** versus **actual** behavior
- It uses operators for **obligation**, **permission** and **prohibition** and mechanisms to handle violations

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## Example

- In the context of a library “when a person  $p$  borrows a book  $b$ , he should return it within 2 weeks” (syntax is not important)

$$[(borrow(p, b))]O(return(p, b))_{\leq 2 \text{ weeks}}$$

- There is no control over the borrower on whether he will comply with this norm or not
- We should add a mechanism to specify what happens in case the person does not return the book within 2 weeks

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# (Standard) Deontic Logic

In One Slide

- Concerned with **moral** and **normative** notions
  - *obligation, permission, prohibition, optionality, power, indifference, immunity, etc*
- Focus on
  - The logical consistency of the above notions
  - The faithful representation of their intuitive meaning in law, moral systems, business organizations and security systems
- Difficult to avoid *puzzles* and *paradoxes*
  - Logical paradoxes, where we can deduce contradictory actions
  - “Practical oddities”, where we can get counterintuitive conclusions
- Approaches
  - *ought-to-do*: expressions consider *names of actions*
    - “The Internet Provider *must send* a password to the Client”
  - *ought-to-be*: expressions consider *state of affairs* (results of actions)
    - “The average bandwidth *must be* more than 20kb/s”

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- Since [Aristotle](#) (384 BC–322 BC) there were some philosophers' writing on obligation, permission and prohibition
- [Leibniz](#) (1646–1716) related obligation, permission and prohibition with logical modalities of necessity, possibility and impossibility
- [Ernst Mally](#) (1926) used the term *deontik* for his “Logic of the Will”
  - Also called it: The logic of what ought to be
  - No mention of Leibniz nor of relation between modal and normative notions
- A lot of discussions in the late 1930s and early 1940s
  - [Jørgen Jørgensen](#) and [Alf Ross](#)

# The Beginnings

- It is accepted that the deontic logic was born as discipline from the following (independent) works
  - G.H. von Wright published the paper “Deontic Logic” (1951)
  - O. Becker (1952, in German)
  - J. Kalinowski (1953, in French)
- All 3 authors explored the analogy between normative and modal concepts
- von Wright (1951)
  - Started by exploring the formal analogy between the modalities “possible”, “impossible” and “necessary” with the quantifiers “some”, “no” and “all”
  - Extended his study to the analogy with the normative notions (the 1951 paper)
- A. Prior (1954) criticized von Wright’s paper
  - How to obtain derived obligations, i.e. *conditional obligations*?
  - von Wright’s answer by adding **relative permission**:
    - $P(p/q)$ : “it is permitted that  $p$  on the condition that  $q$ ”
- Much more followed...

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# Ought-to-do vs. Ought-to-be

- **Ought-to-do**: expressions consider *names of actions*
  - “One ought to close the window”
- **Ought-to-be**: expressions consider *state of affairs* (results of actions)
  - “The window ought to be closed”

# Ought-to-do vs. Ought-to-be

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Why is this so important?

- Some things are easier to represent in one approach and others in the other
  - “The average bandwidth *must be* more than 20kb/s”
  - Sergot’s example on the “strict University code”
- The logical system may have some nicer properties in one or the other approach
  - Paradoxes...

# Why Is This All So Complicated?

- **Norms** as prescriptions for conduct, are not **true** or **false**
  - If norms have no truth-value, how can we reason about them and detect contradictions and define logical consequence?
- According to von Wright: norms and valuations are still subject to logical view
- Consequence: Logic has a wider reach than truth!
- **Prescriptive** vs. **descriptive** view
- Conditional norms
- Meta-norms
- How to represent what happens when an obligation is not fulfilled or a prohibition is violated?
- Paradoxes
- A lot more...

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# Formal Aspects of Deontic Logic

- There are many formal systems for deontic logic
- We will give a flavor of **SDL** (Standard Deontic Logic)
- Usually called the **Old System** of Von Wright
  - $P$ : permission
  - $O$ : obligation
  - $F$ : prohibition

# Standard Deontic Logic

- Takes different modal logics and makes analogies between “necessity” and “possibility”, with “obligation” and “permission”
- It turns out to be difficult!
  - Many of the rules in modal logic do not extrapolate to deontic logic

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## Example

In modal logic:

- If  $\Box p$  then  $p$  (if it is necessary that  $p$ , then  $p$  is true)
- If  $p$  then  $\Diamond p$  (if  $p$  is true, then it is possible)

The deontic analogs:

- If  $O(p)$  then  $p$  (if it is obligatory that  $p$ , then  $p$  is true)
- If  $p$  then  $P(p)$  (if  $p$  is true, then it is permissible)

## Definition

**SDL** consists of the following axioms:

$$(K_O) \quad O(\varphi \Rightarrow \psi) \Rightarrow (O\varphi \Rightarrow O\psi)$$

$$(D_O) \quad \neg O \perp$$

$$(P) \quad P\varphi \Leftrightarrow \neg O\neg\varphi$$

$$(F) \quad F\varphi \Leftrightarrow O\neg\varphi$$

(*Taut*) the tautologies of propositional logic

And two rules:

$$(N_O) \quad \frac{\varphi}{O\varphi}$$

$$(MP) \quad \frac{\varphi \quad \varphi \Rightarrow \psi}{\psi}$$

- SDL has a **Kripke-like modal semantics** based on:
  - A set of possible worlds (with a truth assignment function of propositions per possible world)
  - An accessibility relation associated with the *O*-modality
- The accessibility relation points to **ideal** or **perfect deontic alternatives** of the current world
- To handle **violations** the semantics need to be extended
  - Many extensions have been proposed

# Some Problems with Deontic Logic

- Problems to handle **violations** (exceptions, *contrary-to-duties*, *contrary-to-prohibitions*)
  - A **contrary-to-duty** (CTD) expresses what happen when an obligation is not fulfilled
  - A **contrary-to-prohibition** (CTP) defines what is to be done when a prohibition is violated

## Example

**CTD:** You must send an acknowledgment within 10 minutes after receiving the message. If you don't do that, you must pay double.

**CTP:** You are forbidden to send a message before having acknowledged the reception of the previous answer. If you don't do that, you must pay double.

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- Paradoxes, ...

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# Paradoxes and Practical Oddities

- **Deontic paradoxes.** A paradox is an apparently true statement that leads to a contradiction, or a situation which is counter-intuitive

- *The Gentle Murderer Paradox*

- ① It is obligatory that John does not kill his mother;
- ② If John does kill his mother, then it is obligatory that John kills her gently;
- ③ John does kill his mother.

It could be possible to infer that John is obliged to kill his mother (contradicting 1 above)

- **Practical oddities.** A situation where you can infer two assertions which are contradictory from the intuitive practical point of view, though they might not represent a logical contradiction

- Assume you have the following norms and facts:

- ① Keep your promise;
- ② If you haven't kept your promise, apologize;
- ③ You haven't kept your promise.

It could be possible to deduce that you are both obliged to keep your promise and to apologize for not keeping it

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### Example

- 1 It is obligatory that one mails the letter
- 2 It is obligatory that one mails the letter or one destroys the letter

In SDL these are expressed as:

- 1  $O(p)$
- 2  $O(p \vee q)$

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- 1  $O(p)$
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### Problem

- In SDL one can infer that  $O(p) \Rightarrow O(p \vee q)$

### Example

- 1 You may either sleep on the sofa or sleep on the bed.
- 2 You may sleep on the sofa and you may sleep on the bed.

In SDL this is:

- 1  $P(p \vee q)$
- 2  $P(p) \wedge P(q)$

# Paradoxes

## Free Choice Permission Paradox

### Example

- 1 You may either sleep on the sofa or sleep on the bed.
- 2 You may sleep on the sofa and you may sleep on the bed.

In SDL this is:

- 1  $P(p \vee q)$
- 2  $P(p) \wedge P(q)$

### Problem

- The natural intuition tells that  $P(p \vee q) \Rightarrow P(p) \wedge P(q)$
- In SDL this would lead to  $P(p) \Rightarrow P(p \vee q)$  which is  $P(p) \Rightarrow P(p) \wedge P(q)$
- So  $P(p) \Rightarrow P(q)$
- Thus: *If one is permitted something, then one is permitted anything*

### Example

- 1 It is obligatory I now meet Jones (as promised to Jones)
- 2 It is obligatory I now do not meet Jones (as promised to Smith)

In SDL this is:

- 1  $O(p)$
- 2  $O(\neg p)$

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In SDL this is:

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### Problem

- In natural languages the two obligations are intuitive
- But the logical formulae are inconsistent when put together (in conjunction) in SDL
- In SDL,  $O(p) \Rightarrow \neg O(\neg p)$ , and we get a contradiction

### Example

- 1 It ought to be the case that Jones helps Smith who has been robbed
- 2 It ought to be the case that Smith has been robbed

And one naturally infers that:

Jones helps Smith who has been robbed if and only if Jones helps Smith and Smith has been robbed

In SDL the first two are expressed as:

- 1  $O(p \wedge q)$
- 2  $O(q)$

# Paradoxes

## The Good Samaritan Paradox

### Example

- 1 It ought to be the case that Jones helps Smith who has been robbed
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And one naturally infers that:

Jones helps Smith who has been robbed if and only if Jones helps Smith and Smith has been robbed

In SDL the first two are expressed as:

- 1  $O(p \wedge q)$
- 2  $O(q)$

### Problem

- In SDL one can derive that  $O(p \wedge q) \Rightarrow O(q)$  which is counter-intuitive in natural languages

### Example

- 1 It is obligatory that John does not kill his mother
- 2 If John does kill his mother, then it is obligatory that John kills her gently
- 3 John does kill his mother

In SDL these are expressed as:

- 1  $O(\neg p)$
- 2  $p \Rightarrow O(q)$
- 3  $p$

# Paradoxes

## The Gentle Murderer Paradox

### Example

- 1 It is obligatory that John does not kill his mother
- 2 If John does kill his mother, then it is obligatory that John kills her gently
- 3 John does kill his mother

In SDL these are expressed as:

- 1  $O(\neg p)$
- 2  $p \Rightarrow O(q)$
- 3  $p$

### Problem

- When adding a natural inference like  $q \Rightarrow p$ , one can infer that  $O(p)$  (contradicting 1 above)

### Example

- 1 John ought to go to the party
- 2 If John goes to the party then he ought to tell them he is coming
- 3 If John doesn't go to the party then he ought not to tell he is coming
- 4 John does not go to the party

In SDL these are expressed as:

- 1  $O(p)$
- 2  $O(p \Rightarrow q)$
- 3  $\neg p \Rightarrow O(\neg q)$
- 4  $\neg p$

### Example

- 1 John ought to go to the party
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In SDL these are expressed as:

- 1  $O(p)$
- 2  $O(p \Rightarrow q)$
- 3  $\neg p \Rightarrow O(\neg q)$
- 4  $\neg p$

### Problem

- In SDL one can infer  $O(q) \wedge O(\neg q)$  (due to statement 2)

# Paradoxes: Diagnosis of the Problems

- Part of the problems arise from the following 4 confusions [MWD96]

## Why paradoxes in deontic logic?

- 1 Confusion between **ought-to-do** and **ought-to-be**
  - Take a *pragmatic* point of view: difficult to get a paradox-free logic of norms, ethics, and morality
- 2 Confusion between the formal interpretation and the natural language
  - Example, the logical **or** is usually understood as a **choice**
- 3 Confusion between **ideality** and **actuality**
  - Needs a good treatment of exceptions, CTD's, CTPs, etc
- 4 Confusion between normative notions for abstract contexts (e.g. ethics) and those needed in concrete practical applications
  - In practical applications: not interested on the philosophical problems
  - A concrete application helps getting rid of most paradoxes

[MWD96] J.-J. Meyer, R.J. Wieringa and F.P.M. Dignum. **The role of deontic logic in the specification of information systems.**

## Reminder

- We want to use **deontic e-contracts** to specify and reason about contracts in software systems (e.g., components, services)
- We need a formal system to relate the normative notions of obligation, permission and prohibition
- We want to represent (nested) “exceptions”: Can we represent and reason about what happens when an obligation is not fulfilled or a prohibition is violated?
- We want to avoid the philosophical problems of deontic logic (restrict its use to our application domain)

- G.H. von Wright. **Deontic Logic: A personal view.**
- P. McNamara. **Deontic Logic.** See the entry at the Stanford Encyclopedia of Philosophy (<http://plato.stanford.edu/entries/logic-deontic>)
- J.-J. Ch. Meyer, F.P.M. Dignum and R.J. Wieringa. **The Paradoxes of Deontic Logic Revisited: A Computer Science Perspective.**
- J.-J. Meyer, R.J. Wieringa and F.P.M. Dignum. **The role of deontic logic in the specification of information systems.**