ZMT 1

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1 On the pumping lemma for CFL

The best formulation of the pumping lemma, which indicates also how the proof works, seems to be the following.

Lemma: Given a context-free grammar $G = (V, T, P, S)$ there exists N such that if $|t| \geqslant N$ and t in $L(G)$ we can write $t = uvwxy$ such that there is a variable symbol A satisfying
$S \Rightarrow^* uAy$
$A \Rightarrow^* vAx$
$A \Rightarrow^* w$
with $vx \neq \epsilon$ and $|vwx| \leqslant N$

It follows from this that we have uv^kwx^ky in $L(G)$ for all k.

What I forgot to notice in the lecture last time is that this holds also for $k = 0$.

Having this in mind it is not difficult to show from this that the language

$$L = \{a^nb^nc^m \mid n \leqslant m\}$$

is *not* context-free.

Indeed, if it were, we could find N as in the pumping lemma. Consider then $t = a^Nb^Nc^N$ which is in $L(G)$. If we write $t = uvwxy$ we see that we have a contradiction if vwx is inside a^Nb^N. We conclude that we should have vwx inside c^N. But then we have a contradiction *for* $k = 0$ because uwy cannot be in L since there are less c then a in this word.