A remark on impredicativoty

The first version of the calculus of constructions (March 1984) was based on combinators. It was a rather direct extension of de Bruijn language PAL. The syntax was the following

$$t, A \ ::= \ k \mid [x:A]t \mid \ * \qquad\qquad k \ ::= \ c \mid x \mid k\ t$$

A context Γ, Δ is a list of type declarations $x_1 : A_1, \ldots, x_n : A_n$ and the rules are

$$\frac{x : A \ in\ \Gamma}{\Gamma \vdash x : A} \qquad \frac{c : A}{\Gamma \vdash c : A} \qquad \frac{\Gamma \vdash k : [x:A]B \qquad \Gamma \vdash t : A}{\Gamma \vdash k\ t : B(t)} \qquad \frac{\Gamma, x : A \vdash B : *}{\Gamma \vdash [x:A]B : *}$$

The rule of context extensions are the following

$$\frac{\Gamma \vdash A : *}{\Gamma, x : A \vdash *} \qquad \frac{}{\vdash *} \qquad \frac{\Gamma \vdash C}{\Gamma, x : C \vdash *} \qquad \frac{\Gamma, x : A \vdash C}{\Gamma \vdash [x:A]C}$$

Finally, if we have $x_1 : A_1, \ldots, x_n : A_n \vdash t : B$ we can introduce a new constant

$$c : [x_1 : A_1] \ldots [x_n : A_n]B$$

with the computation rule

$$c\ x_1\ \ldots\ x_n = t$$

Examples

We define $N : *$

$$N = [A : *][a : A][f : [x : A]A]A$$

We have $A : *, a : A, f : [x:A]A \vdash a : A$.
Hence we can introduce $0 : N$ by the equation $0\ A\ a\ f = a$.

We have $n : N, A : *, a : A, f : [x:A]A \vdash f\ (n\ A\ f\ a) : A$.
Hence we can introduce $S : [n : N]N$ by the equation $S\ n\ A\ f\ a = f\ (n\ A\ f\ a)$.