A Combinatorial Multi-Armed Bandit Approach for Stochastic Facility Allocation Problem

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Outline

- Introduction
- Problem Formulation
- Solution of the Problem
- Simulation
- Future Work

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Introduction

- **Background of Facility Allocation**:
  - Strategic placement of resources in various fields: urban planning, telecommunications, computing infrastructure.
  - Focus on optimizing spatial resources in dynamic, uncertain conditions.
- **Problem Complexity**:
  - Decision-making is iterative, aiming to maximize total reward over multiple rounds.
  - Challenges in environments with variable demands, like emergency services and telecommunications.
  - Combinatorial nature: multiple facilities are decided upon simultaneously.

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Problem Formulation

- **Model Setup**:
  - **Grid Layout**: 1×1 square divided into N cells (perfect square).
  - **Population Density**: Each cell i has an unknown fixed density $D(i)$.
- **Facility Allocation**:
  - **Round-by-Round Decision**: Allocate K facilities at cell centers per round, represented as $\boldsymbol{F}(t) = \{f_1(t), \dots, f_K(t)\}$.
  - **Unique Positioning**: No two facilities share the same location in the same round.
- **Voronoi Partitioning**:
  - Determines which facility point each cell is closest to, using either Manhattan or Euclidean distance.
  - Cells are assigned to the nearest facility, breaking ties randomly.

Problem Formulation

- **Attraction Probability**:
  - Probability $p_{i,j}(t)$ of attracting an individual from cell i to facility j inversely proportional to their distance.
  - Modeled as: $\frac{\alpha}{d(f_j(t),i)+1}$, where α is a tunable factor and d is the chosen distance metric.
- **Expected Population Attraction**:
  - Each round models population attraction as a binomial random variable:
    $$X_i(t) \sim \ Binomial(D(i), p_{i,j}(t)).$$
  - Expected attracted population from cell i to facility j: $E[X_i(t)] = \ \sum_{j \leq K} D(i)p_{i,j}(t)$.
- **Regret Minimization Objective**:
  - **Regret Definition**: Difference between optimal and actual attracted population over rounds.
  - **Optimization Goal**: Minimize cumulative regret by selecting $\boldsymbol{F}(t)$ to maximize total expected population attraction.

Problem Formulation

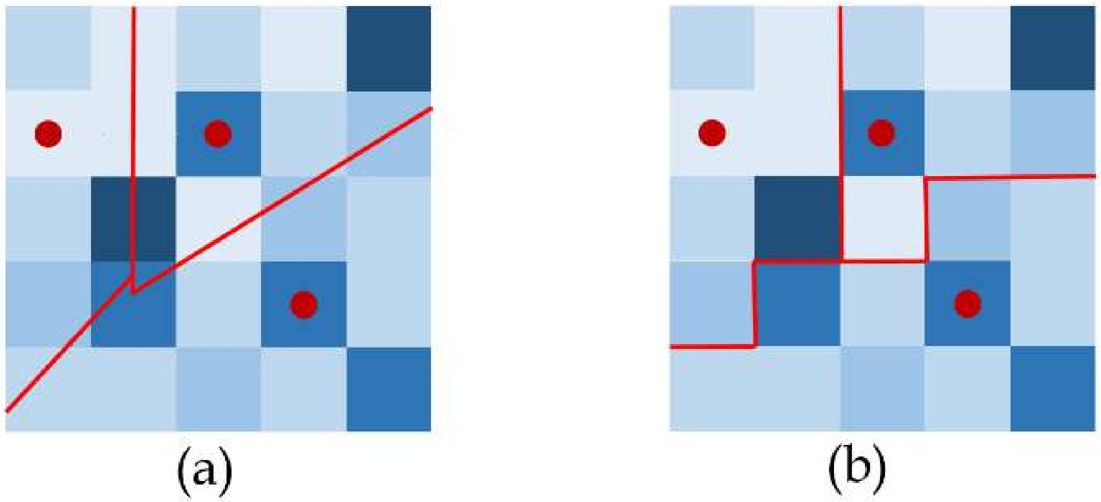

Figure 1: An illustration showing the effect of the choice of distance metric on the Voronoi partition $V_j(t)\ \forall j \le K$. The background color represents the value of the underlying population density of the cells $D(i)$: (a) Euclidean distance metric; (b) Manhattan distance metric.

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Solution of the Problem

Algorithm 1 Geometric-UCB for facility allocation

Input: $D(i)\ \forall i \leq N, K$, distance metric.
Output: $\mathbf{F}(t) \quad \forall t = 1, 2, \ldots, T$.
Initialization: $X_i(0) \leftarrow 0 \forall i \leq N, \hat{\mu}(\mathbf{F}, t) \leftarrow 0$ and $N_{\mathbf{F}}(t) \leftarrow 0 \forall \mathbf{F}$

```
for t = 1, 2, ..., T do
    for all possible allocations F do
        Evaluate UCB(F, t) from Equation 4.
    Choose F(t) based on Equation 5.
    Perform the Voronoi partition based on F(t) and the
        chosen distance metric to get V_j(t) for all j ≤ K.
    Observe X_i(t) ∀i ≤ N and update μ̂(F, t) and N_F(t) ∀F
        based on Equations 6-7.
return F(1), F(2), ..., F(T).
```

$$UCB(\mathbf{F}, t) = \hat{\mu}(\mathbf{F}, t) + \sqrt{2 \log t / N_{\mathbf{F}}(t)}, \tag{4}$$

$$\mathbf{F}(t) = \arg\max_{\mathbf{F}} \left(\hat{\mu}(\mathbf{F}, t) + \sqrt{2 \log t / N_{\mathbf{F}}(t)} \right). \tag{5}$$

$$\hat{\mu}(\mathbf{F}, t+1) = \frac{N_{\mathbf{F}}(t)\hat{\mu}(\mathbf{F}, t) + \sum_{j=1}^{K} \sum_{i \in V_j(t)} X_i(t)}{N_{\mathbf{F}}(t) + 1}. \tag{6}$$

$$N_{\mathbf{F}}(t+1) = \begin{cases} N_{\mathbf{F}}(t) + 1 & \text{if } \mathbf{F}(t) = \mathbf{F} \\ N_{\mathbf{F}}(t) & \text{otherwise} \end{cases}. \tag{7}$$

THEOREM 5.1. *Algorithm 1 guarantees a regret bound of:*

$$R(T) \leq 2\sqrt{2N \log T} \left(1 + 1/\sqrt{N}\right).$$

Solution of the Problem

- **Algorithm Choice**:
  - Utilizes a Combinatorial Upper Confidence Bound (C-UCB) algorithm.
  - Balances exploration (gaining new information) and exploitation (using known high-reward locations).
- **Algorithm Overview**:
  - **Expected Attraction**: Computes expected total population attraction for different facility sets, $\boldsymbol{F}(t)$.
  - **UCB Formula**: Incorporates both past data and an exploration bonus to guide allocation decisions.
- **Algorithm Execution**:
  - **Initialization**: Sets initial conditions for all variables and parameters.
  - **Iteration Process**: Evaluates and chooses facility sets based on their upper confidence bounds across all rounds.
  - **Voronoi Partitioning**: Performed each round to determine the influence area of each facility based on chosen distance metric.
  - **Observation and Update**: Records results from the current allocation to refine future decisions.

Solution of the Problem

- **Key Features of Geometric-UCB**:
  - Uses real-time data to dynamically adjust decisions.
  - Aims to maximize total attraction over time, minimizing regret.
  - Suitable for scenarios where the number of facilities (K) is small, making complex computations tractable.
- **Computational Complexity**:
  - **Time Complexity**: Dominated by evaluating all potential allocations ($O(T \times N^K)$) and computing Voronoi partitions each round ($O(T \times K^2)$).

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Simulation

- **Experimental Settings Overview**:
  - **Facility Numbers**: $K = 3$ or 4 to manage computational feasibility.
  - **Probability Parameter**: α varied from 0.1 to 1.0 to test different attraction levels.
  - **Distance Metrics**: Both Manhattan and Euclidean used to examine adaptability.
- **Data Used for Simulation**:
  - **Real-World Traces**: Population density data from the United States, discretized into 36 or 49 cell grids.
  - **Synthesized Data**: Generated datasets with population densities drawn from a normal distribution to test across varied scenarios.

Simulation

- **Algorithm Comparison**:
  - **Epsilon-Greedy Algorithm**: Examines balance between exploration and exploitation, with $\epsilon = 0.25$.
  - **Thompson Sampling**: Assesses performance against a probabilistic method that uses Bayesian inference for decision-making.
  - **Random Selection**: Provides a baseline by randomly choosing facility locations, ignoring prior data.
- **Goals of Comparative Evaluation**:
  - Test the Geometric-UCB's efficiency against established algorithms.
  - Identify strengths and potential areas for improvement in different settings.
  - Validate robustness and adaptability of Geometric-UCB under varied experimental conditions.

Simulation

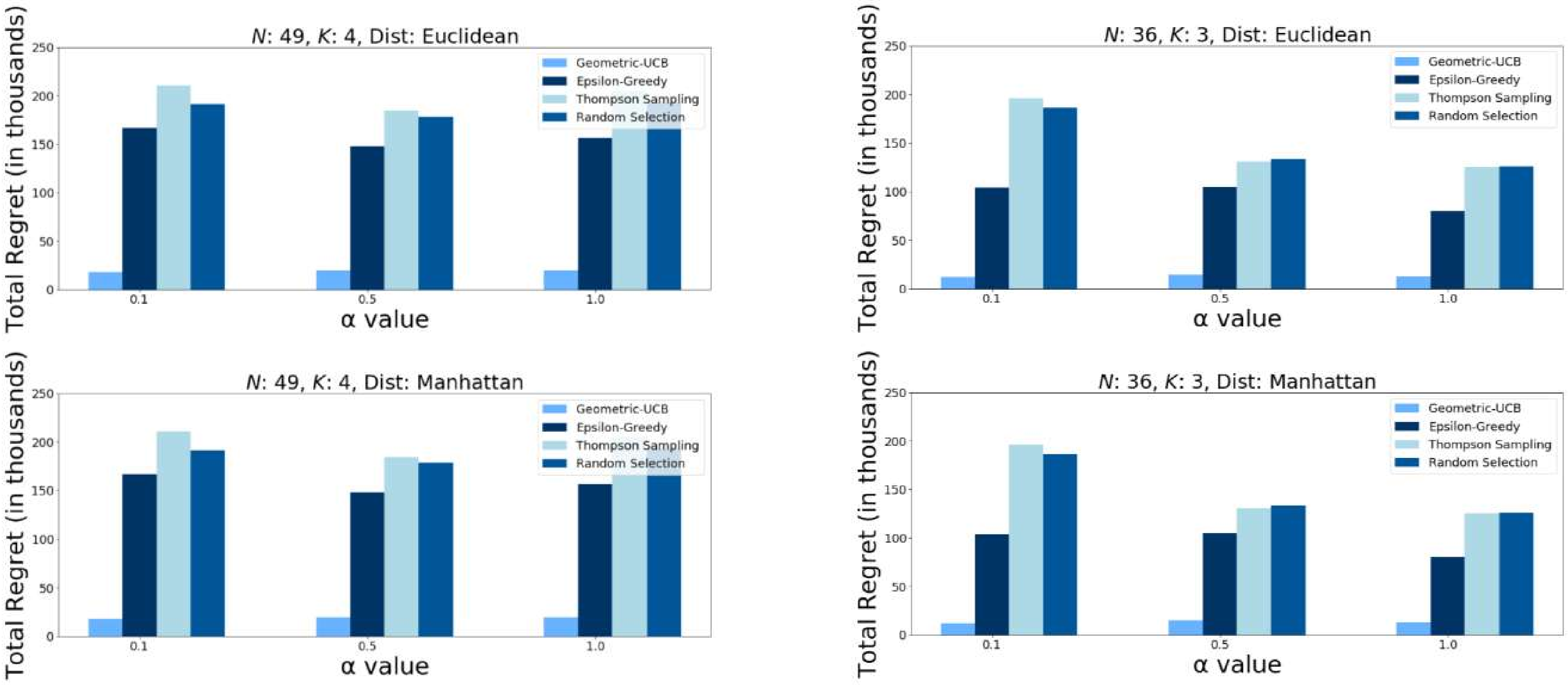

Figure 3: Total regret value for different algorithms under synthesized data with different α values. $\mu_D = 5000, \sigma_D = 100$.

Simulation

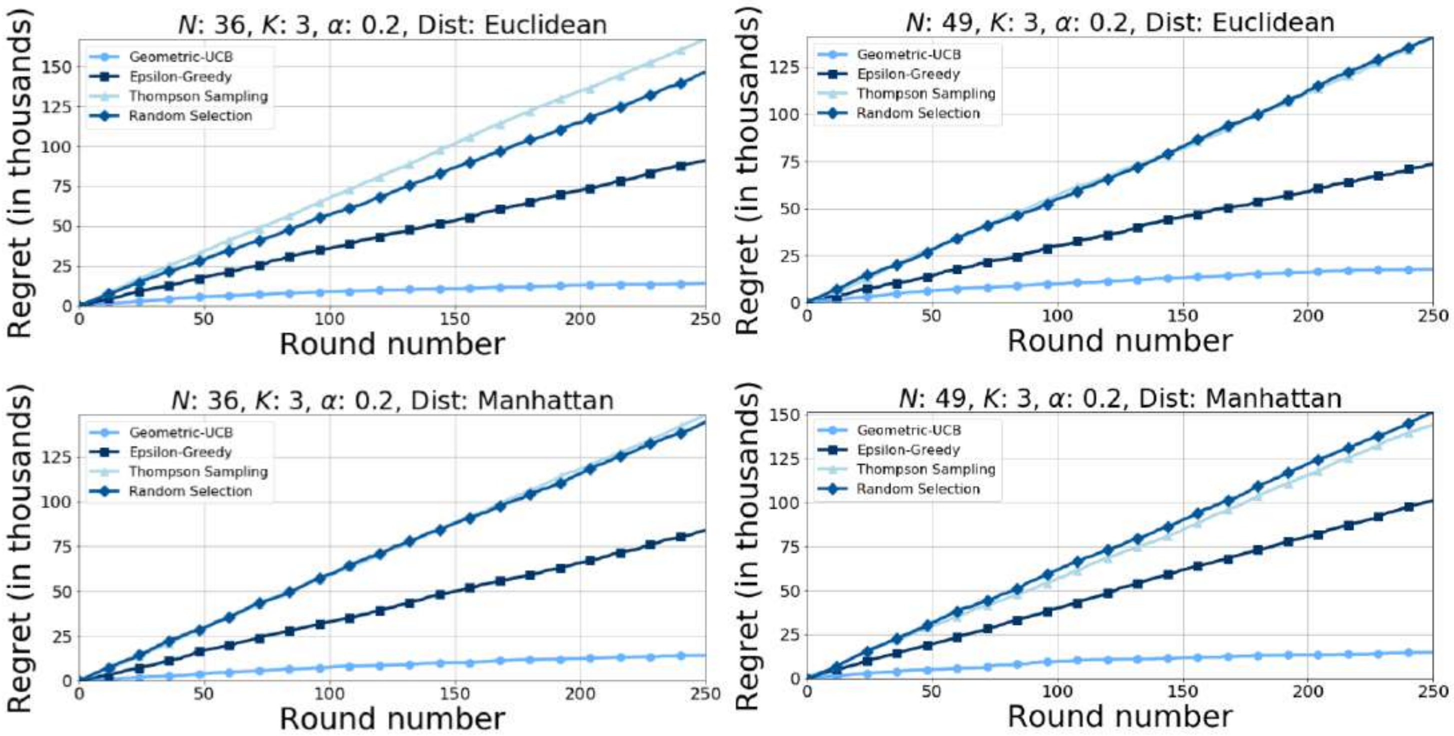

Figure 4: Regret value for different algorithms under synthesized data with different N values. $\mu_D = 5000, \sigma_D = 100$.

Simulation

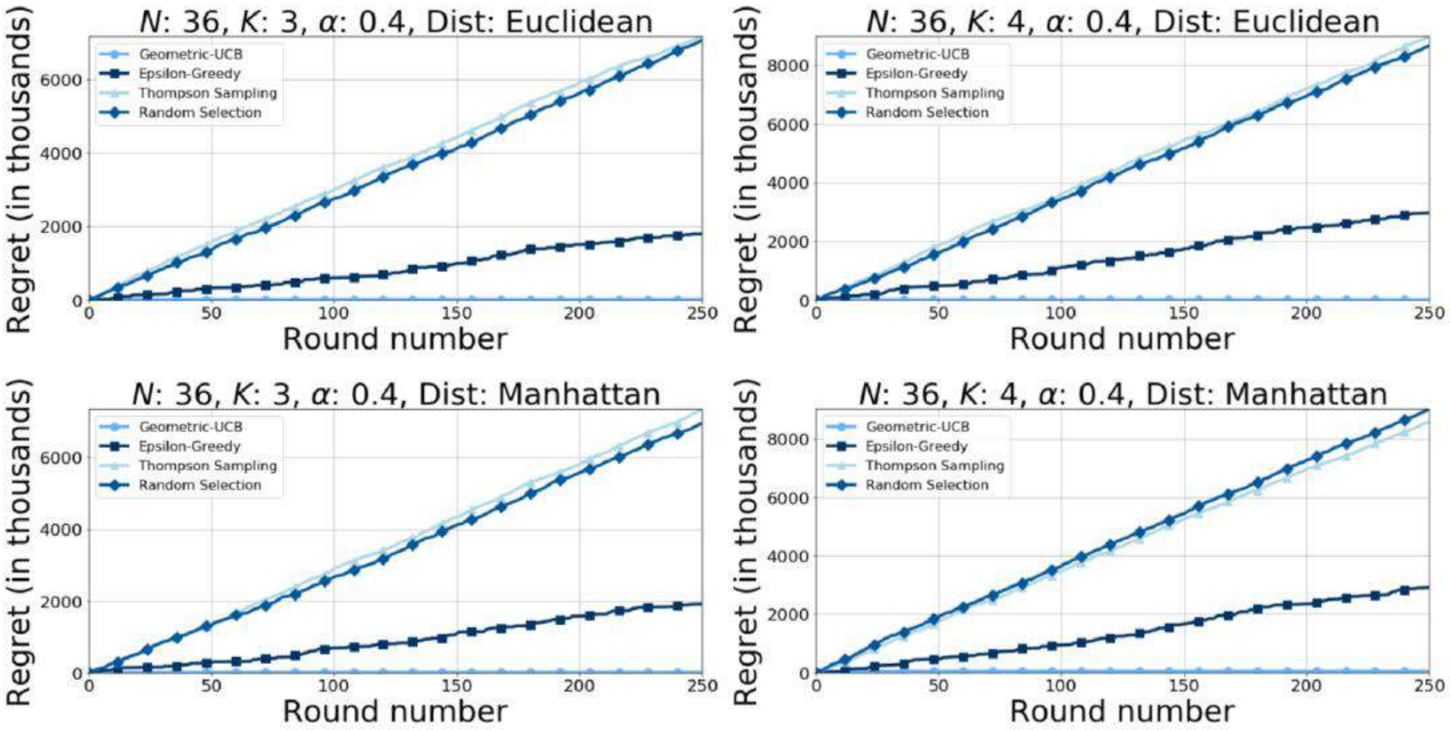

Figure 5: Regret value for different algorithms under real-world traces with different K values.

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Future Work

- **Expanding Dimensions**:
  - Explore the applicability of the Geometric-UCB algorithm in higher-dimensional spaces.
  - Test the scalability and computational feasibility as dimensions increase.
- **New Performance Measures**:
  - Investigate other metrics beyond regret to assess the algorithm's effectiveness.
  - Consider factors like computational efficiency, convergence speed, and robustness under varying conditions.
- **Refinement of Probability Parameter (α)**:
  - Develop adaptive strategies for tuning α dynamically based on observed attraction levels.
  - Enhance the algorithm's responsiveness to changes in population density and attraction patterns.

Conclusion

- **Key Contributions**:
  - Introduced a novel Geometric-UCB algorithm tailored for the stochastic facility allocation problem.
  - First application of CMAB techniques in 2-dimensional spaces with uncertain population distributions.
- **Algorithm Advantages**:
  - Efficiently balances exploration and exploitation to maximize total population attraction.
  - Demonstrated adaptability with both Manhattan and Euclidean distances in facility allocation.
- **Validation through Simulations**:
  - Tested on both real-world data and synthesized datasets to verify effectiveness and efficiency.
  - Outperformed traditional algorithms like Epsilon-Greedy and Thompson Sampling in various setups.

Thank you!

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