

# **An Algebraic Approach to the Analysis of Constrained Workflow Systems**

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## **What is a workflow system?**

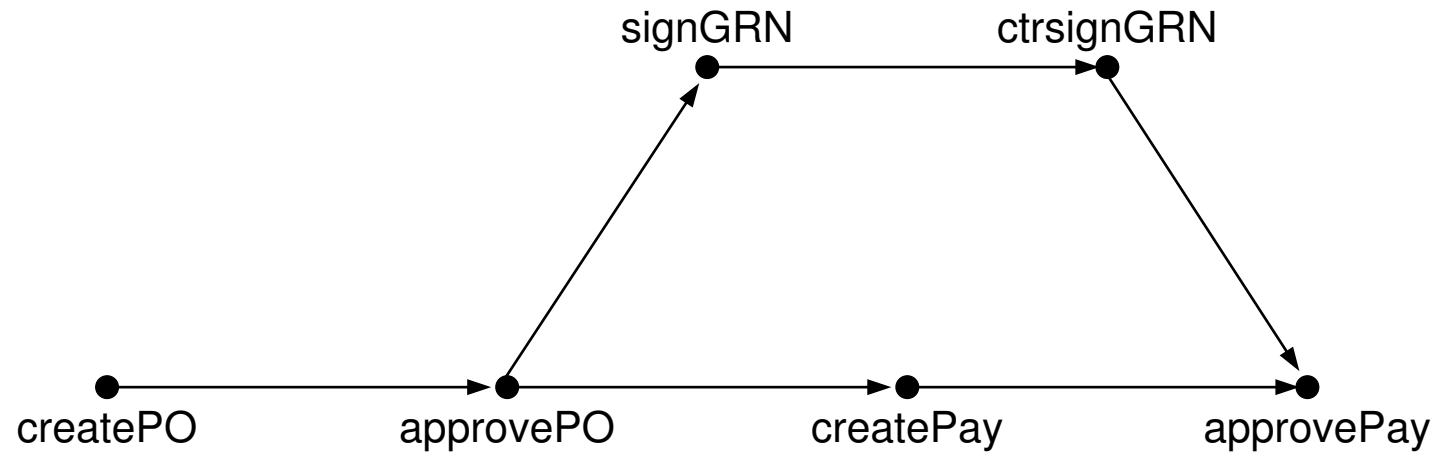
A representation of an organizational or business process

Typically specified as a set of tasks and a set of dependencies between the tasks

Users of the system are authorized to perform certain tasks

Role-based access control is a popular choice for enforcing authorization requirements in workflow systems

## Example – A purchase order workflow system



## Constrained workflow systems

It may be necessary to impose constraints on who can perform a task given that a prior task has been performed by a particular individual

These constraints may exist to prevent fraud or to implement a particular feature of a business process

- We don't want the same person who creates the purchase order to approve the purchase order
- We want the GRN to be signed by the person that ordered the goods

These constraints supplement the security policy defined by authorization information

## The problem defined

Design an abstract machine (reference monitor) that decides whether a user request to execute a task in a workflow instance should be granted

A necessary condition for the request to be granted is that the user has appropriate authorization

Granting the request **must not**

- cause the violation of any constraint
- result in a workflow instance that is unsatisfiable

# Motivation

Existing research into constraints in workflow systems

- relies on ad hoc methods
- assumes an underlying computational model
- assumes an underlying access control model

I think the subject can (and should) be approached using

- as few assumptions as possible
- as simple and expressive a framework as possible
- mathematical structures and techniques

## Outline of talk

- Basic concepts and definitions
- Combining constraints
- Satisfiability
- Building a reference monitor for workflow systems
- Conclusion

## A model for constrained workflow systems (1)

A *workflow specification* is a partially ordered set of tasks  $T$

- If  $t < t'$  then  $t$  must be performed before  $t'$  in any instance of the workflow

A *workflow authorization schema* is a pair  $(T, A)$ , where

$$A \subseteq T \times U$$

- If  $(t, u) \in A$  then  $u$  is authorized to perform  $t$  in any instance of the workflow
- Typically  $A$  will not be given explicitly and will be derived from other information (role assignments for example)



## A model for constrained workflow systems (2)

An *entailment constraint* has the form  $(D, (t, t'), \rho)$ , where  $\rho \subseteq U \times U$  and  $t \not\preceq t'$

- $D \subseteq U$  is called the *domain* of the constraint
- $\rho$  is used to determine which users can perform  $t'$  once  $t$  has been performed by a user in  $D$ 
  - If  $u \in D$  performs  $t$  and  $u' \in U$  performs  $t'$ , then  $(D, (t, t'), \rho)$  is *satisfied* iff  $(u, u') \in \rho$

A *constrained workflow authorization schema* is a triple  $(T, A, C)$ , where  $C$  is a set of entailment constraints

## Examples

Define

$$O' = \{(u, v) : u, v \in U, u \neq v\}$$

$(D, (t, t'), O')$  is a separation of duty constraint

- if  $u \in D$  performs  $t$ , then  $u$  cannot perform  $t'$
- $(U, (\text{createP0}, \text{apprP0}), O')$

## Examples

Define

$$0' = \{(u, v) : u, v \in U, u \neq v\} \quad 1' = \{(u, u) : u \in U\}$$

$(D, (t, t'), 0')$  is a separation of duty constraint

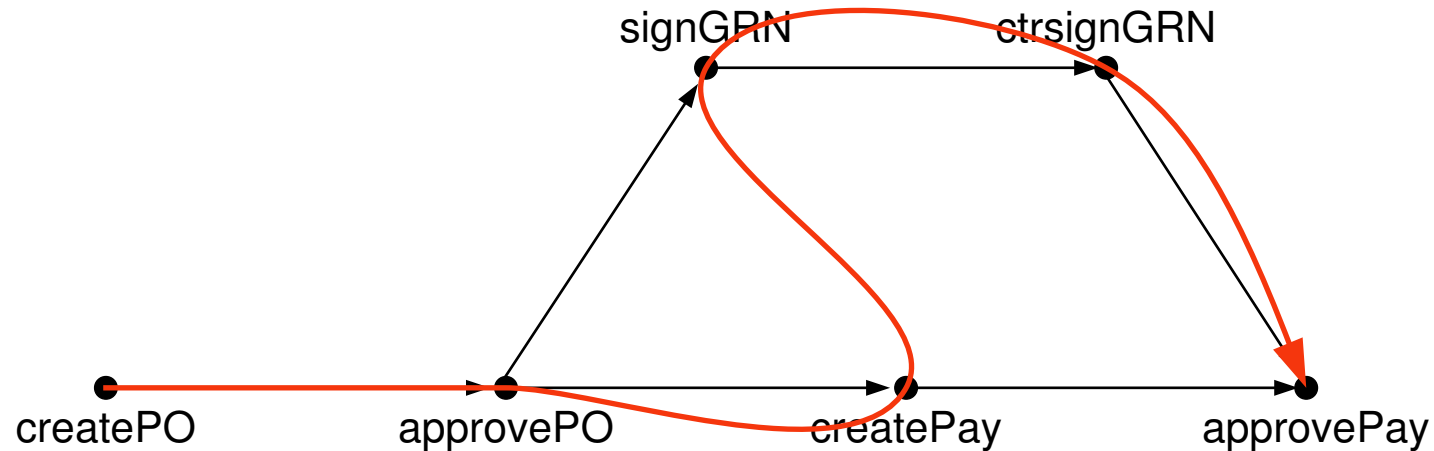
- if  $u \in D$  performs  $t$ , then  $u$  cannot perform  $t'$
- $(U, (\text{createP0}, \text{apprP0}), 0')$

$(D, (t, t'), 1')$  is a “binding of duty” constraint

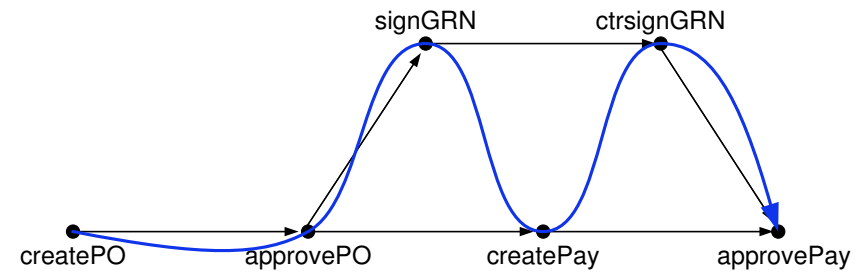
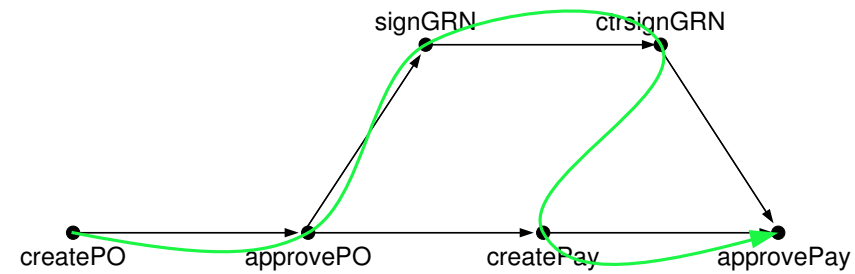
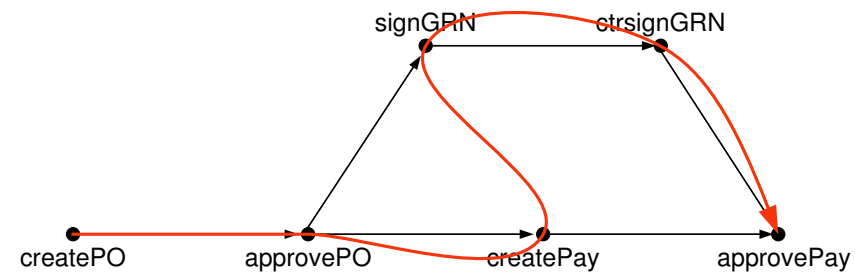
- if  $u \in D$  performs  $t$ , then  $u$  must perform  $t'$
- $(U, (\text{createP0}, \text{signGRN}), 1')$

## Linear extensions

A linear extension of a workflow specification  $T$  represents a possible sequence of execution of the tasks in  $T$



# Linear extensions



## Execution schedules

An execution schedule for  $(T, A, C)$  is a pair  $(L, \alpha)$ , where

- $L$  is a linear extension of  $T$
- $\alpha : T \rightarrow U$  assigns users to tasks

such that

- each user that performs a task in the schedule appropriately authorized

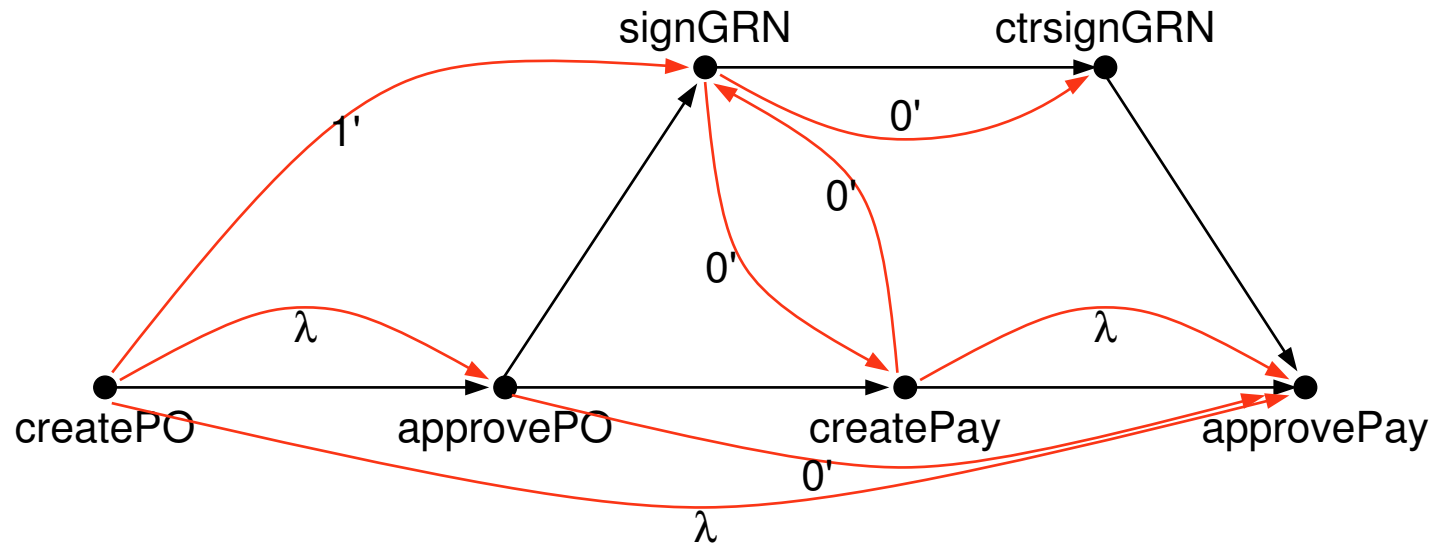
$$\forall t \in T, (t, \alpha(t)) \in A$$

- all constraints are satisfied

$$\forall (D, (t, t'), \rho), \alpha(t) \in D \text{ implies } (\alpha(t), \alpha(t')) \in \rho$$

# The entailment graph

Each constraint is visualized as a labelled edge in a directed graph whose nodes are tasks



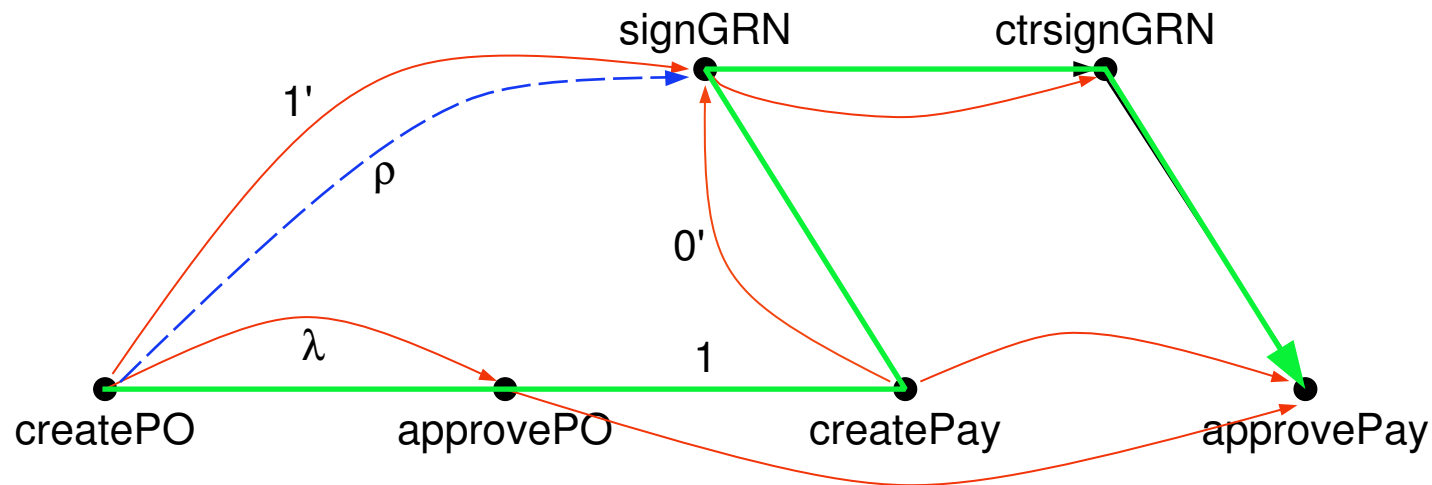
$(u, u') \in \lambda$  if  $u$  is less senior than  $u'$

## Paths in the entailment graph

The ordering on the tasks can be modelled using the relation  $1 = U \times U$

Paths represent composition of constraints

Multiple paths between two nodes can be reduced to a single constraint;  $\rho = \lambda 10' \cap 1'$





## Manipulating constraints

- Make  $D = U$  for all constraints

$$\rho' = \rho \cup \{(u, u') : u \notin D, u' \in U\}$$

- Compute single relation for each pair of tasks  $(t, t')$

$$\rho'' = \rho \cap \rho'$$

- Compose constraint  $(U, (t, t'), \rho)$  with constraint  $(U, (t', t''), \rho')$

$$\rho'' = \{(u, u'') : \exists u' \in U, (u, u') \in \rho, (u', u'') \in \rho'\}$$

- Overlay authorization information with constraints

$$\rho' = \rho \cap \{(u, u') : (t, u), (t', u') \in A\}$$

## Consequences

For any constrained workflow authorization schema  $(T, A, C)$ , there exists a schema  $(T, T \times U, C^*)$  such that

- For all  $t, t' \in T$  such that  $t \not\preceq t'$  there exists a constraint  $c \in C^*$  of the form  $(U, (t, t'), \rho)$
- Every execution schedule of  $(T, A, C)$  is an execution schedule of  $(T, T \times U, C^*)$  and vice versa

## **Interesting questions**

Is a constrained workflow authorization schema satisfiable?

Is a particular instance of a workflow schema satisfiable?

Given a satisfiable workflow authorization schema, is it possible to design a reference monitor so that every instance of that schema is satisfiable?

## An observation

Suppose that  $(T, A, C)$  is a constrained workflow authorization schema and that a minimal element  $t_0 \in T$  is performed by  $u$

This gives rise to a new CWAS  $(T, A', C)$ , where

$$A' = \{(t_0, u)\} \cup \{(t, u) \in A : t \neq t_0\}$$

More generally, let  $I$  be an instance of  $(T, A, C)$ , where  $I$  is an order ideal in  $T$ , and let  $I(t)$  denote the user who performed task  $t$

Then  $(T, A|I, C)$  is a CWAS, where

$$A|I = \{(t, I(t)) : t \in I\} \cup \{(t, u) \in A : t \notin T \setminus I\}$$

## Consequences

The same decision procedure can be used to answer the questions

- Is a CWAS satisfiable?
- Is an instance of a CWAS satisfiable?

A reference monitor that guarantees every workflow instance completes must check that

- if the request were to be granted, the resulting CWAS is satisfiable
- (the request is authorized and does not violate any constraint)

## Two strategies

Compute the “closure” of the entailment graph

- Not acyclic
- Difficult to distinguish between walks and paths
- Problem is NP-complete

Enumerate every linear extension

- Treat each linear extension as a workflow schema
- Polynomial in the number of tasks
- Form single workflow schema (taking “intersection” of schemata for linear extensions)

## A reference monitor

Let  $I$  be an instance of  $W = (T, A, C)$  and let  $I \cup \{t\}$  be an order ideal in  $T$

A request by  $u$  to execute  $t$  in this instance of  $W$  is granted by a completion compliant reference monitor if

- there exists an execution schedule for  $W|I$  such that  $u$  executes  $t$  and
- there exists an execution schedule for  $W|(I \cup \{(t, u)\})$

In other words, we simply run the algorithm for the workflow  $W|(I \cup \{(t, u)\})$  before granting the request  $(t, i, u)$

If the request is granted, the next request must be evaluated for the workflow  $W|(I \cup \{t, u\})$

## **Advantages of this approach**

Independent of underlying computational model and access control model

- Has generality that other approaches lack
- Can be implemented in a variety of different ways
- Uniform treatment of constraints (many existing approaches are ad hoc)

Simple and rigorous

- Design of reference monitor is simpler
- Overall understanding of mechanisms is improved
- Computational complexity of reference monitor is reduced



## Future work

Main priority is to develop model to incorporate multiple instances of tasks

- The specification is a poset  $T$  and a function  $f : T \rightarrow \mathbb{N}$  indicating the number occurrences of each task
- A constraint still has the form  $((D, (t, t'), \rho),$  but  $t$  may equal  $t'$ 
  - A task  $t$  may be repeated  $f(t)$  times but must be performed by the same (different, etc.) user