Composable Non-interactive Zero-knowledge Proofs in the Random Oracle Model

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Based on joint work with abhi shelat (Asiacrypt ’22)

In this talk…

- Zero-knowledge proofs (of knowledge)
  — Understand and use their security guarantees

- A taste for how they are designed and analysed
  — Provably secure composition
  — Random Oracle Model

- [Ks 22] Uncover a gap in the literature that was glossed over as folklore—turns out to permit a new kind of attack
  Briefly discussion on how we fix it

Quick Disclaimer

- **What will be covered**:
  Intuitive abstract idea of how to construct composition-safe ZK, how our attack works

- **What won't be touched**:
  Formalism of definitions, concrete instantiations, efficiency (this is to help understanding, not to hand-wave; please ask if something is unclear!)

Composable Non-interactive Zero-knowledge Proofs in the Random Oracle Model

Zero-knowledge Proofs

- Very powerful cryptographic primitive, introduced by [Goldwasser Micali Rackoff 85]

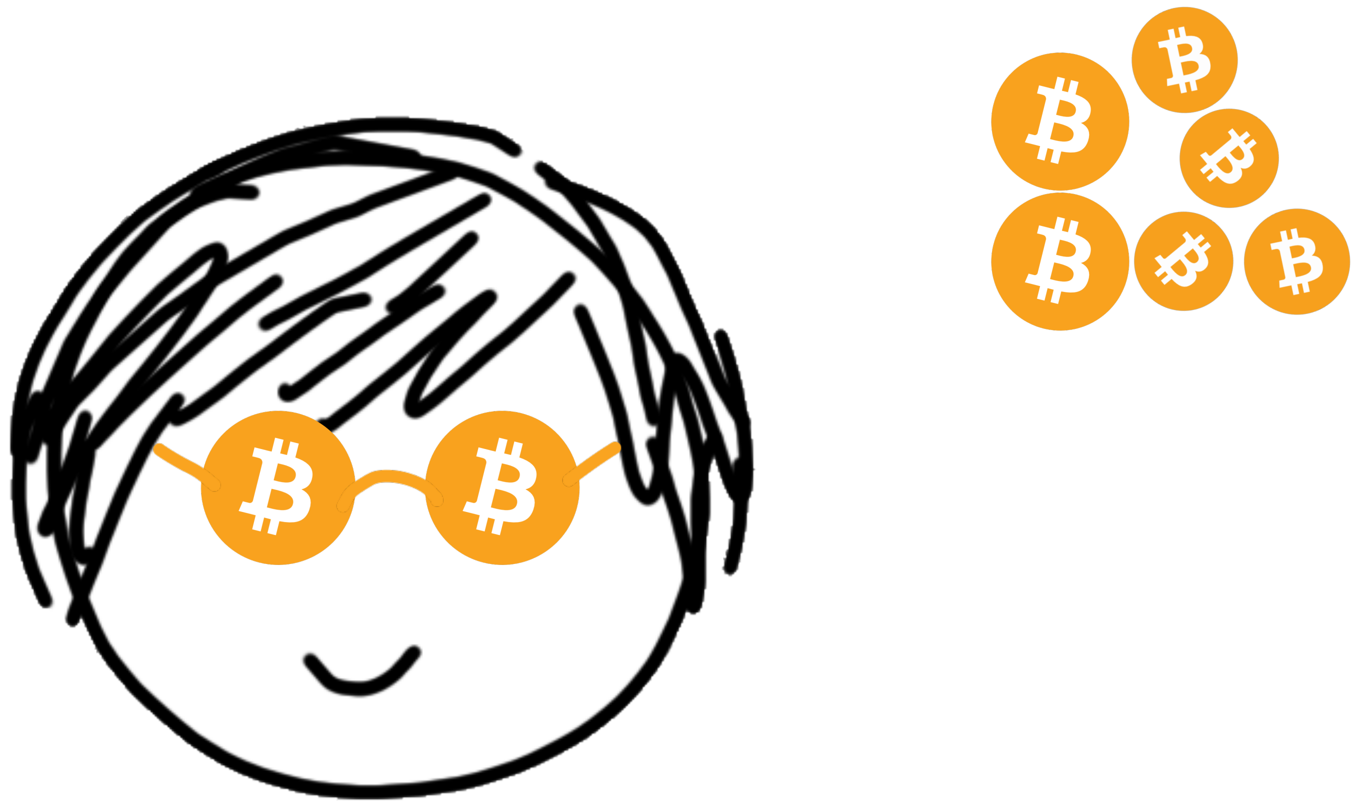
- Intuition: Prover convinces a Verifier of a statement, without revealing “why” it’s true.

  - Prover typically needs to use some secret information

  - Verifier obtains no useful information about Prover’s secrets

Zero-knowledge Proofs

- Simple application: proof of possession (key ownership)



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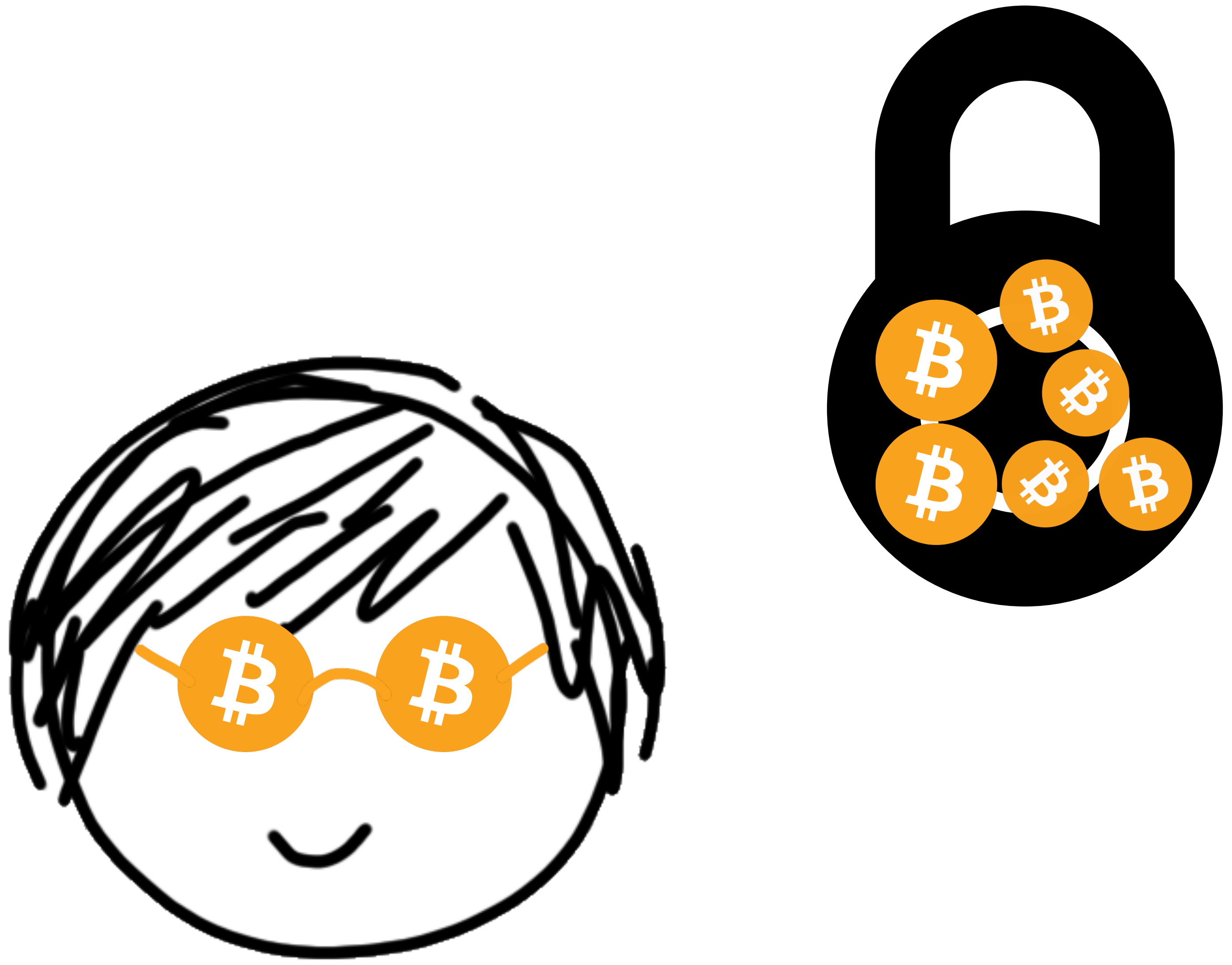

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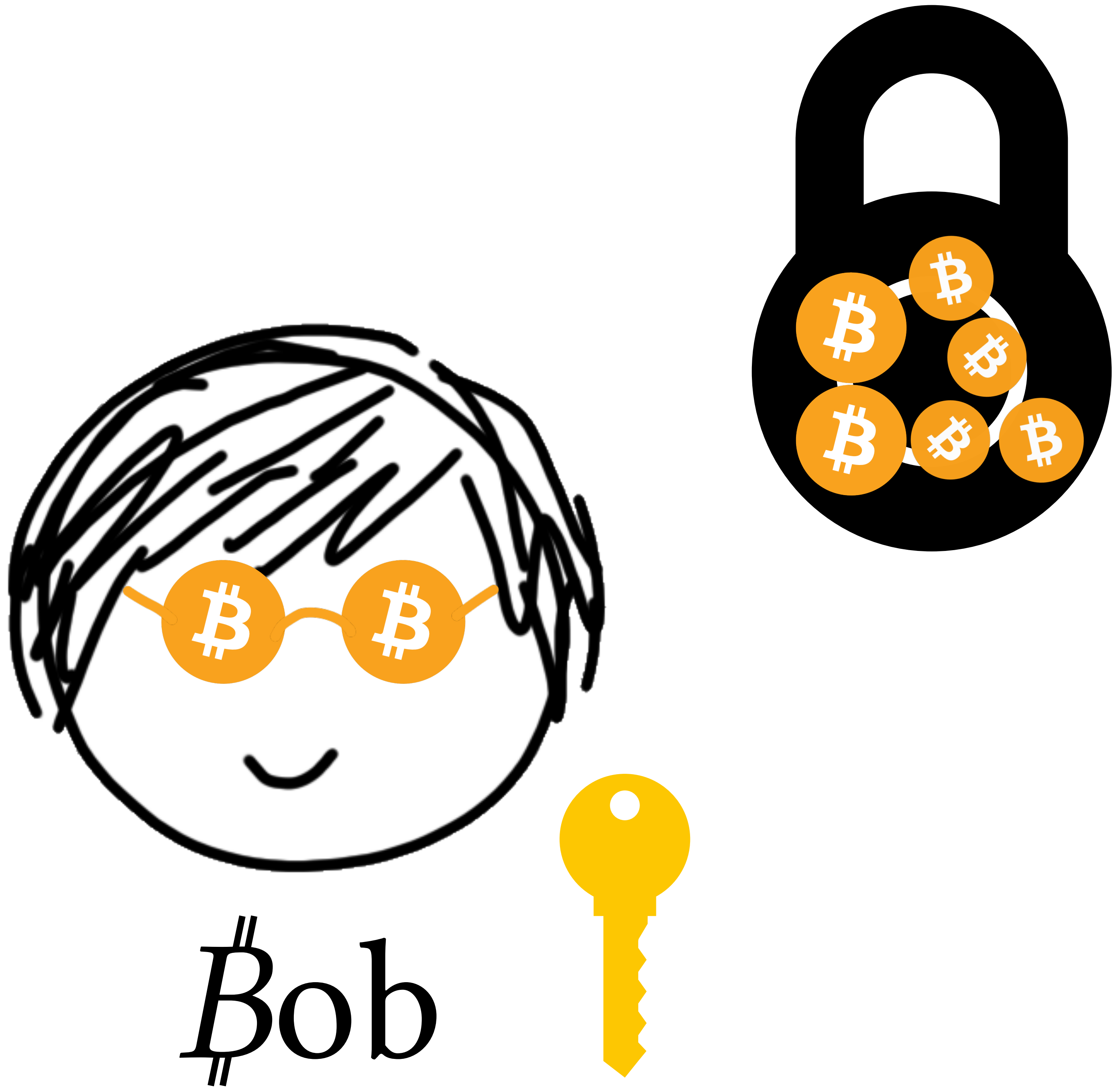

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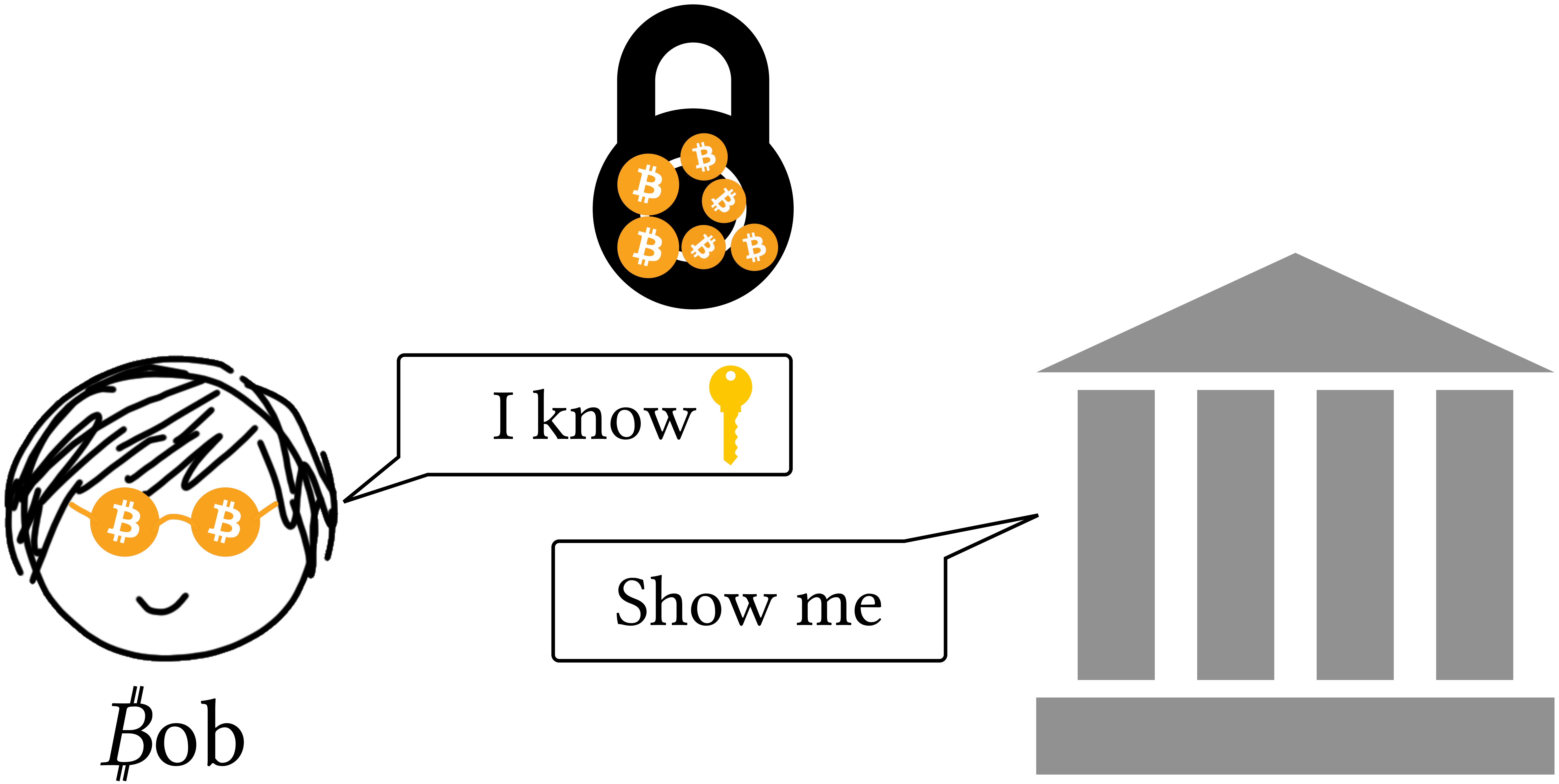

I know 🔑

Show me

Ƀob

Zero-knowledge Proofs

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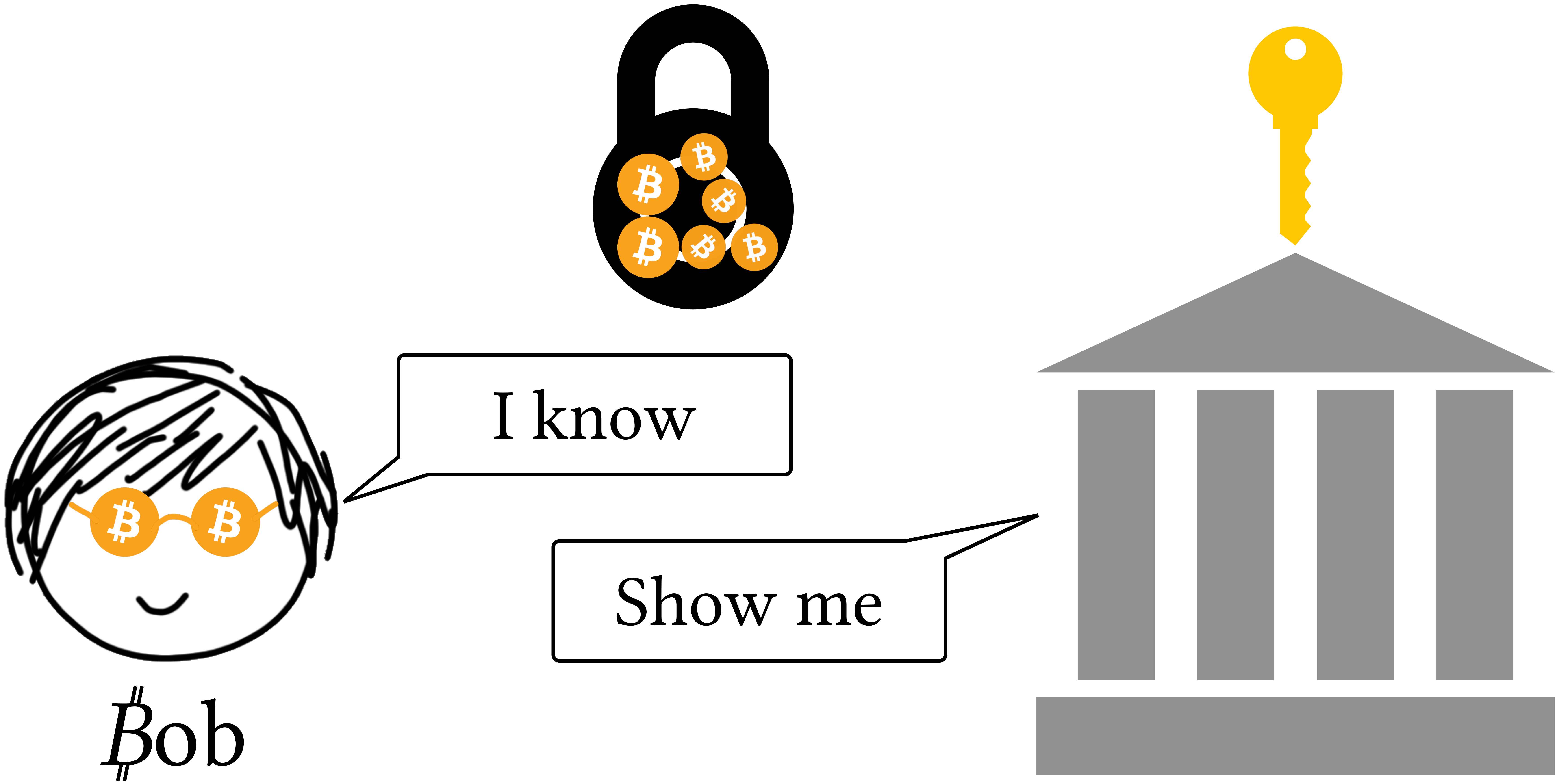

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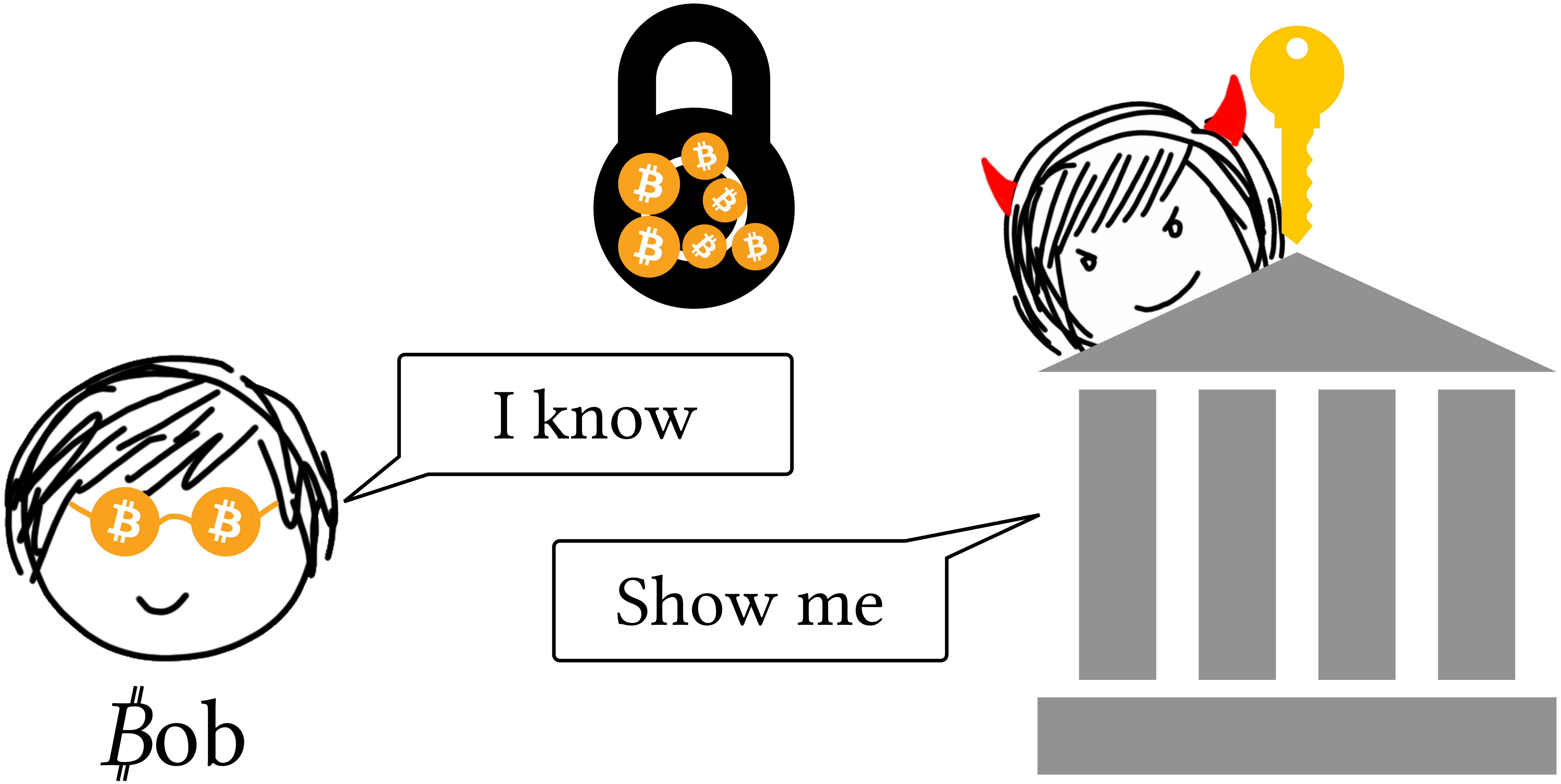

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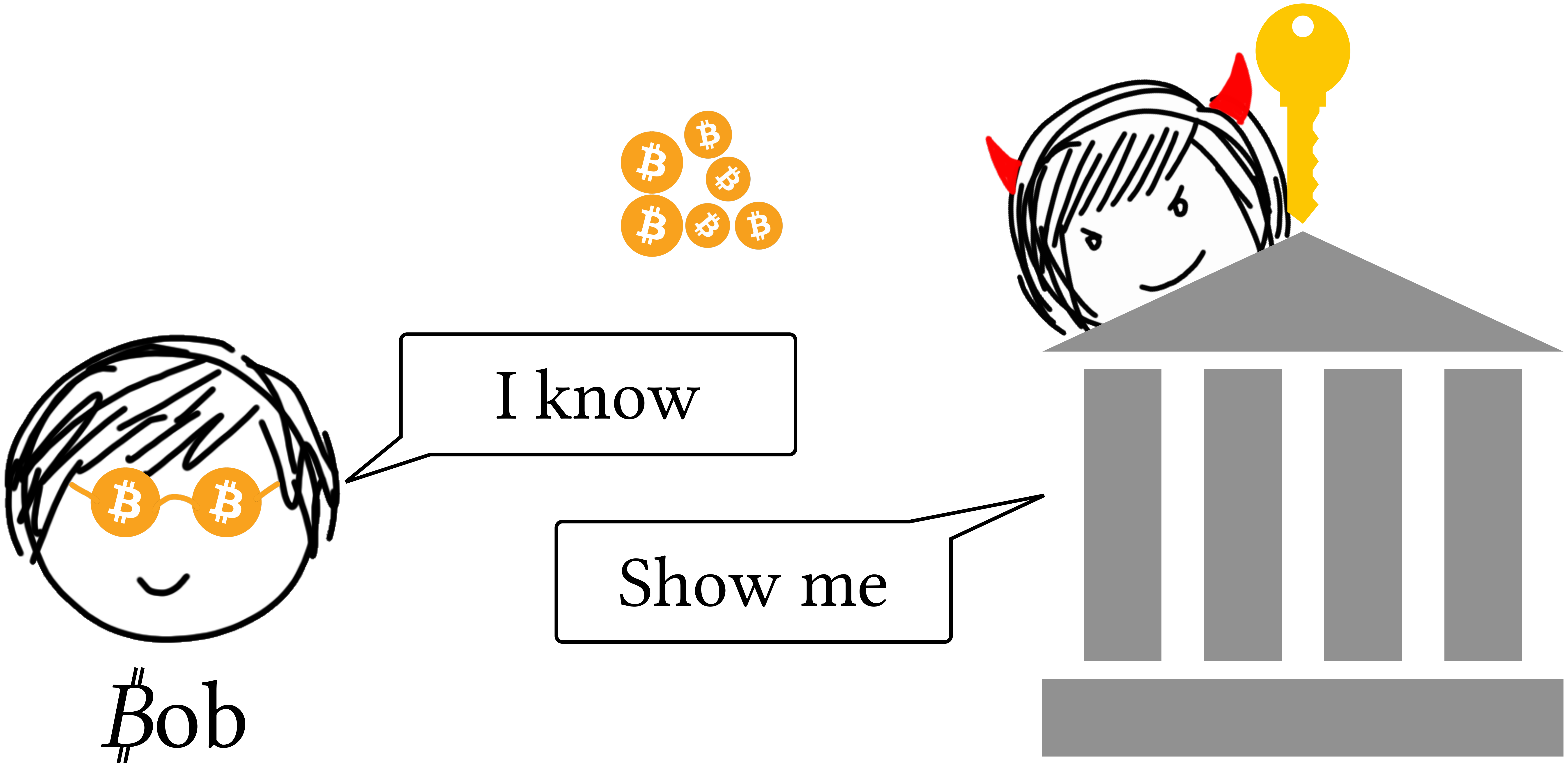

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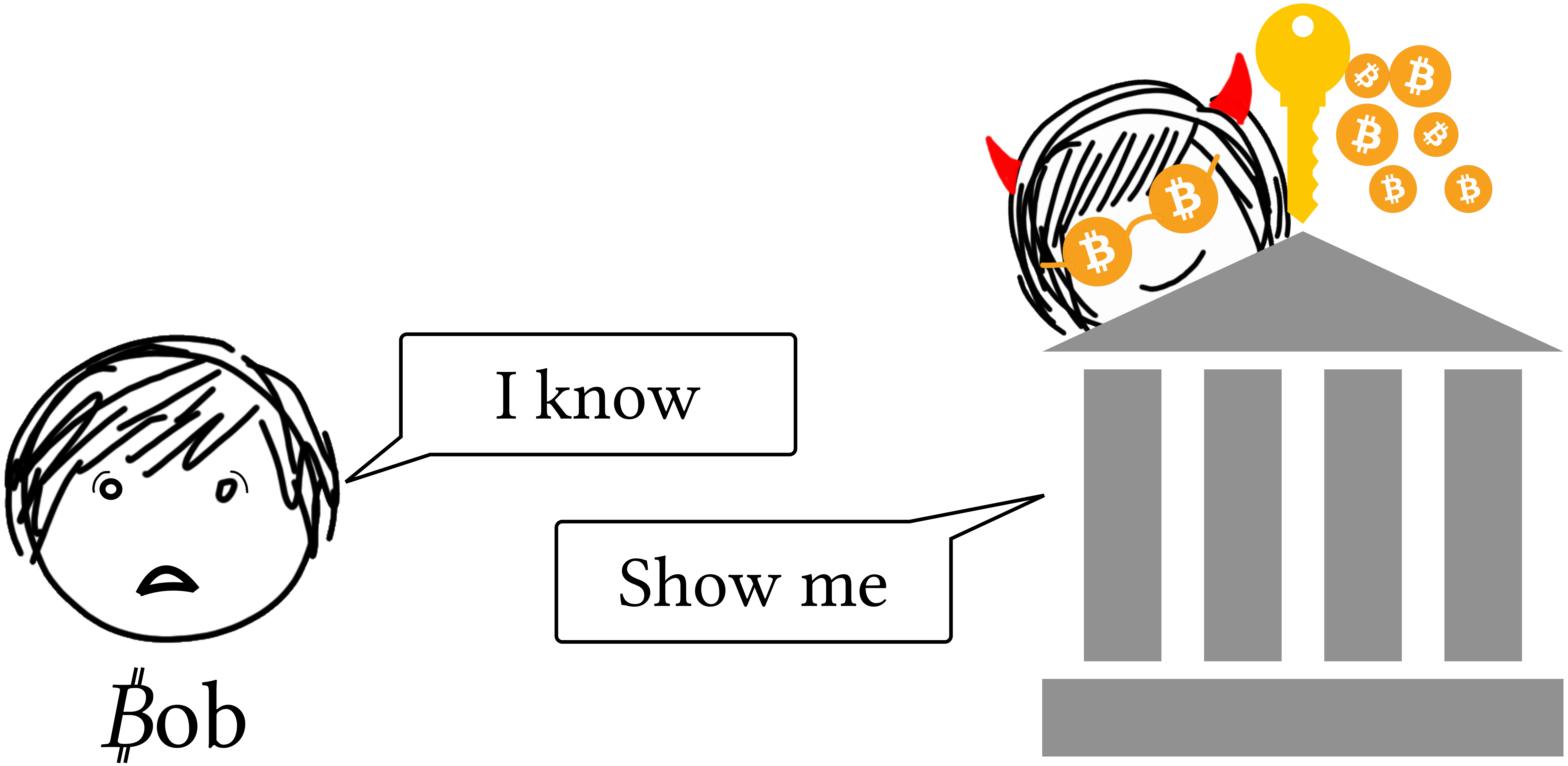

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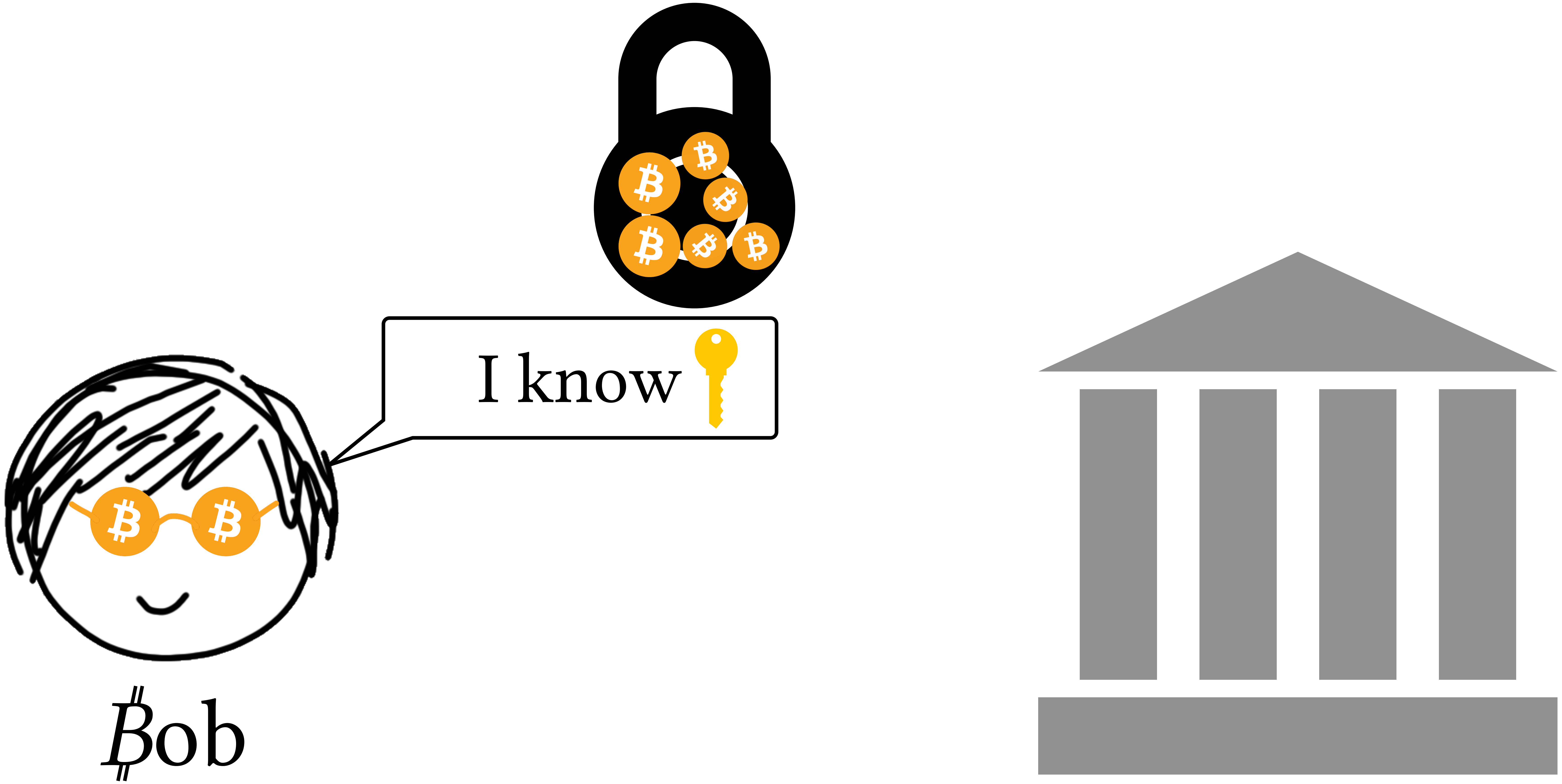

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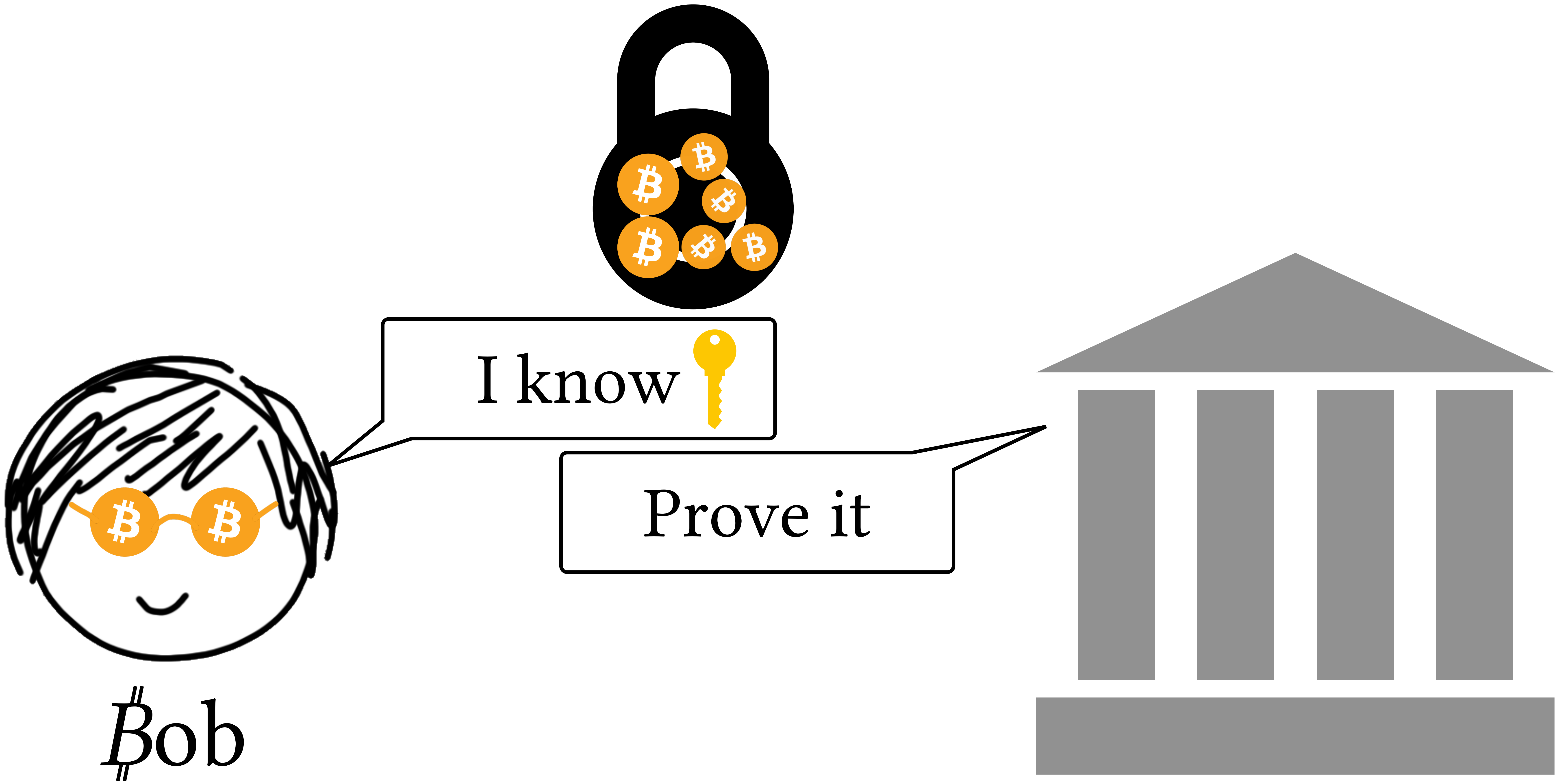

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Prove it

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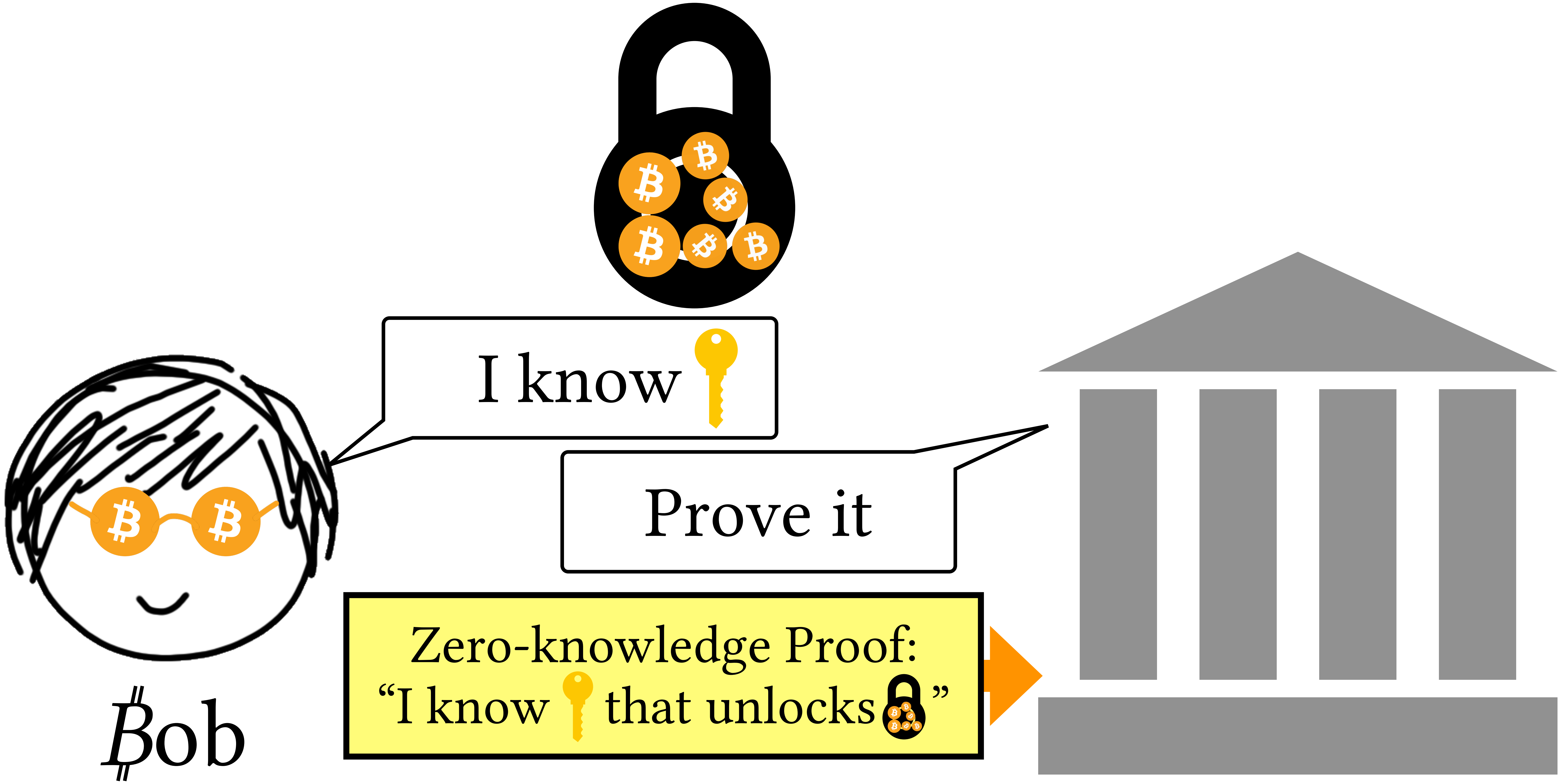

I know 🔑

Prove it

Zero-knowledge Proof:
"I know 🔑 that unlocks 🔒"

₿ob

Defining Zero-knowledge Proofs

- ZK is intuitive: No information about the key should be leaked by the proof
- But what does it mean to "know" something?
- "Proof of Knowledge" is formalized by an "extractor" Ext

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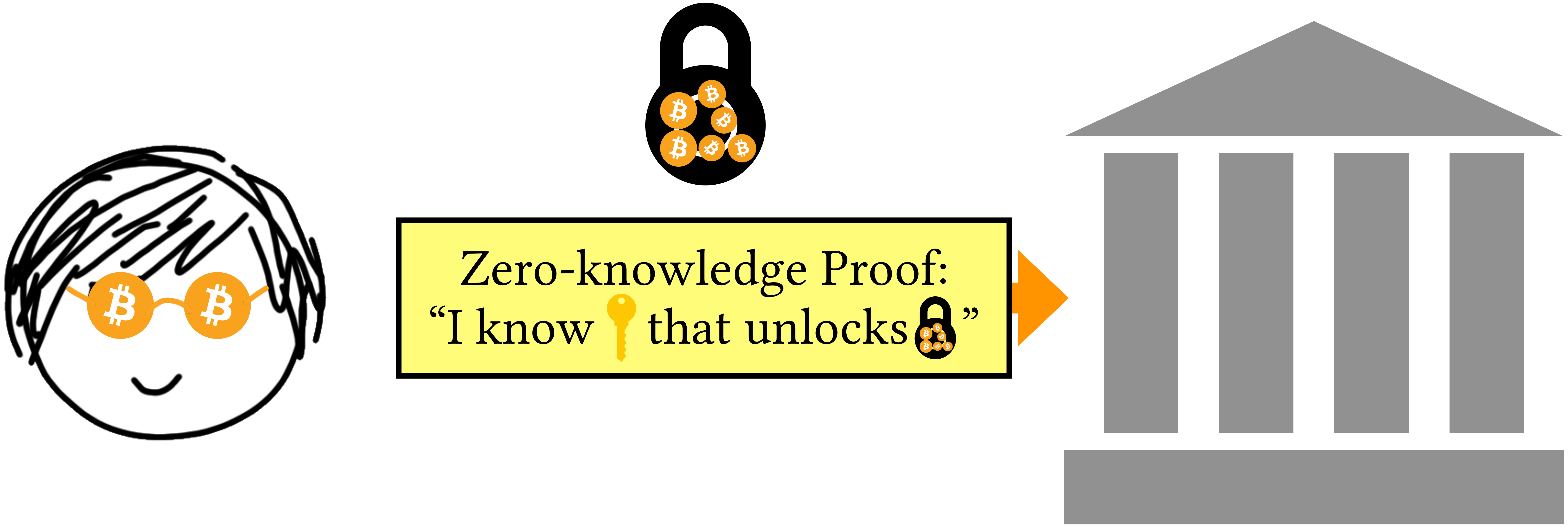

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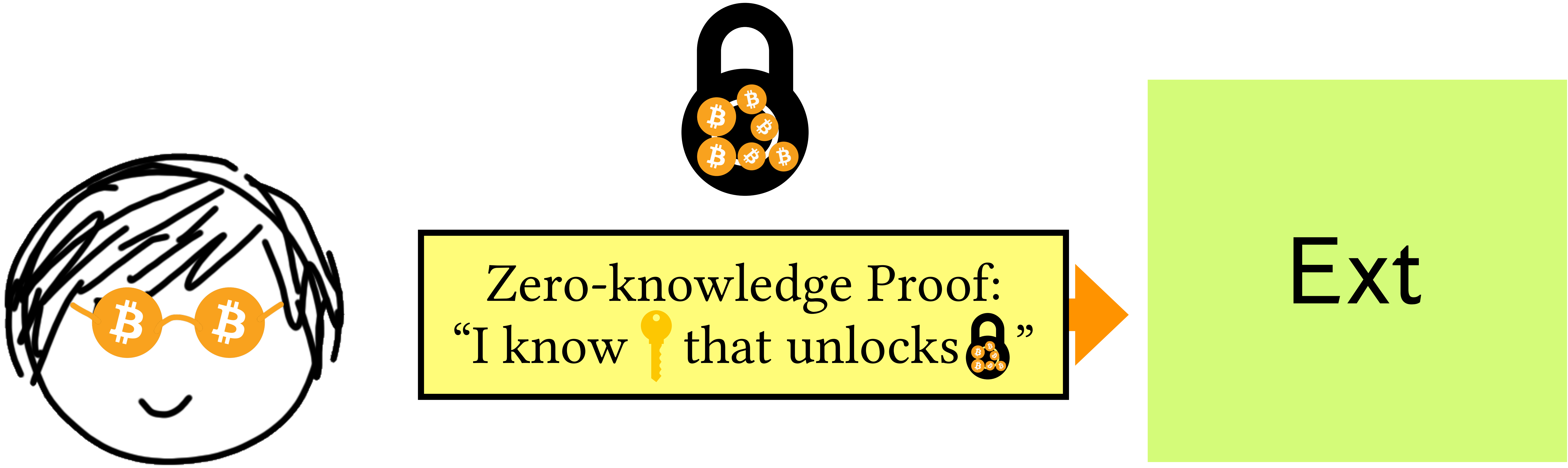

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Ext

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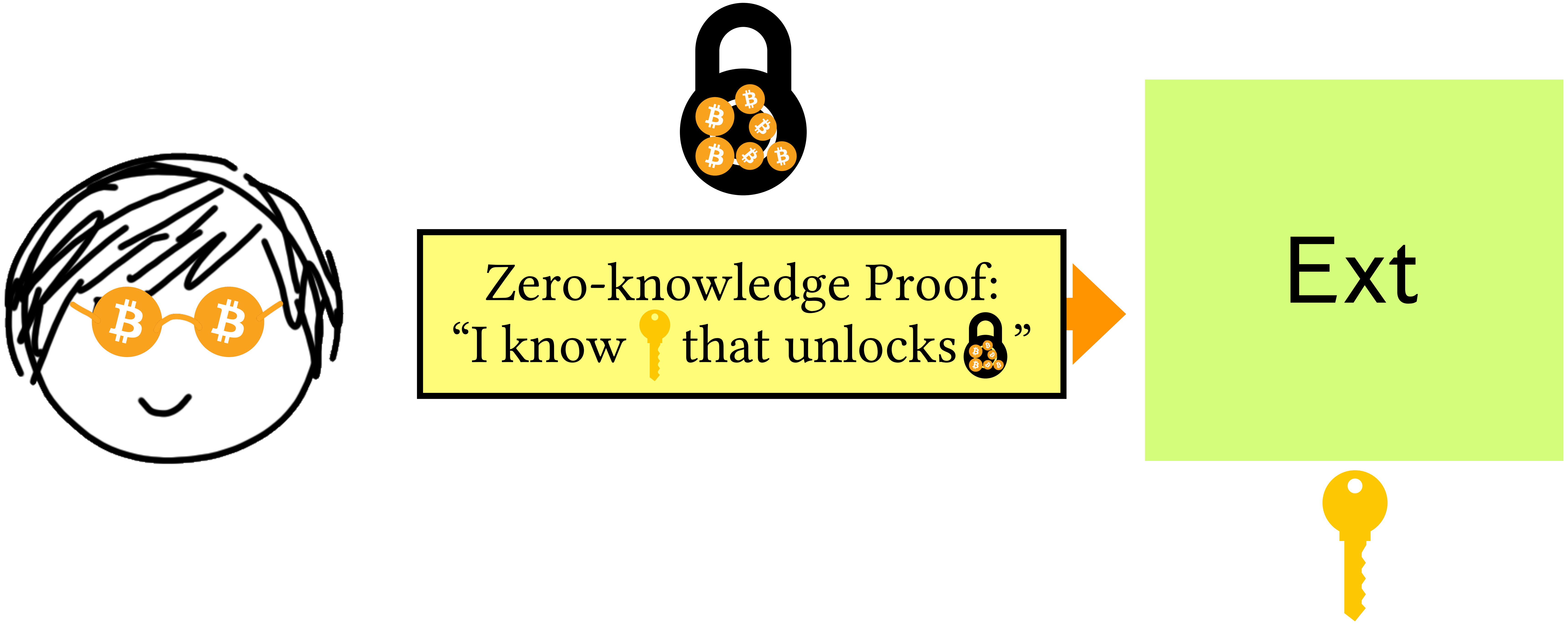


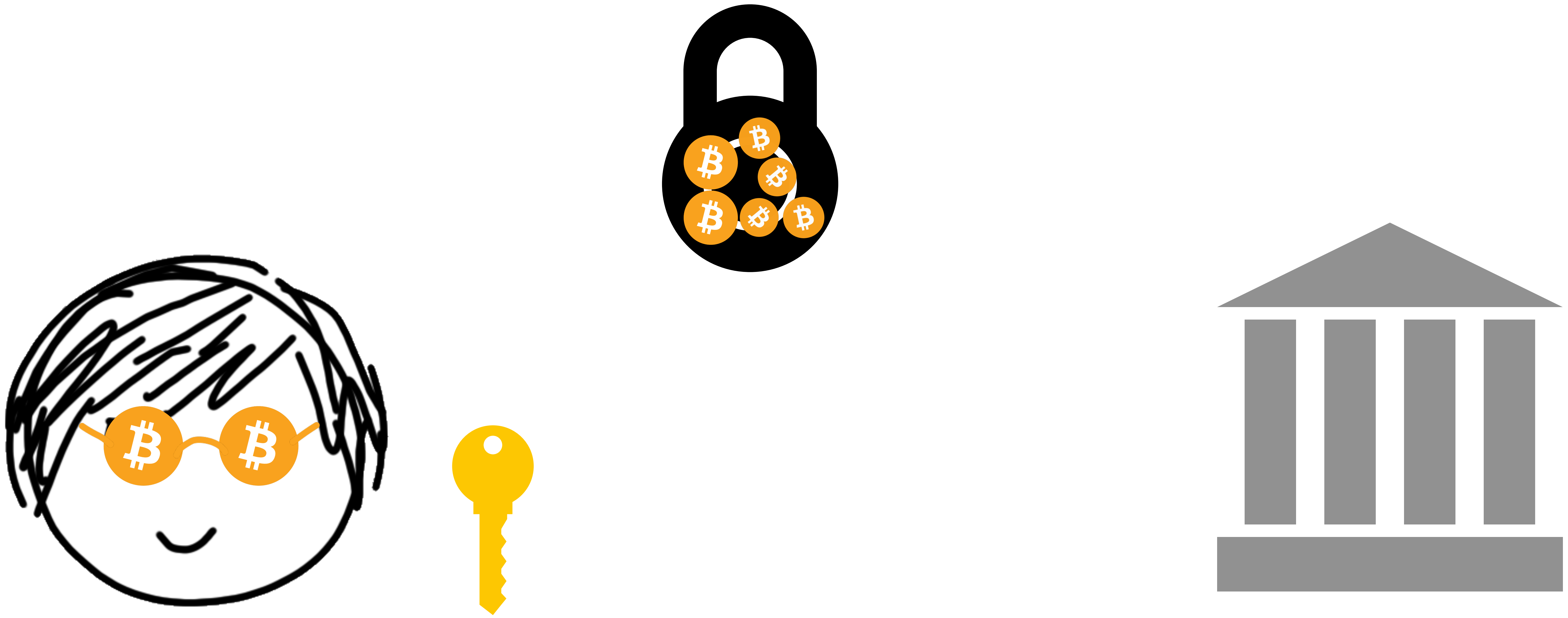
Zero-knowledge Proof:
"I know 🔑 that unlocks 🔒"

Why is Ext special?

- Clearly, Ext must not be an algorithm that just anybody can run
- Ext has carefully chosen special privileges:
  - Powerful enough to accomplish extraction
  - Still meaningful as a security claim
- We will look at a certain type of ZK proof to build intuition

Σ Protocols

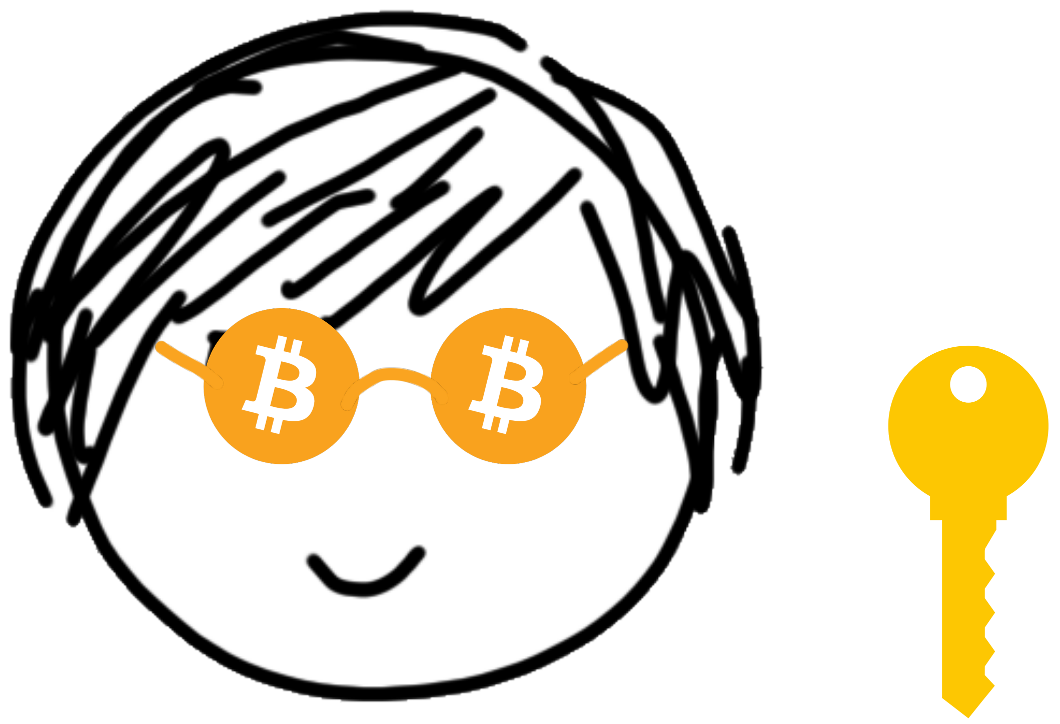
[Damgård 02]



Σ Protocols

[Damgård 02]

$X =$



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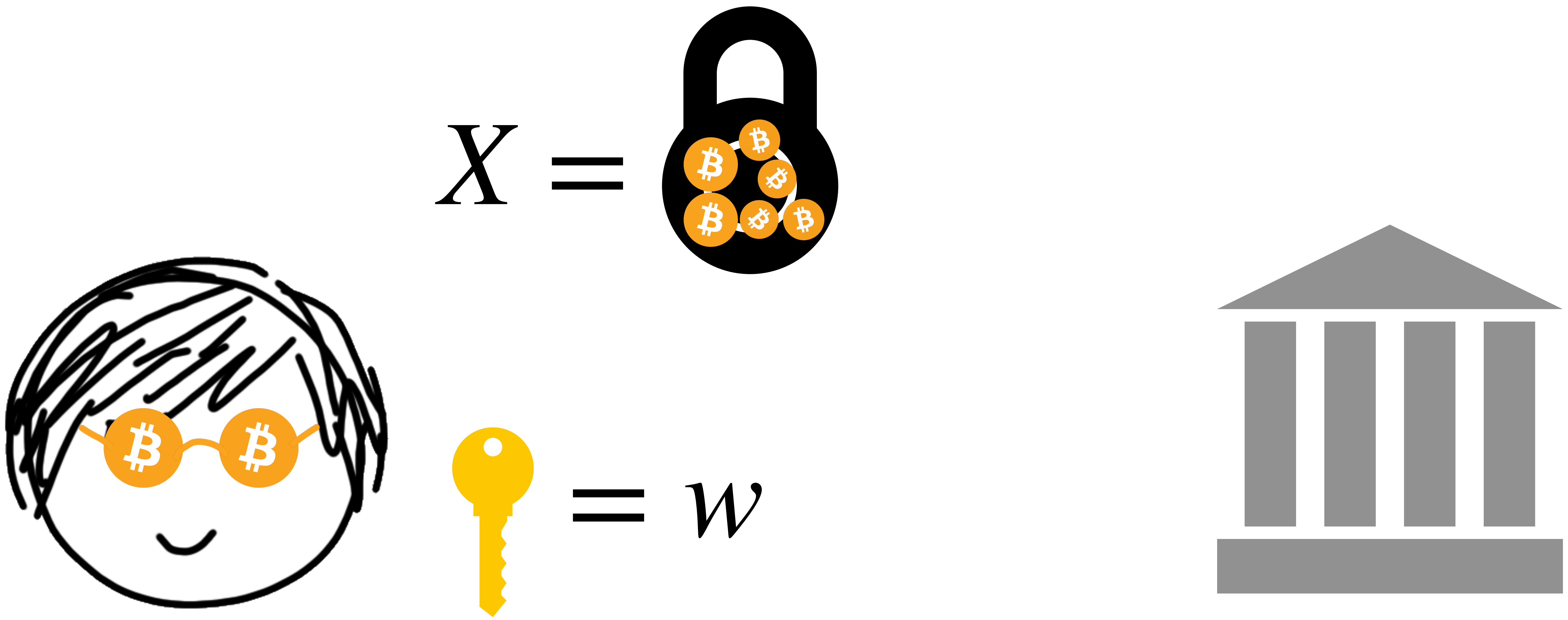

$X =$ 🔒

🔑 $= w$

Σ Protocols

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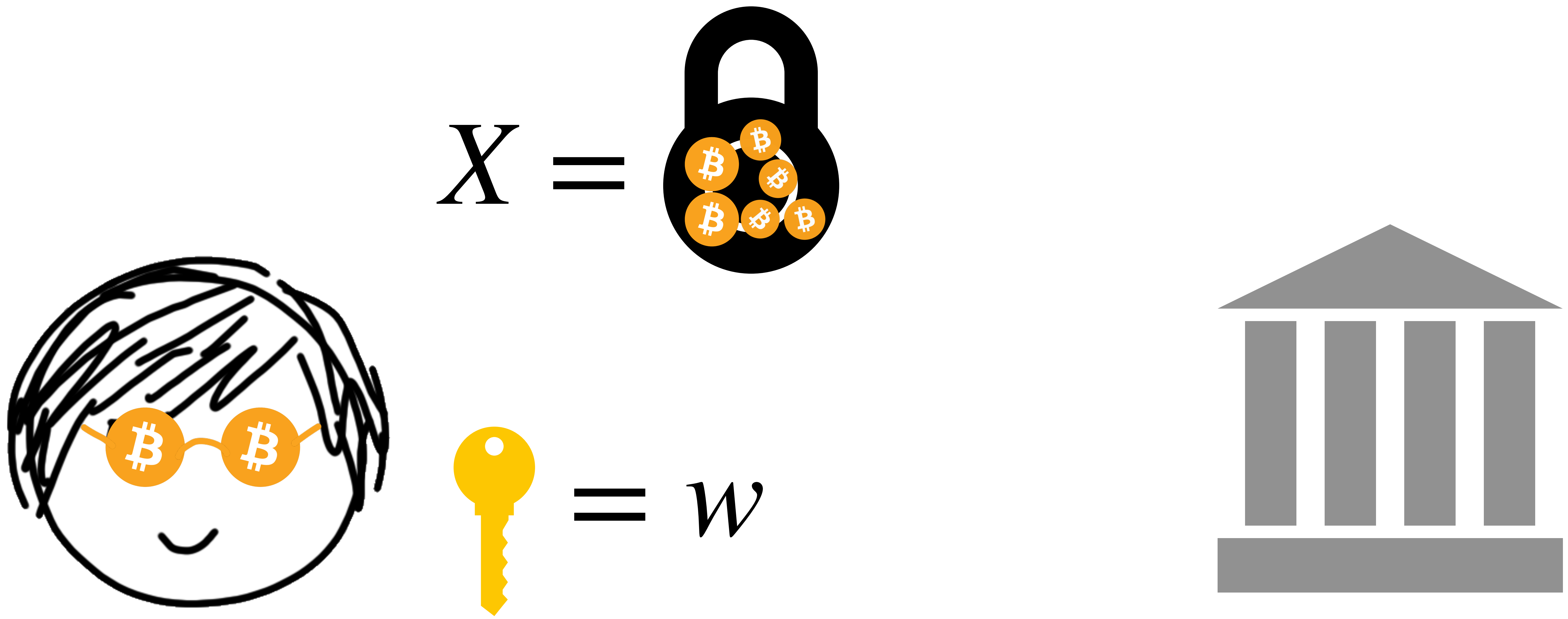

$X =$

$= w$

$P(X, w)$

Σ Protocols

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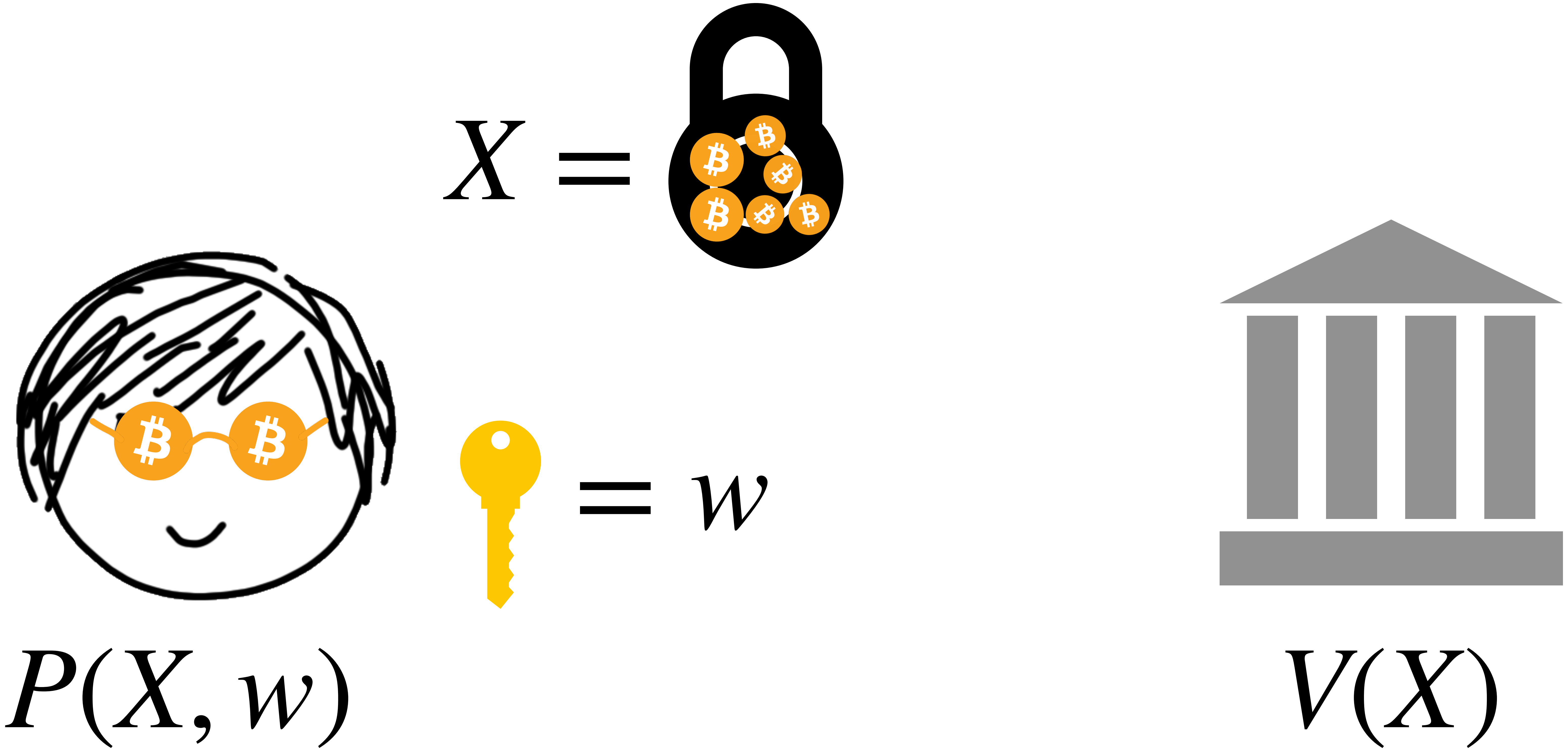

$X =$

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$P(X, w)$

$V(X)$

Σ Protocols

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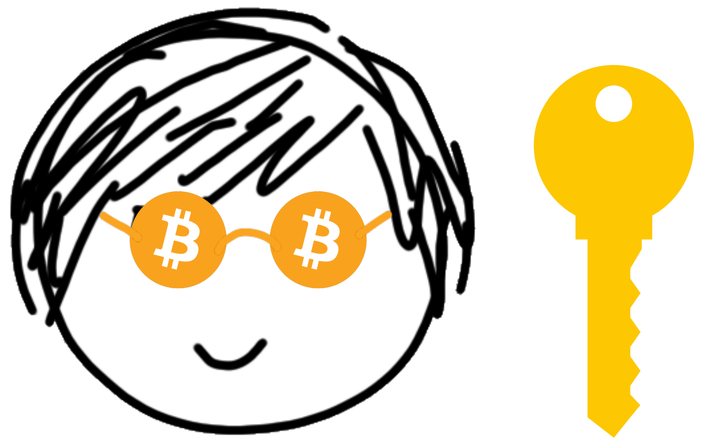

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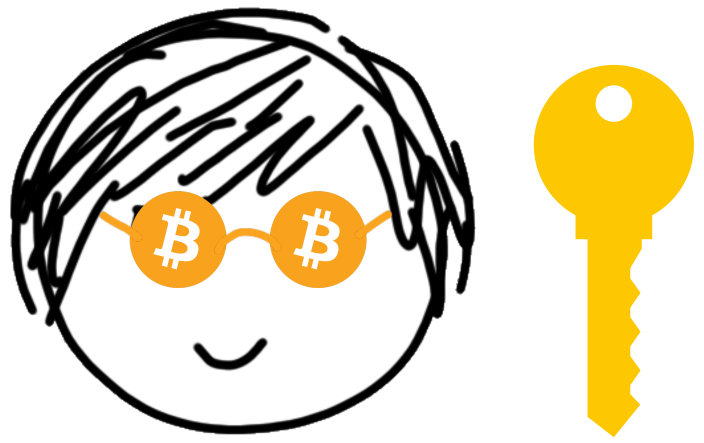

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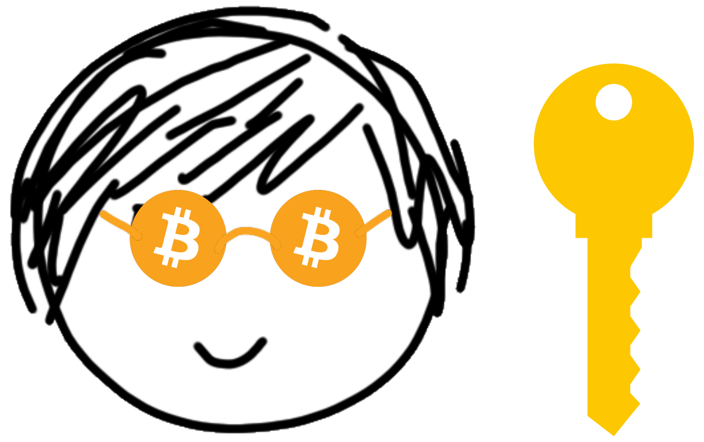

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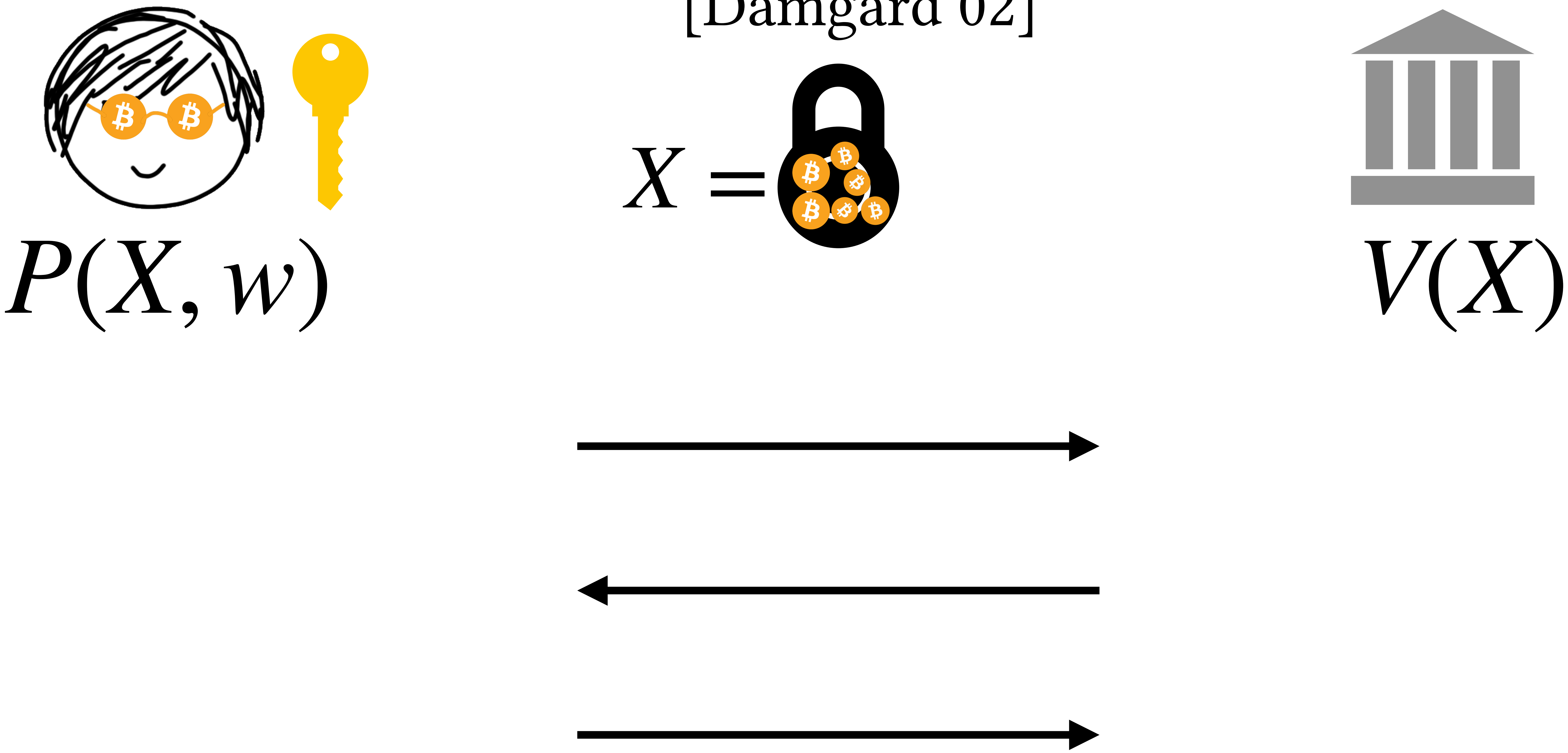

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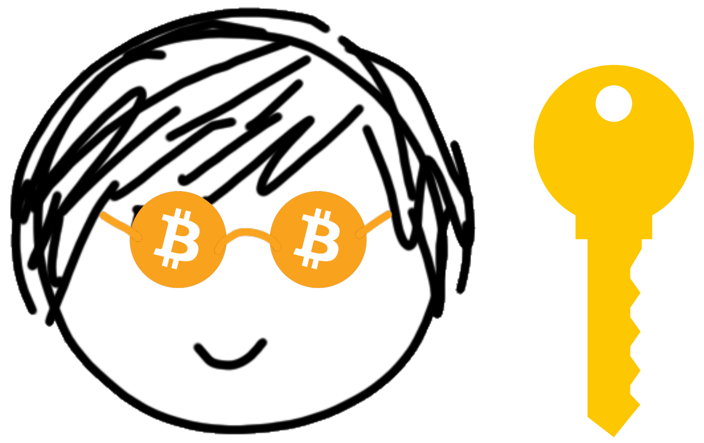

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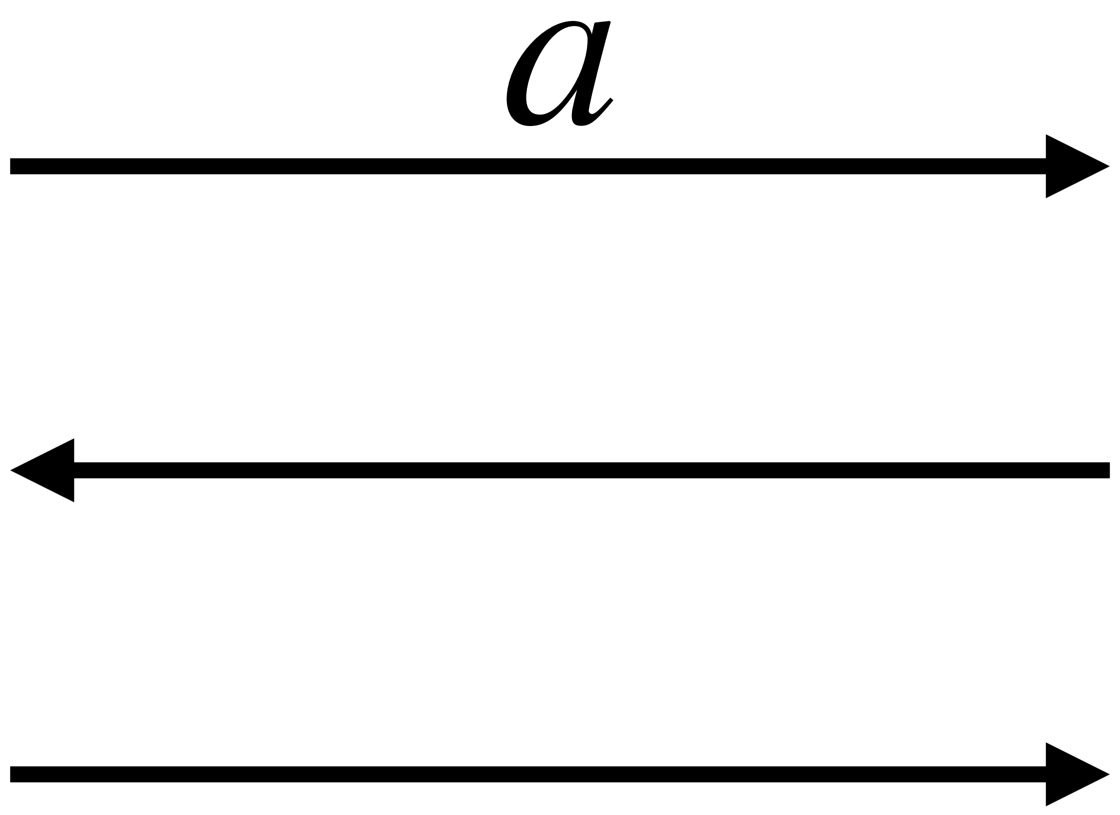

a

Σ Protocols

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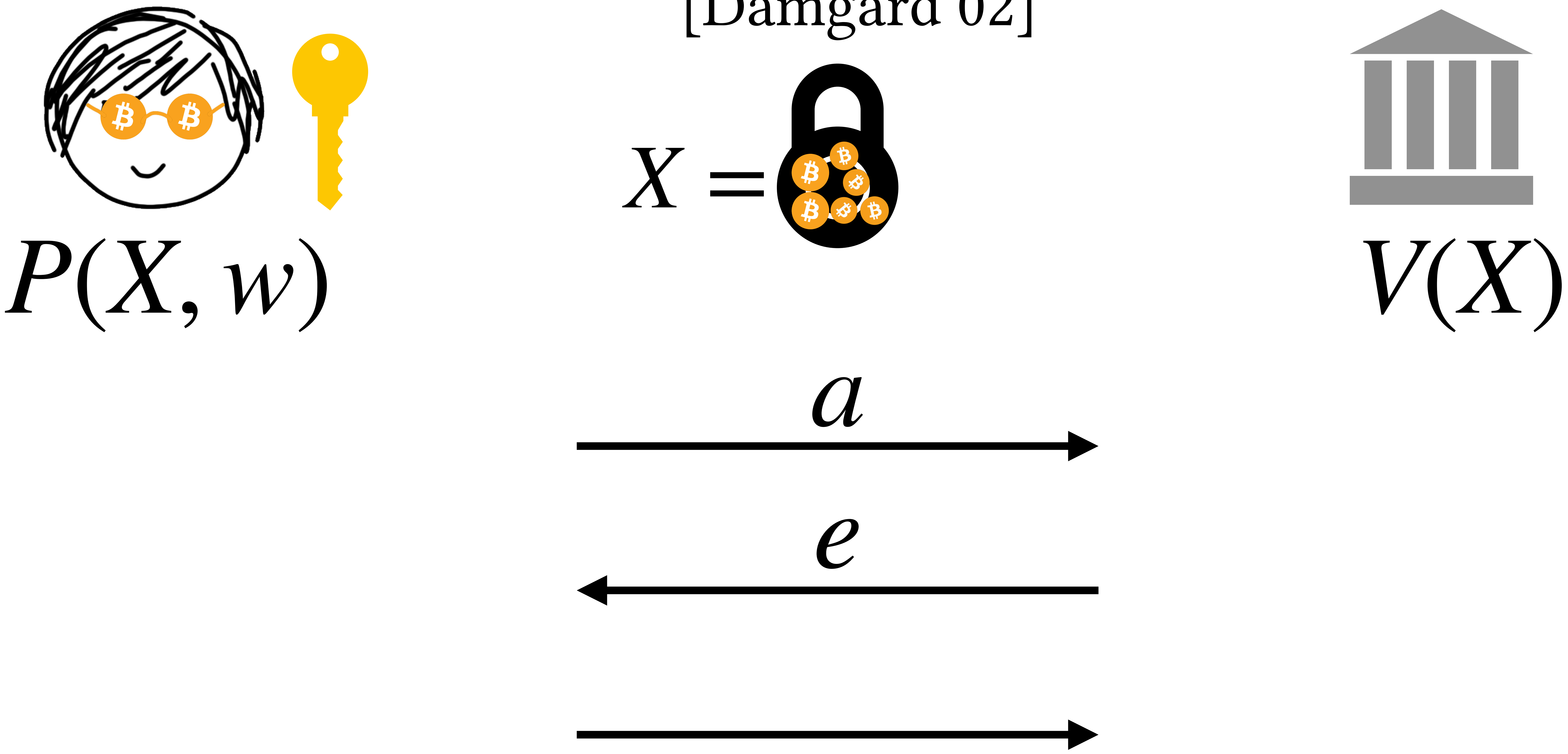

$P(X, w)$

$X =$

$V(X)$

a

e

Σ Protocols

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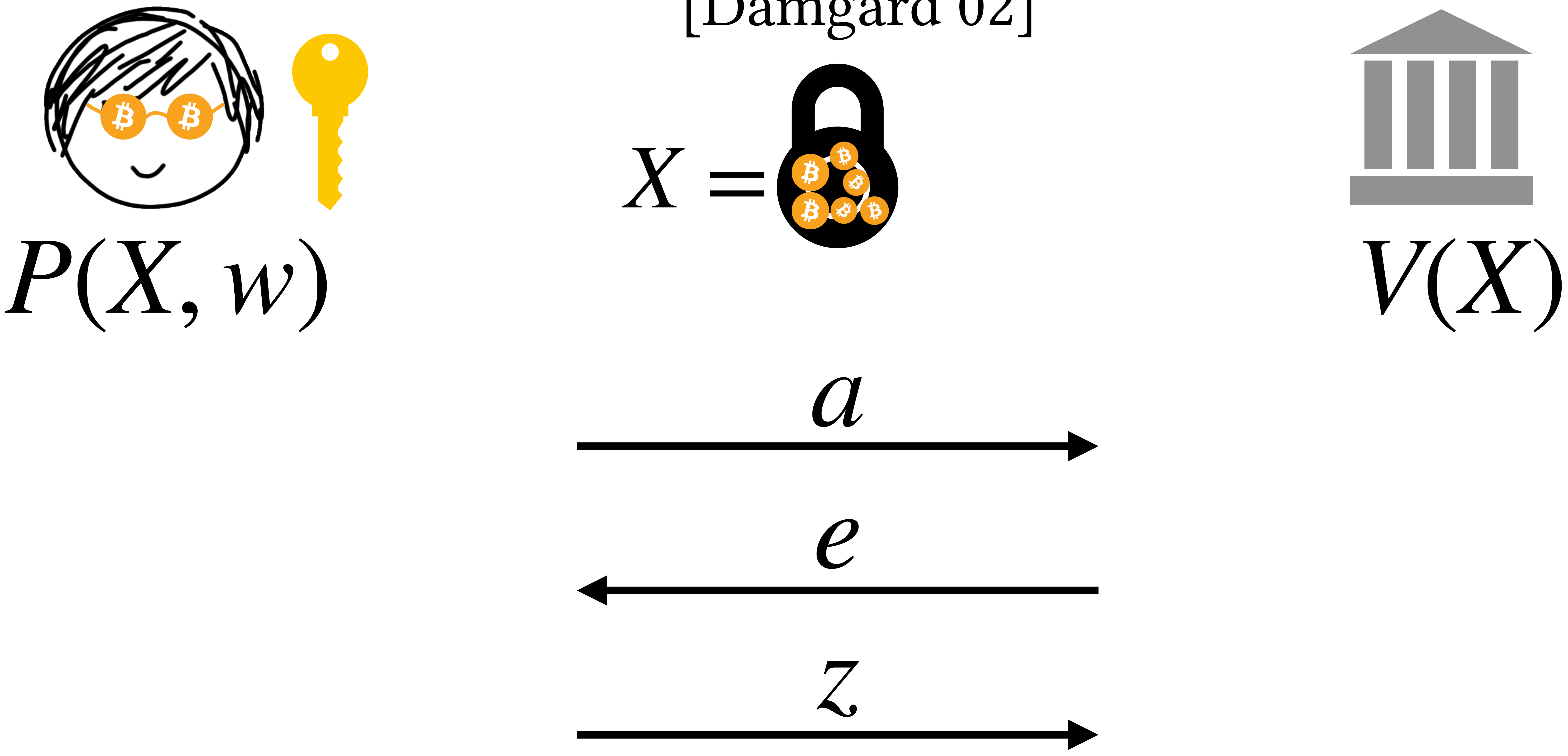

$P(X, w)$

$X =$

$V(X)$

$$P \xrightarrow{a} V$$

$$P \xleftarrow{e} V$$

$$P \xrightarrow{z} V$$

Σ Protocols

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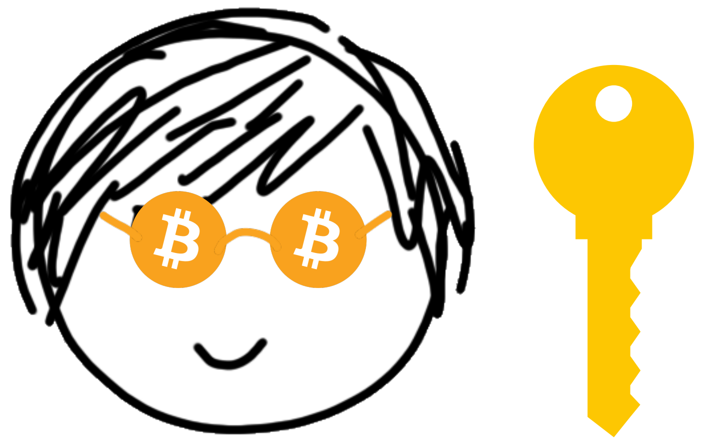

$P(X, w)$

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Commitment $\quad a \quad \longrightarrow$

$\longleftarrow \quad e$

$z \quad \longrightarrow$

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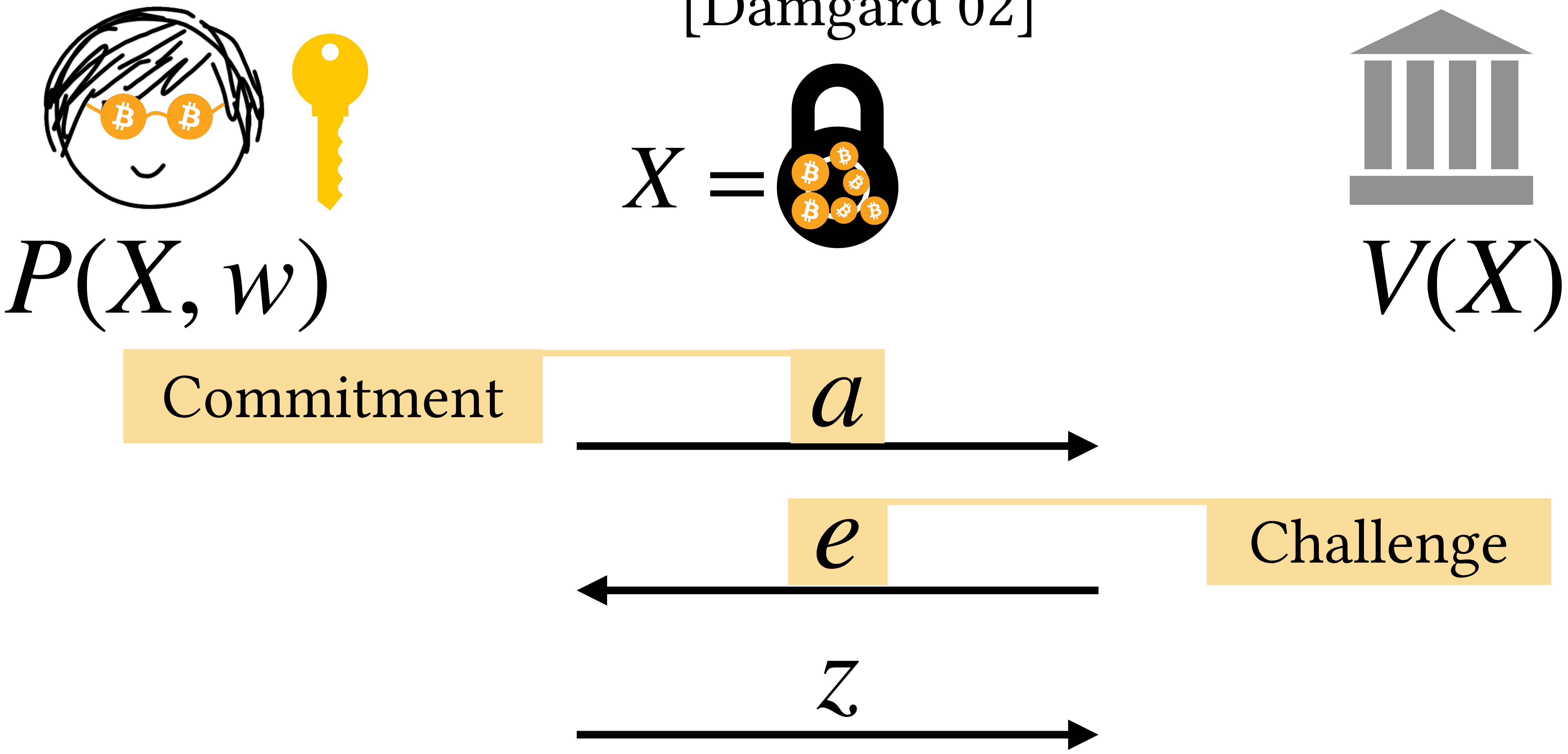

$P(X, w)$

$X =$

$V(X)$

Commitment a

e Challenge

z

Σ Protocols

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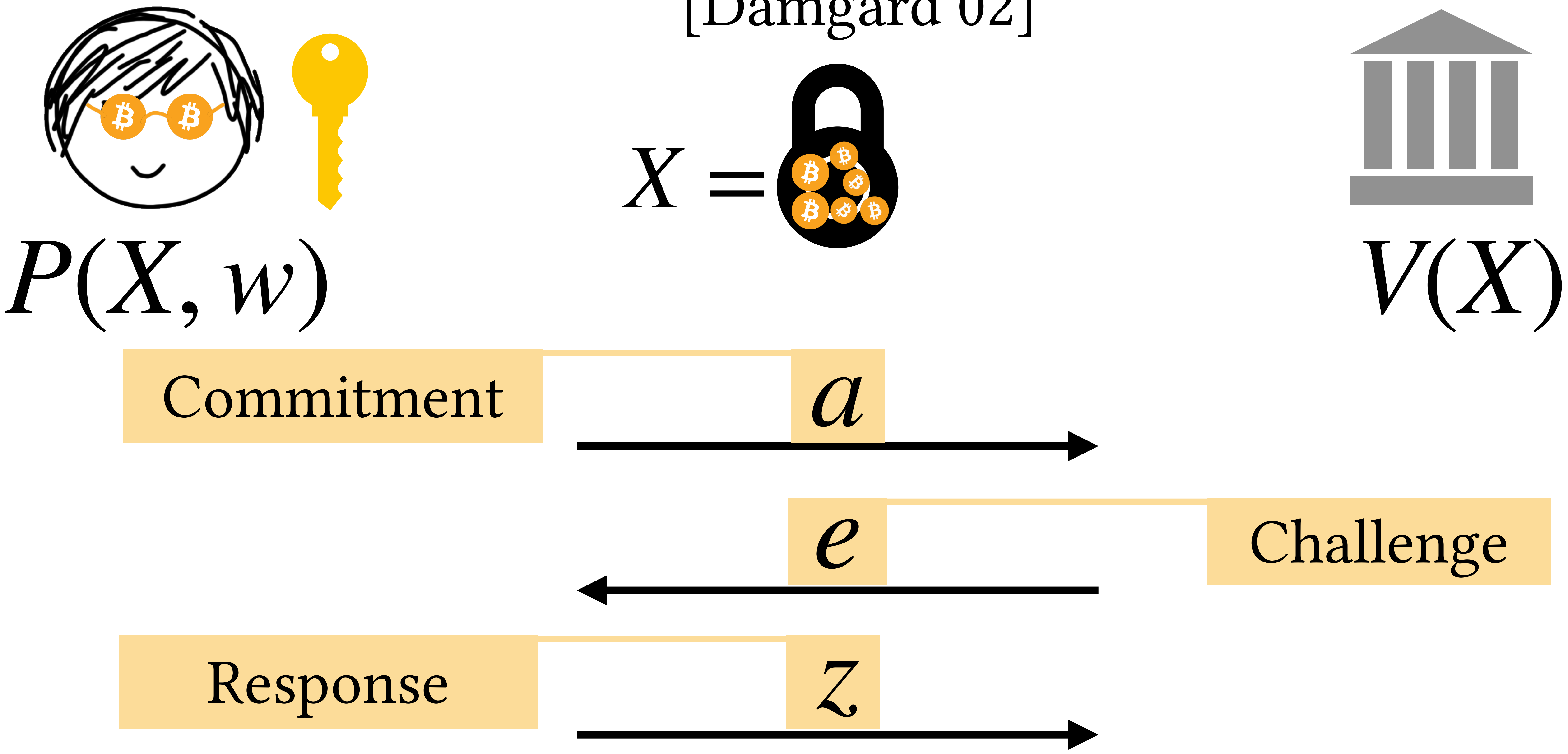

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Commitment a

e Challenge

Response z

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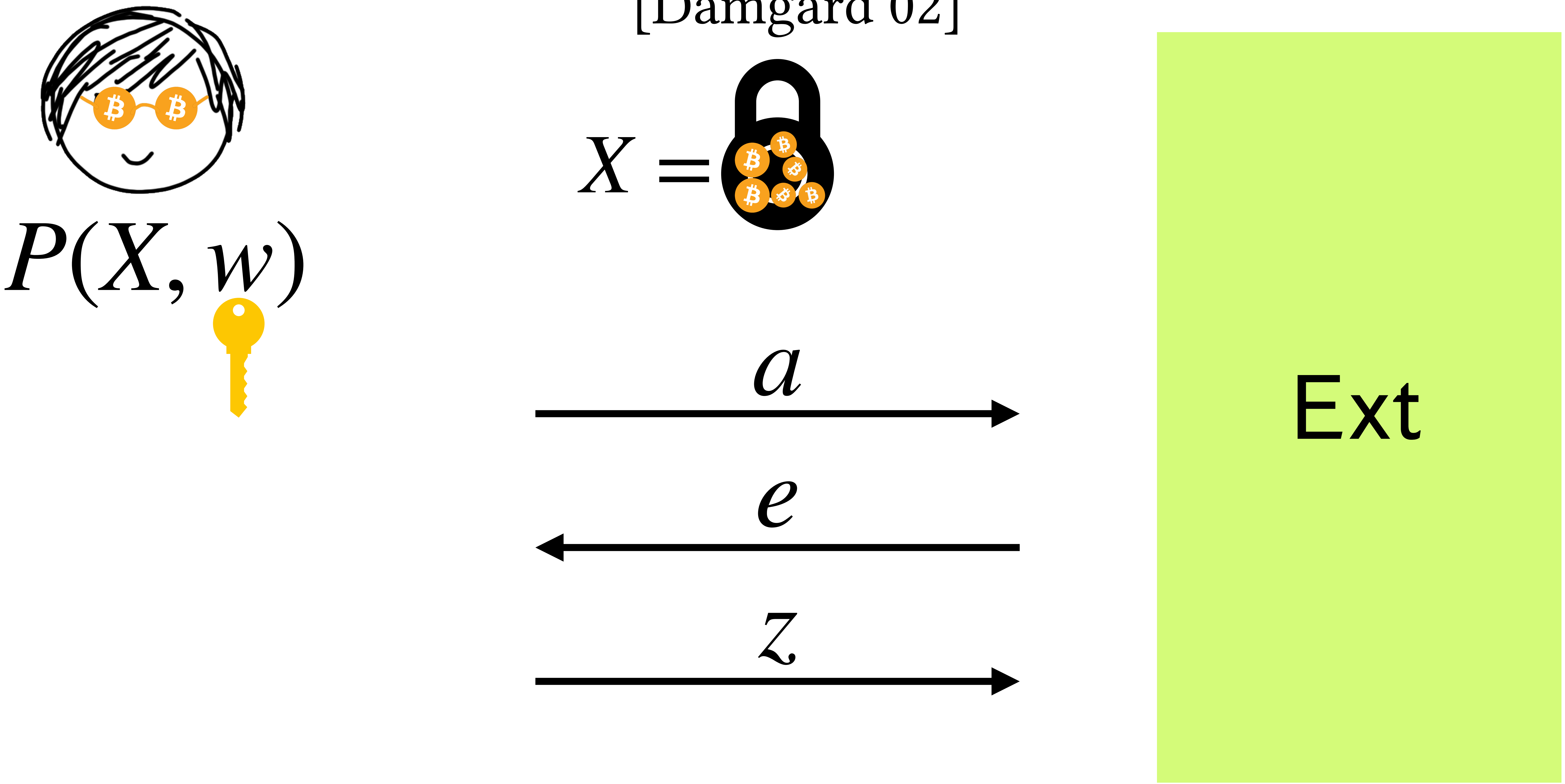

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a

e

z

Ext

Σ Protocols

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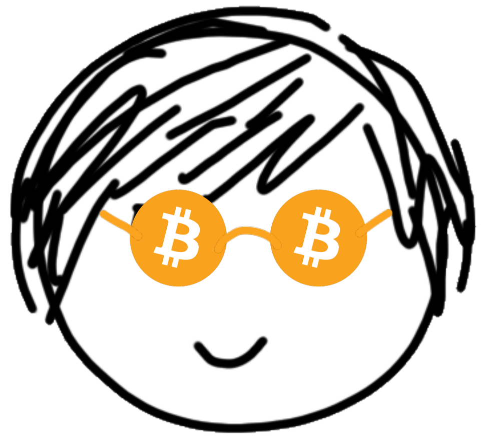

$P(X, w)$

$X =$

$$\xrightarrow{a}$$

Ext

$e \quad z$

Σ Protocols

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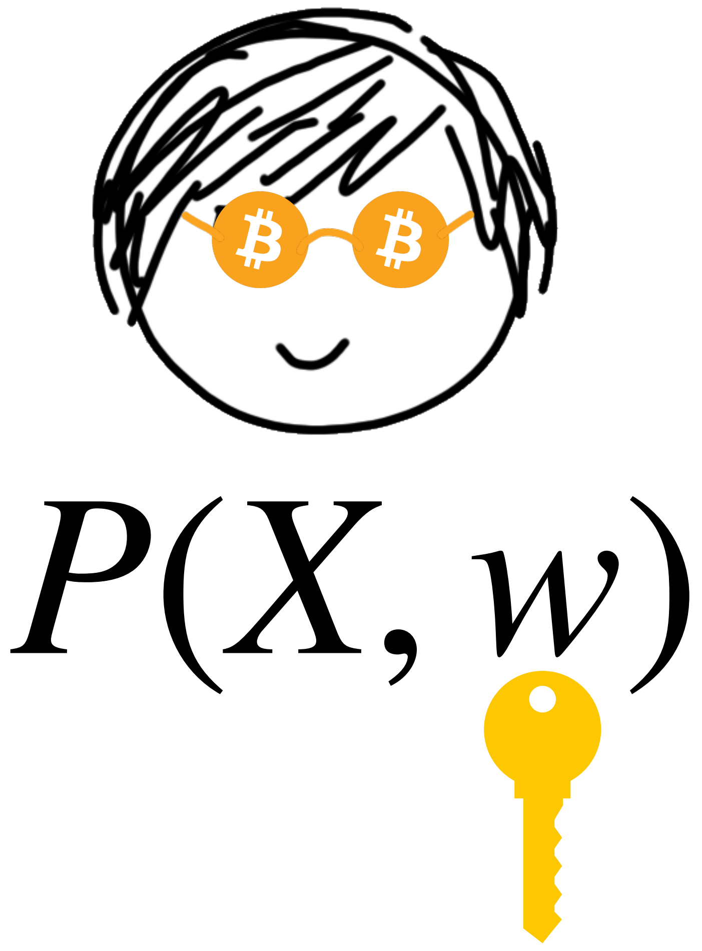

$P(X, w)$

$X =$

$$\xrightarrow{\quad a \quad}$$

$$\xleftarrow{\quad e' \quad}$$

Ext

$e \quad z$

Σ Protocols

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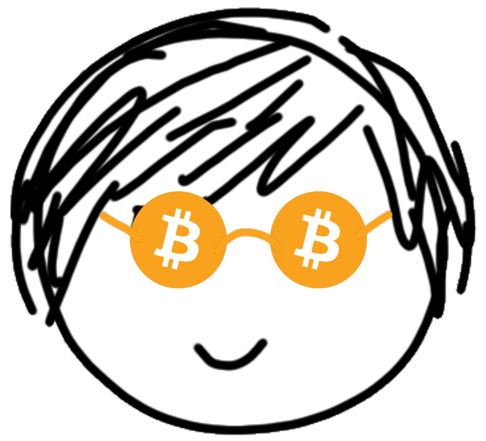

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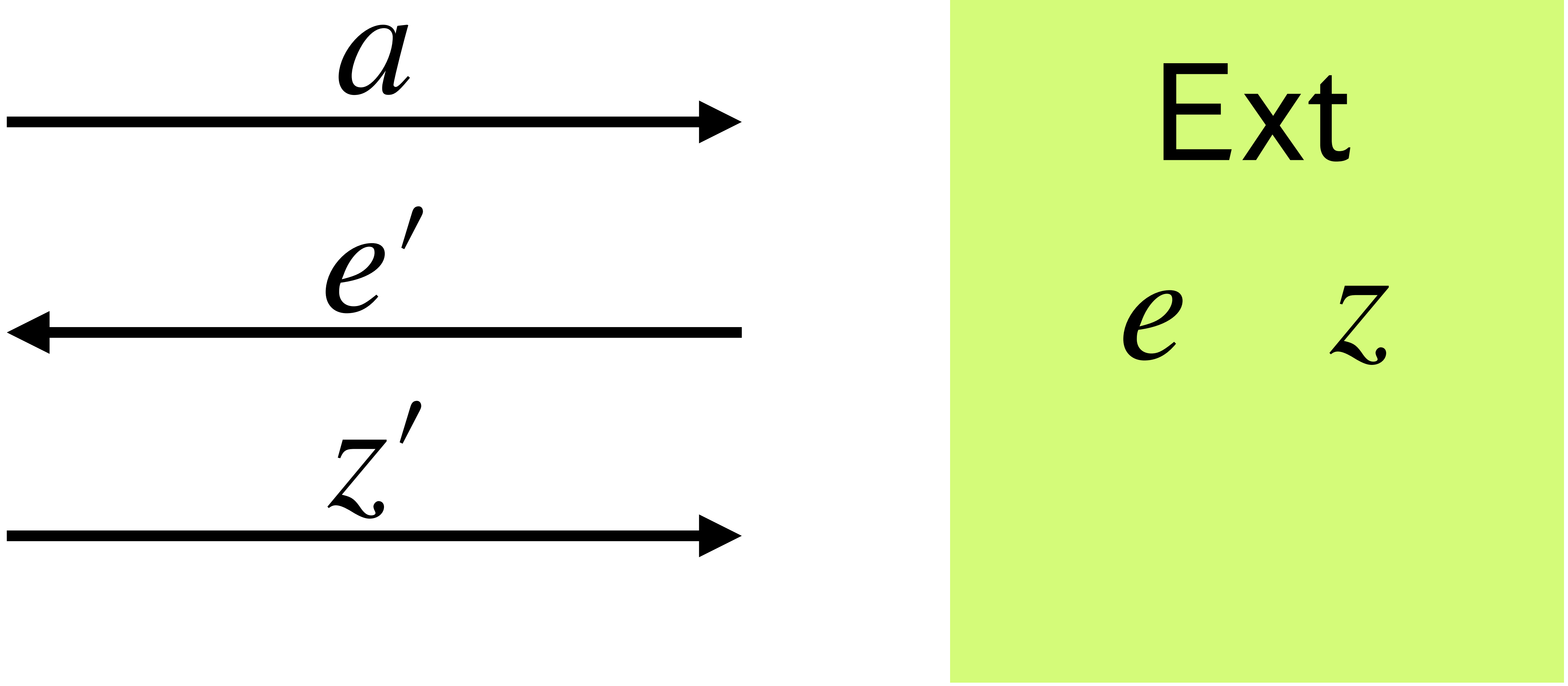

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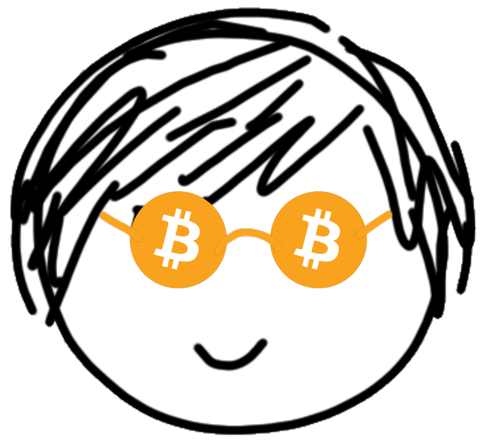

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$e \quad z$

$e' \quad z'$

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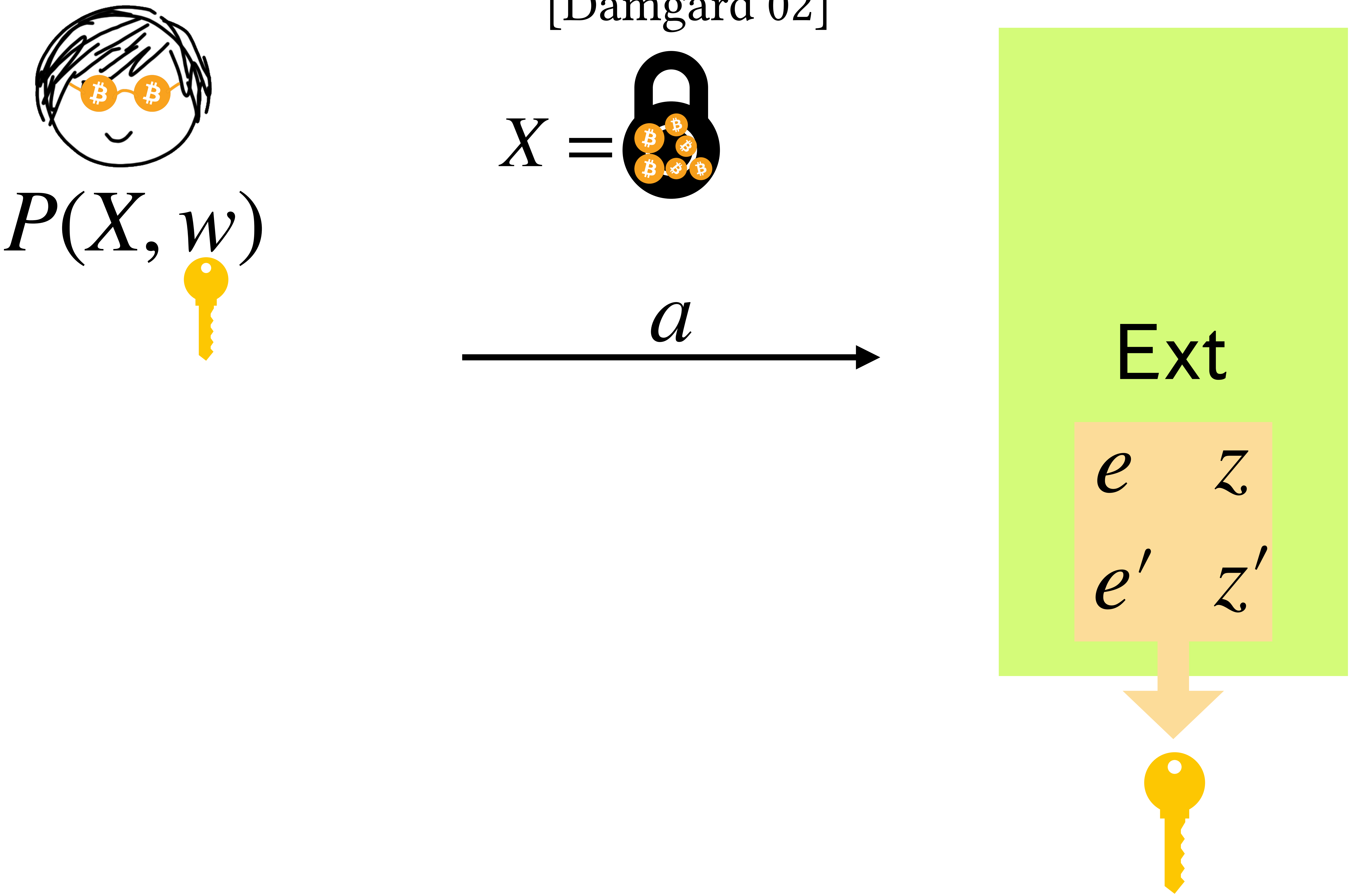

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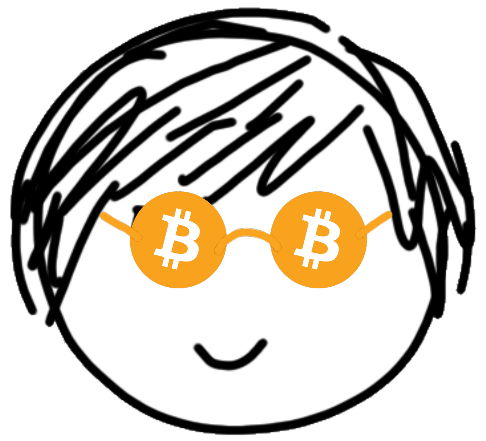

$P(X, w)$

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$$\xrightarrow{\quad a \quad}$$

Ext

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$e' \quad z'$

<u>Toy example</u>

$z = we + f(a)$

$z' = we' + f(a)$

solve for w

Σ Protocols

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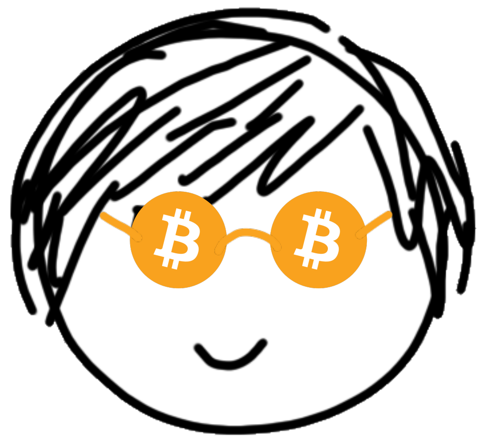

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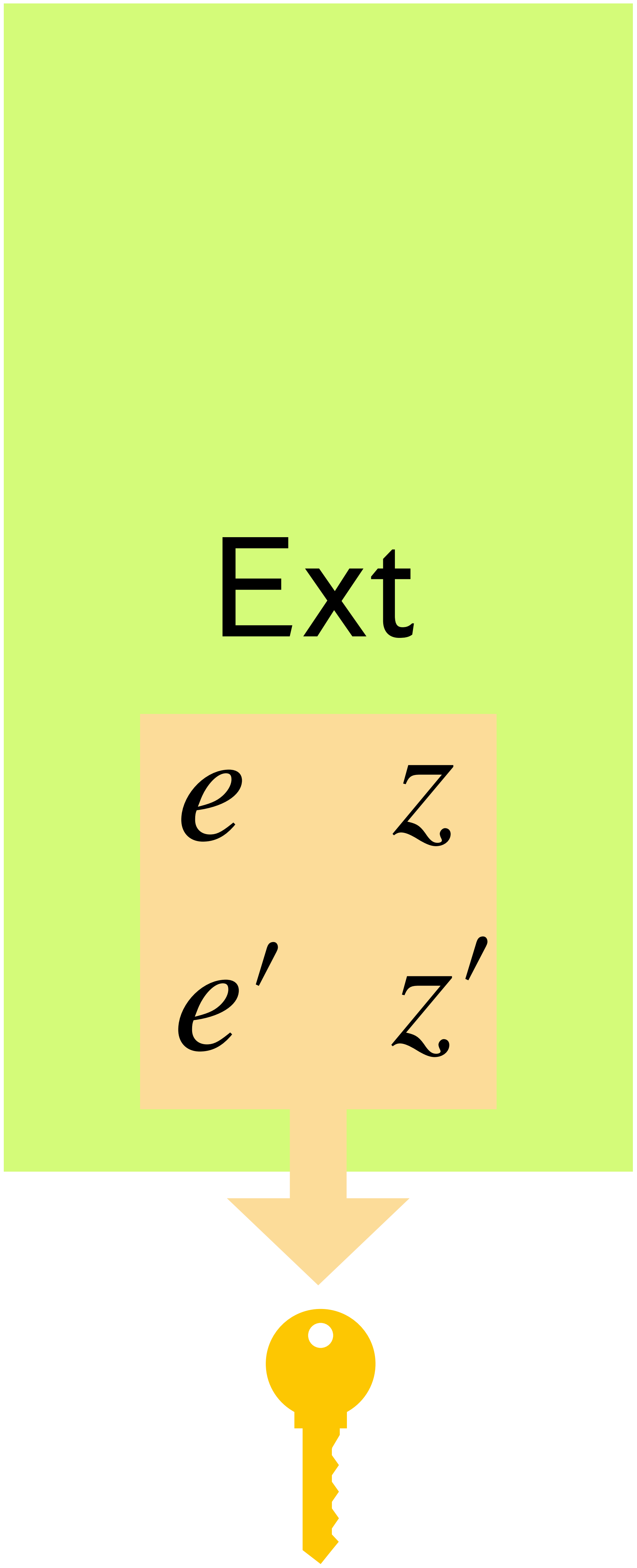

Ext

$e \quad z$

$e' \quad z'$

This is a useful protocol feature to keep in mind

<u>Toy example</u>

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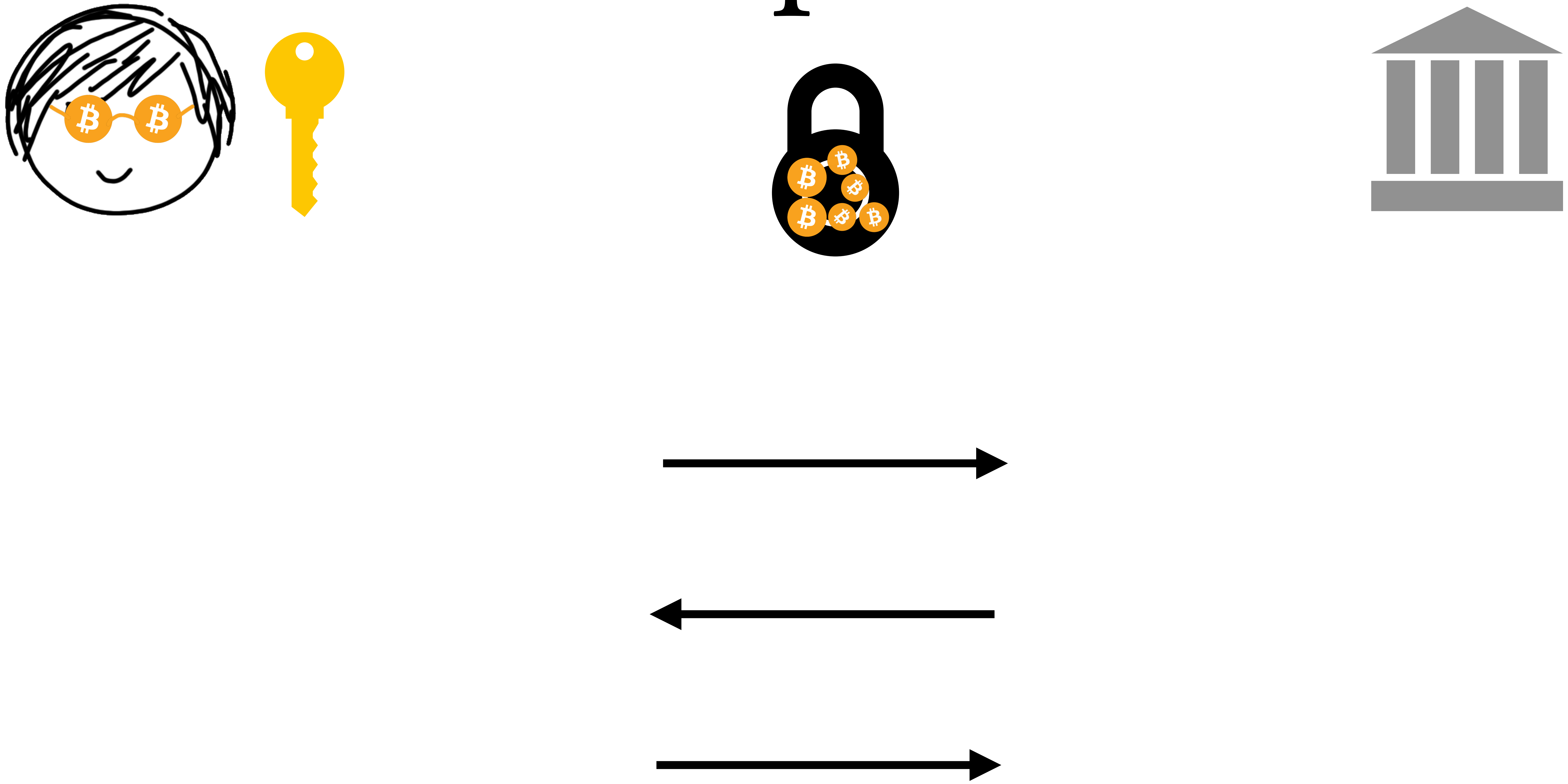
Composable Non-interactive Zero-knowledge Proofs in the Random Oracle Model

Composable?



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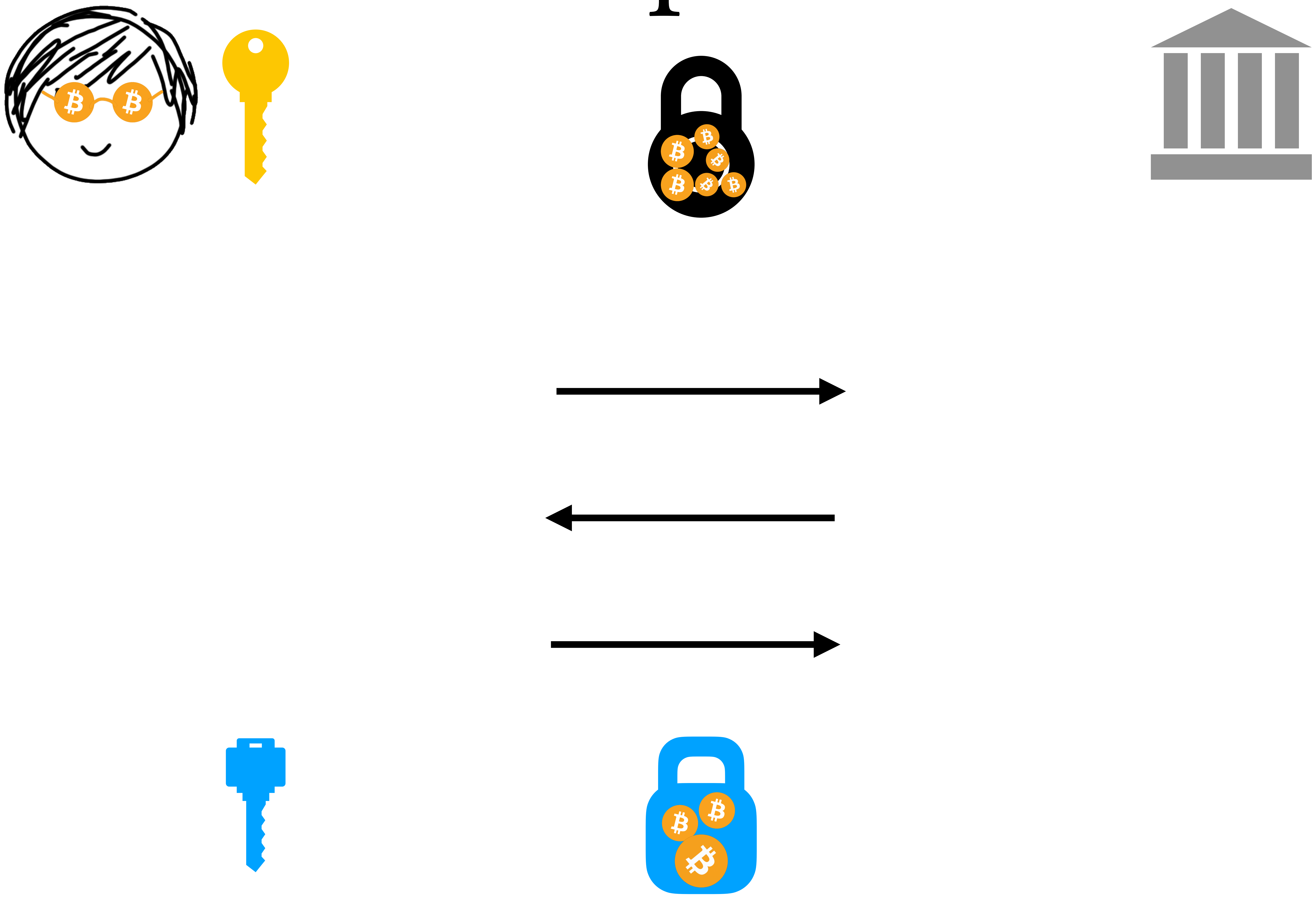

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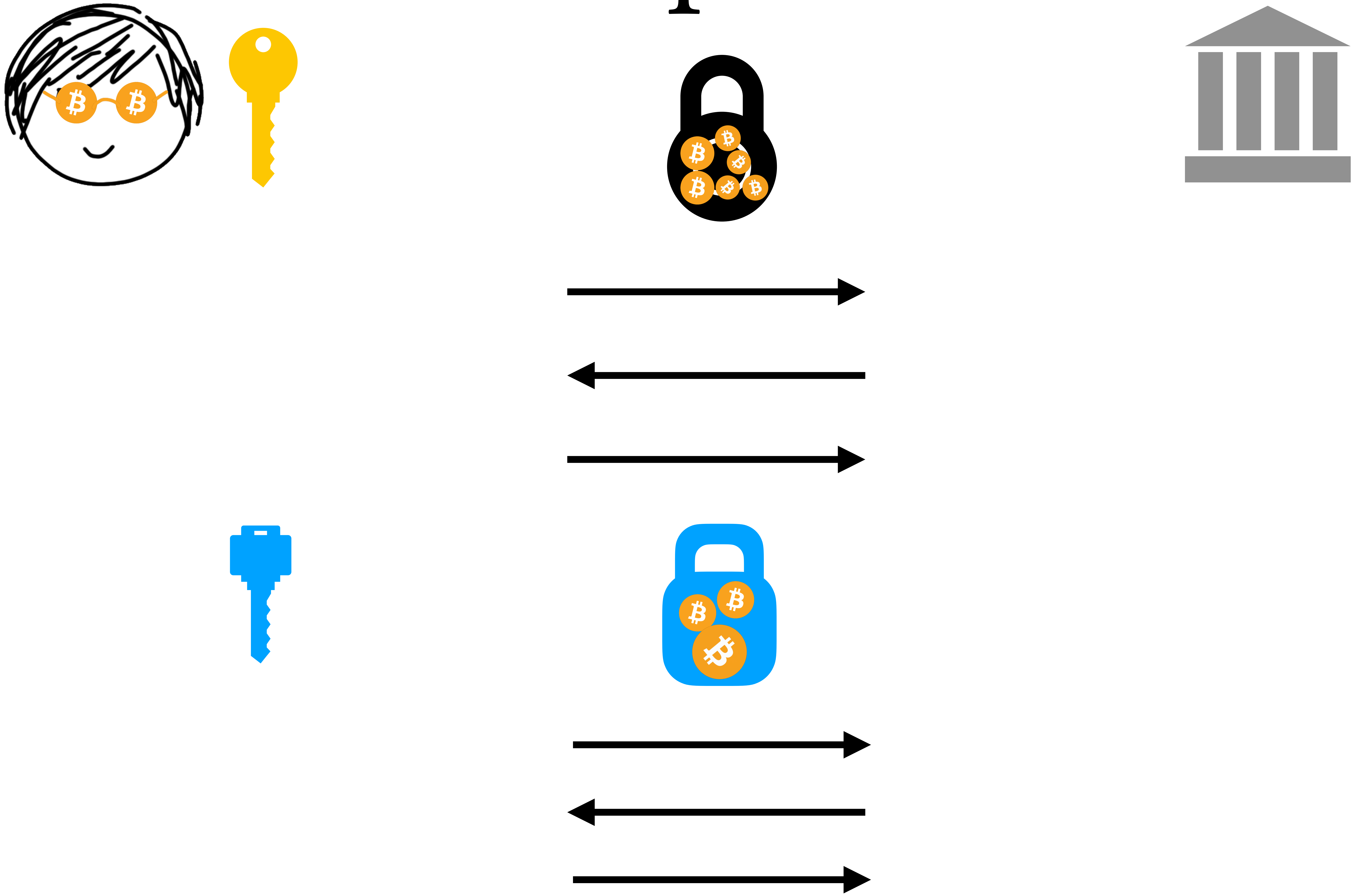

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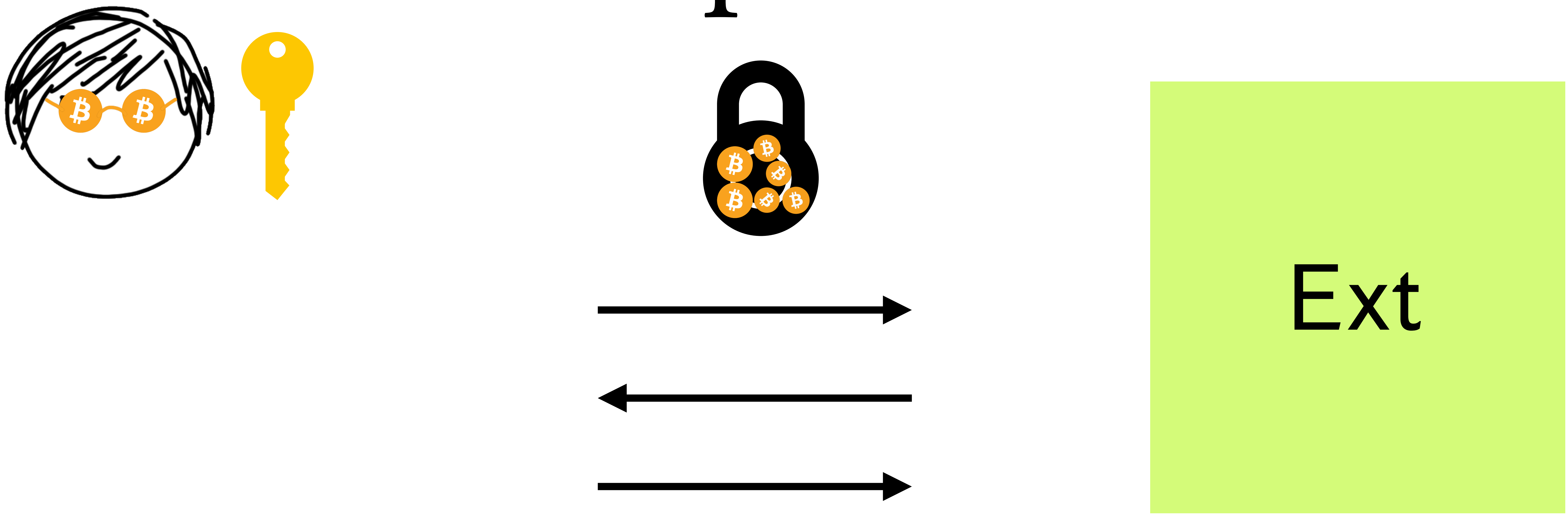

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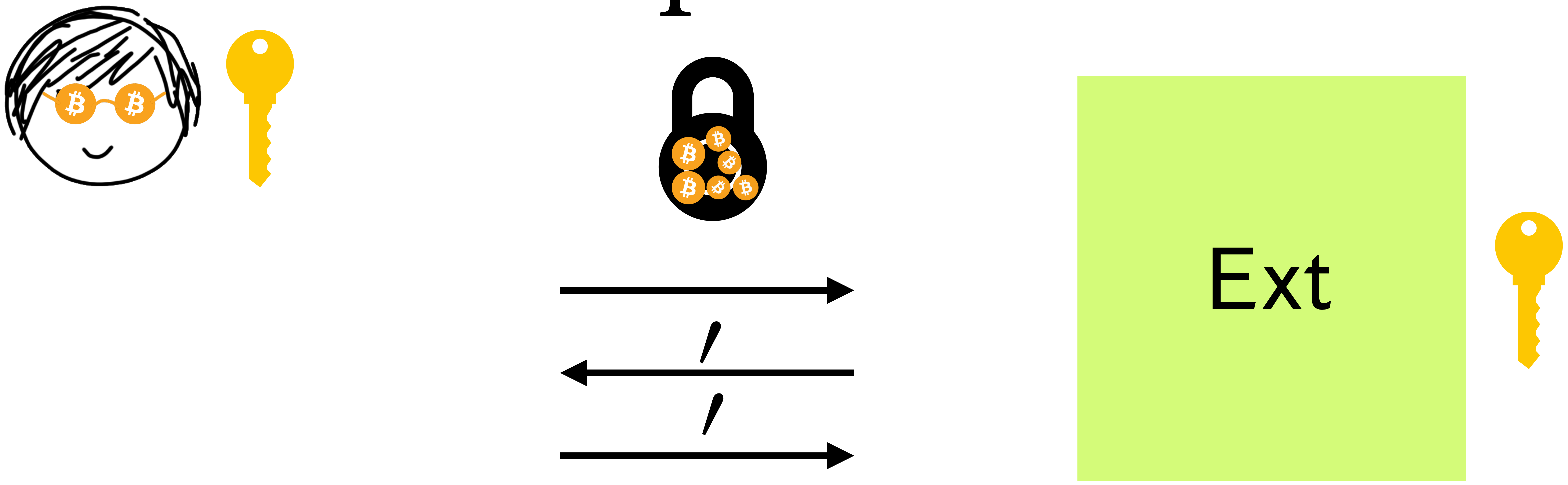

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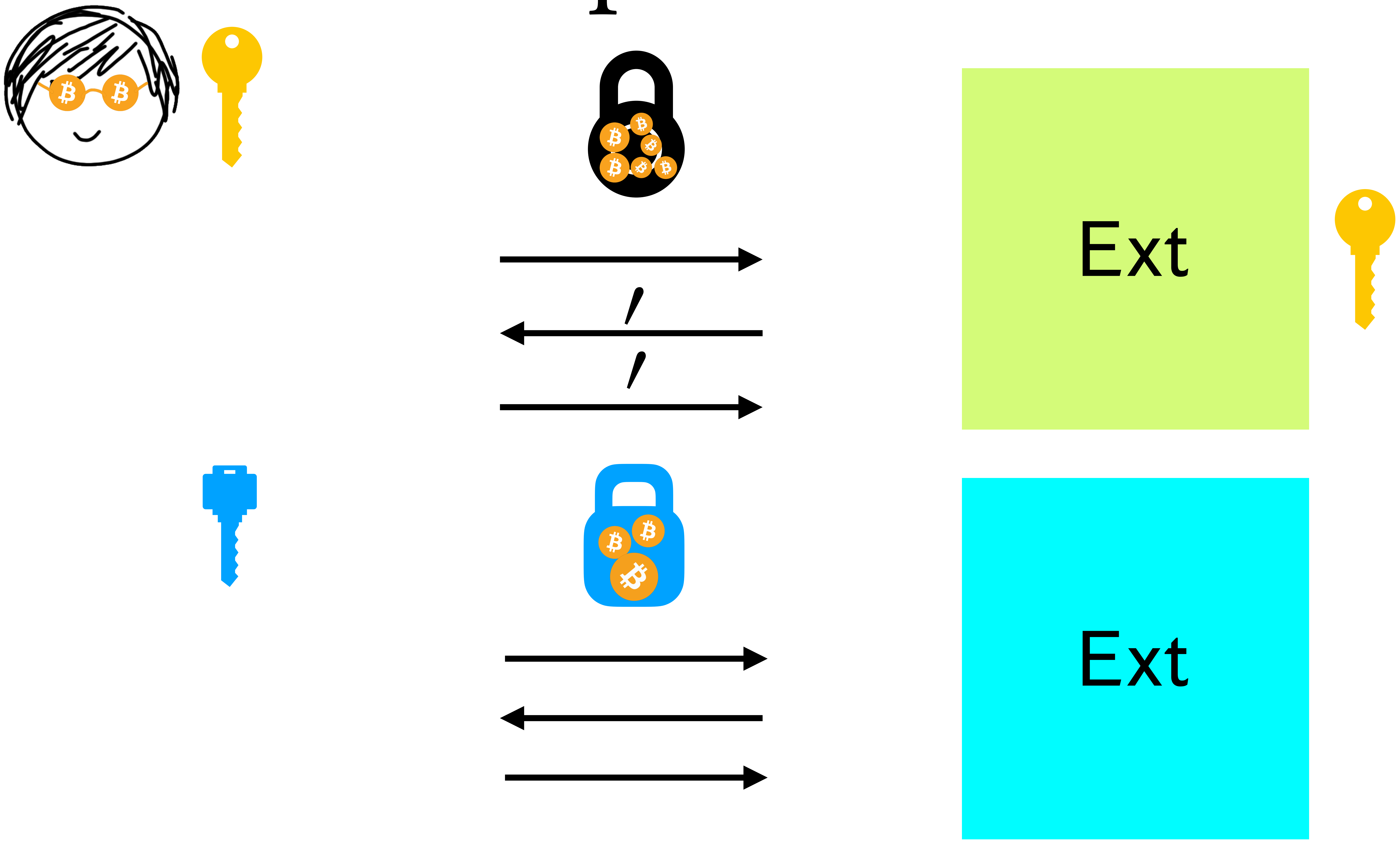

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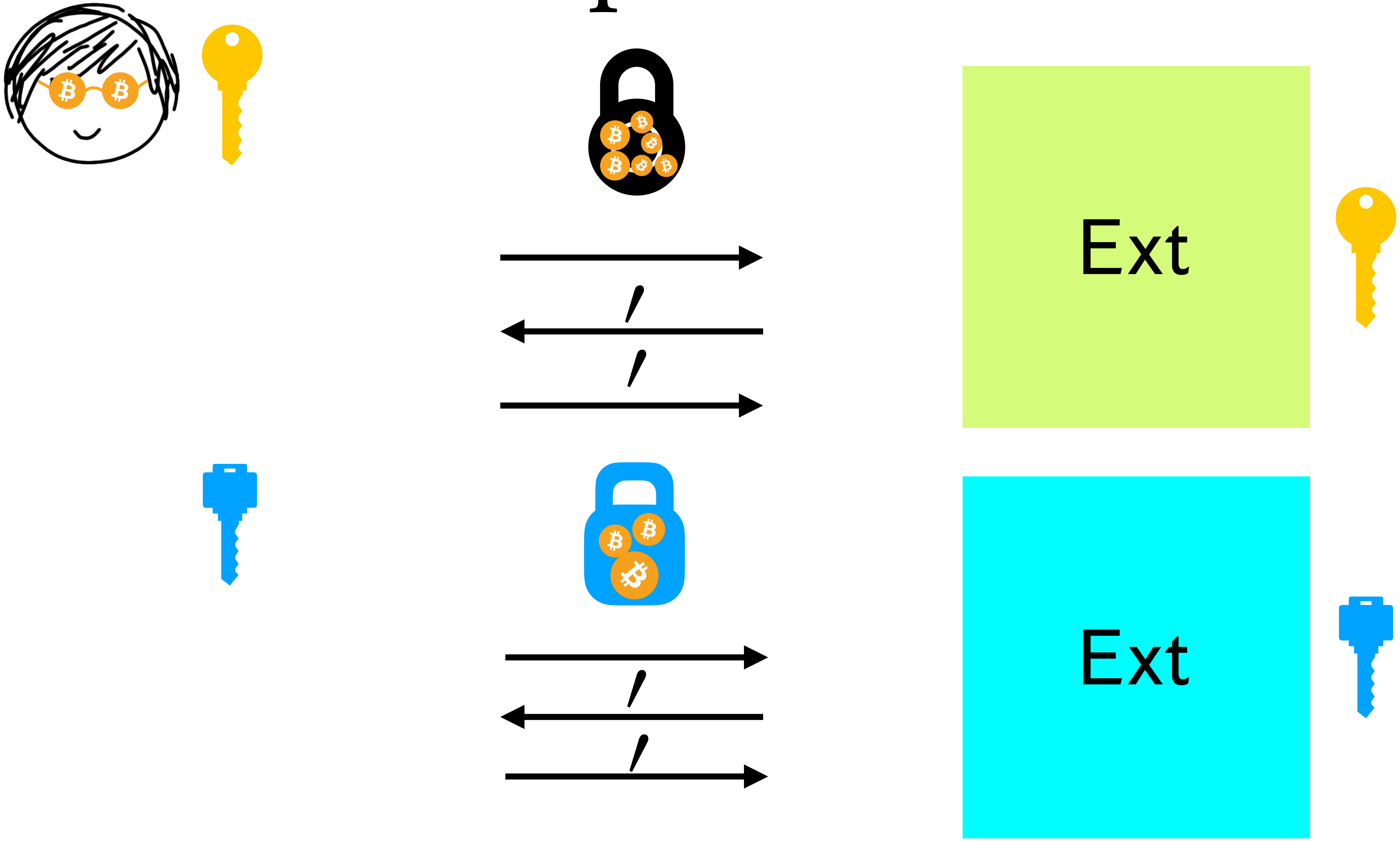

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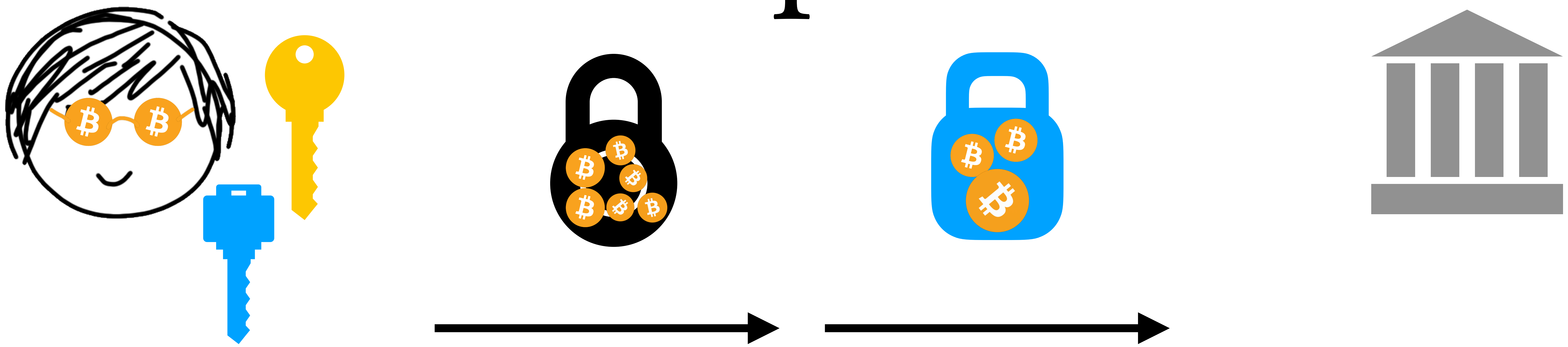

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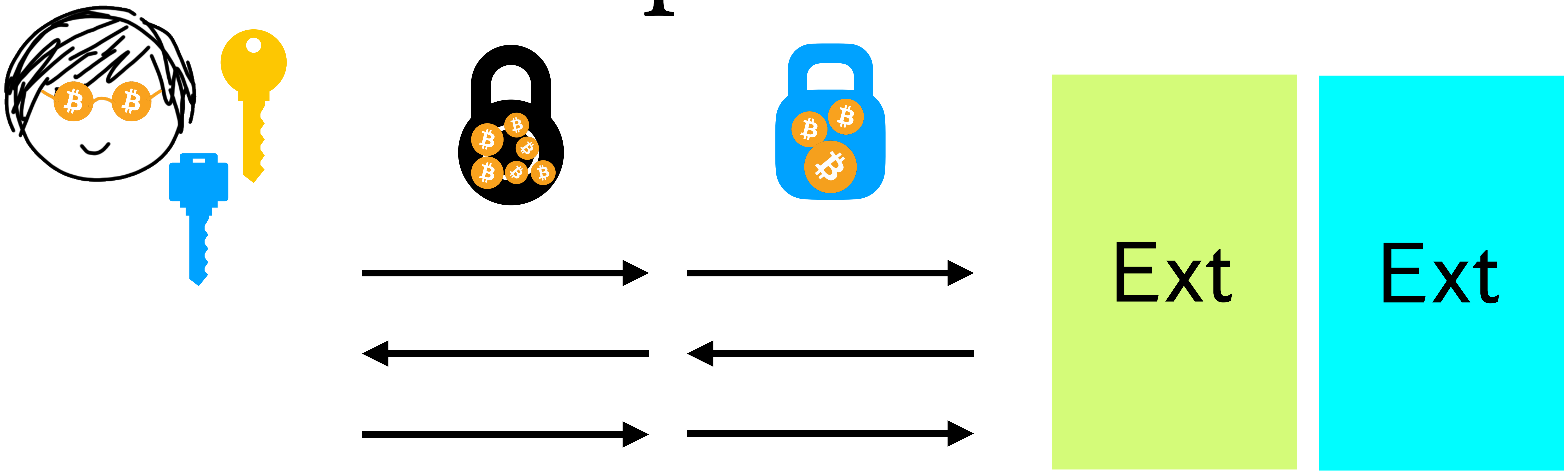
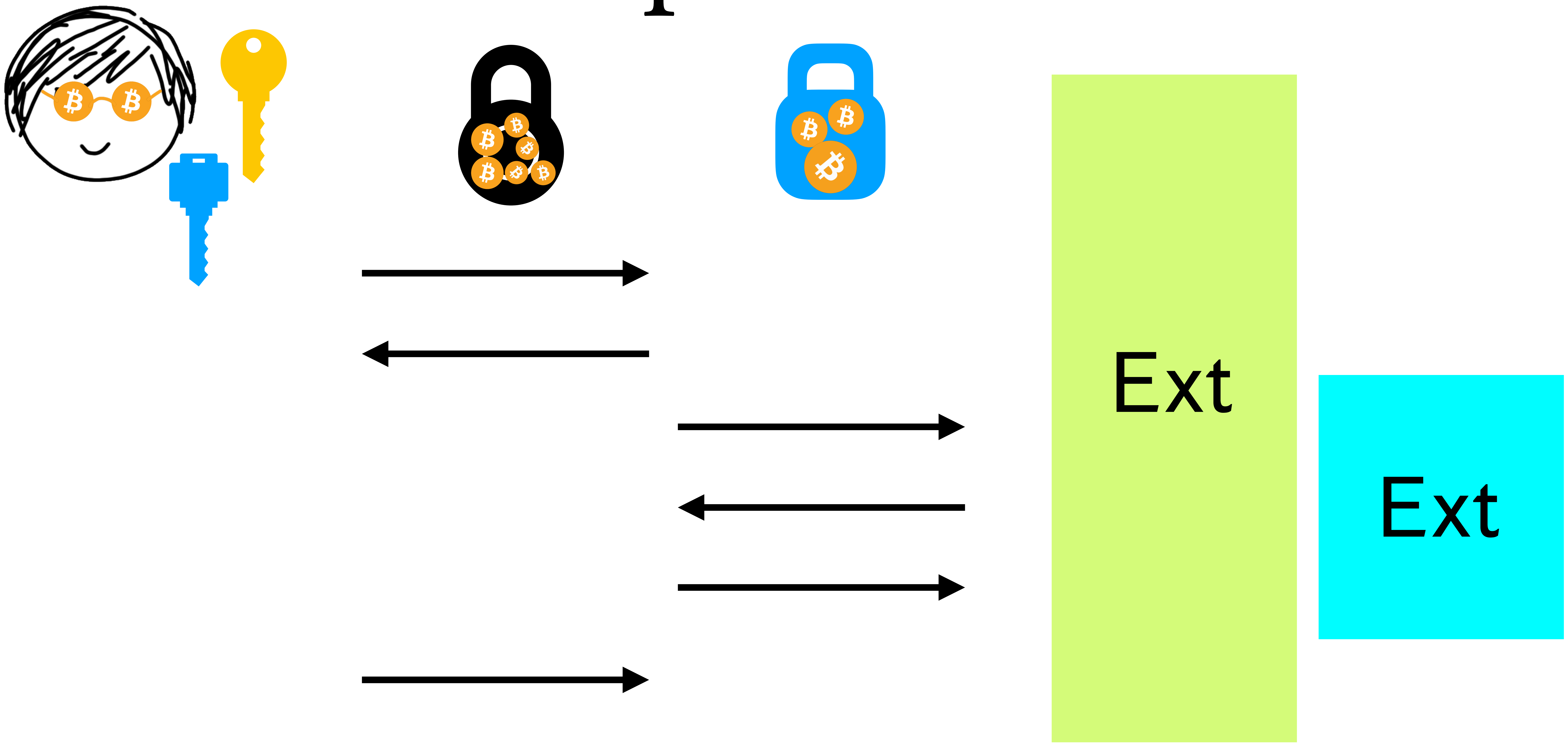


Ext

Ext

Composable?



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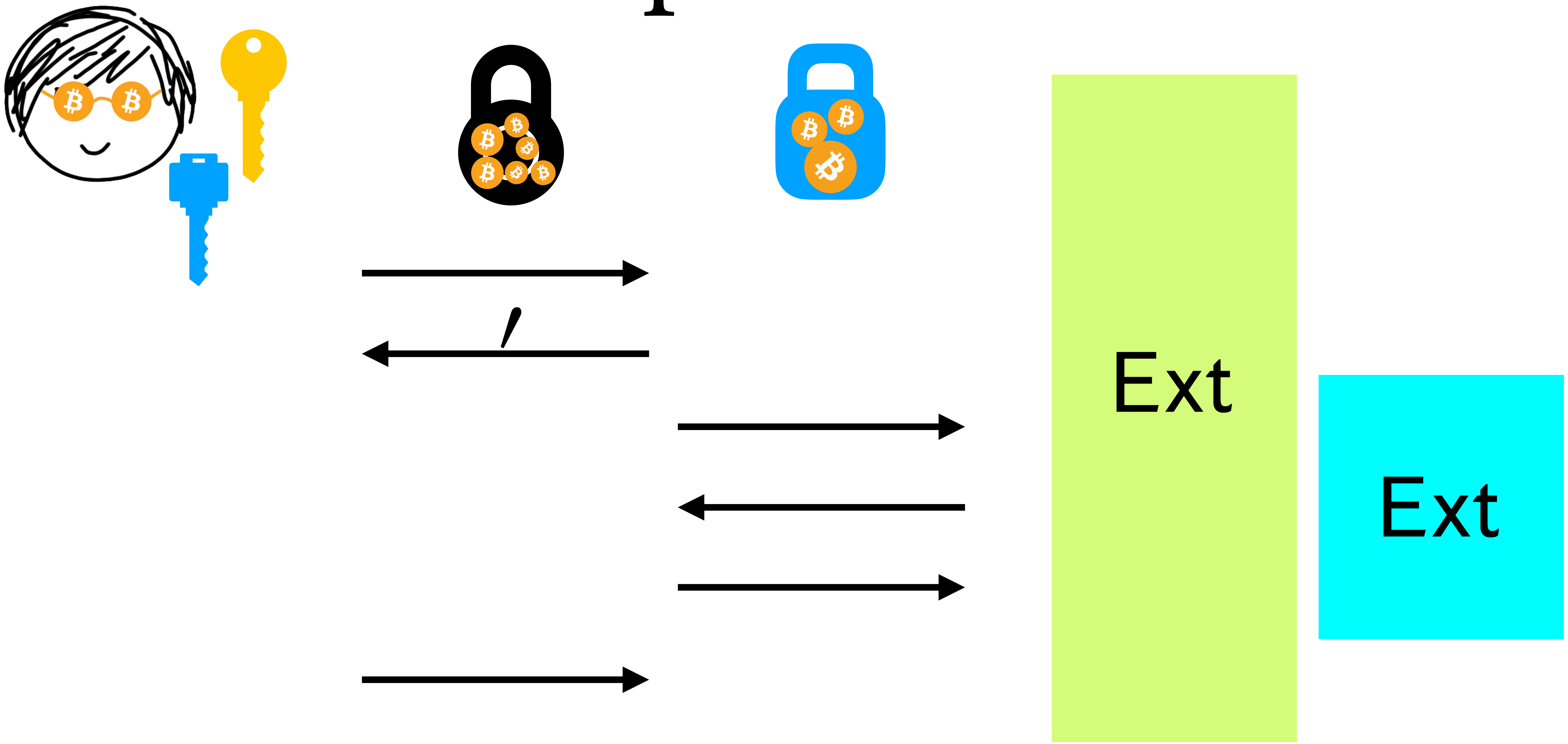

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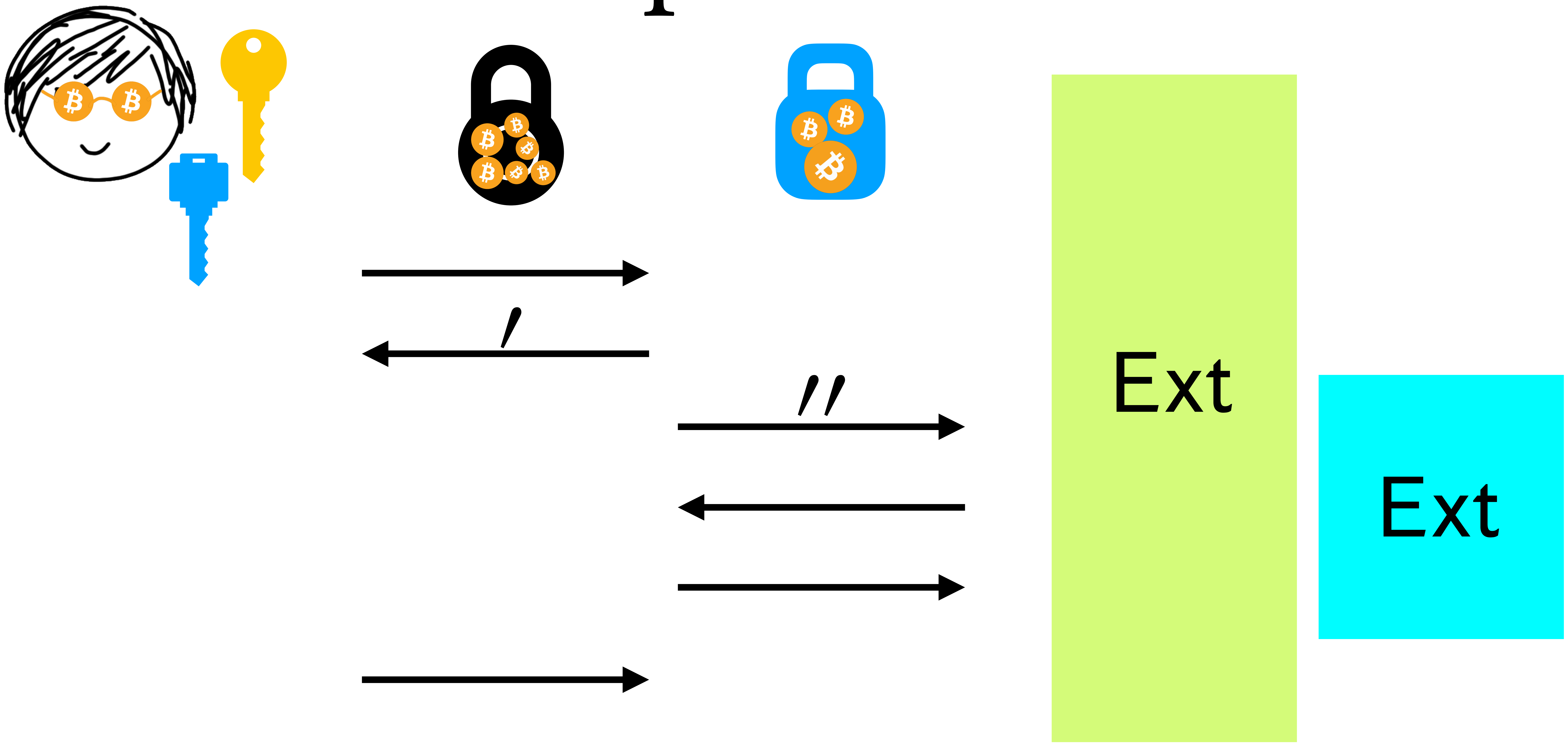

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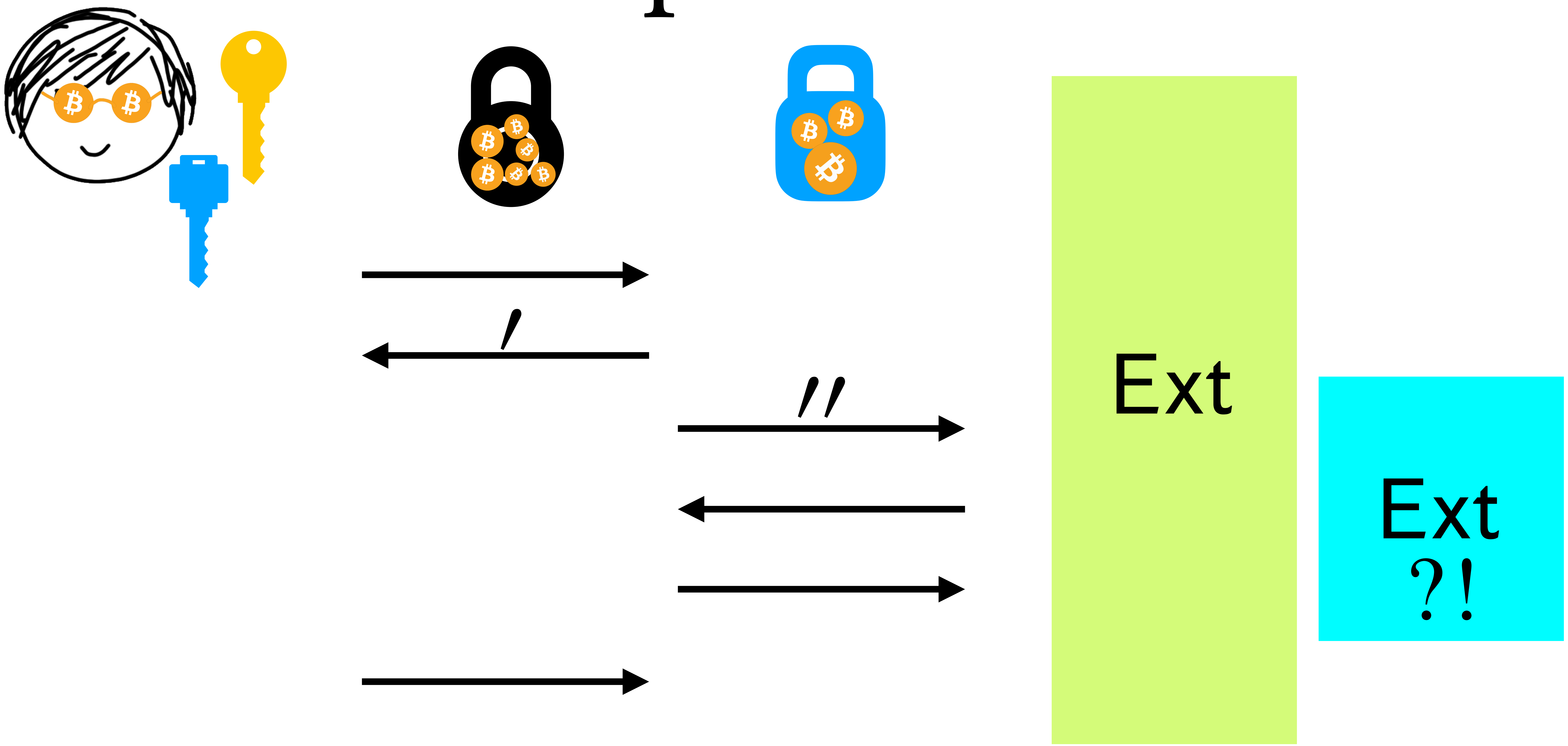

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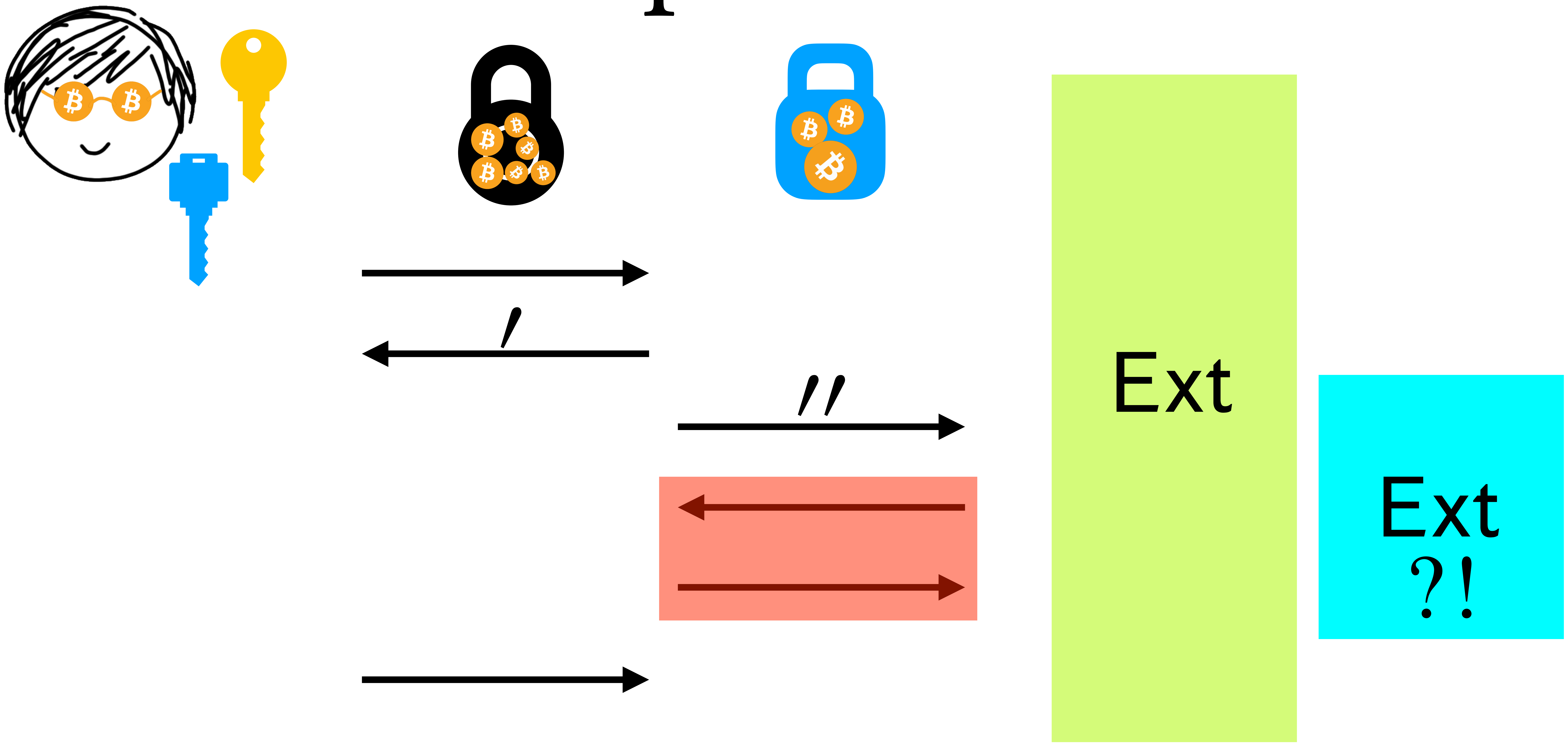

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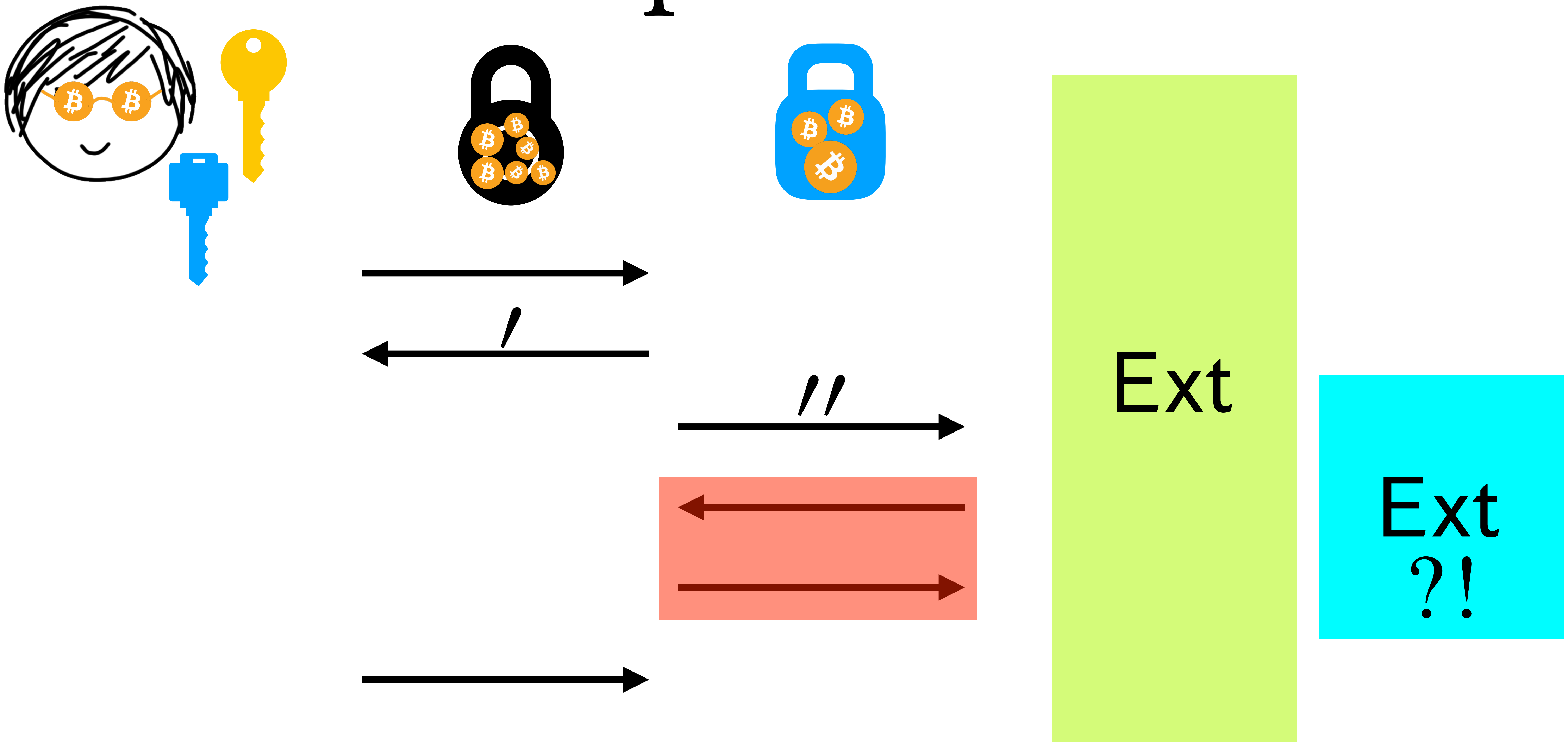


Rewinding extraction strategies are bad for concurrent composition

Straight-line Extraction

- What special privileges can we grant Ext that compose nicely?

- One option is a “Common Reference String”
  - i.e. system parameter for which Ext has a backdoor
  - Well studied, theoretically sound
  - Unsatisfying in practice; trusted generator needed

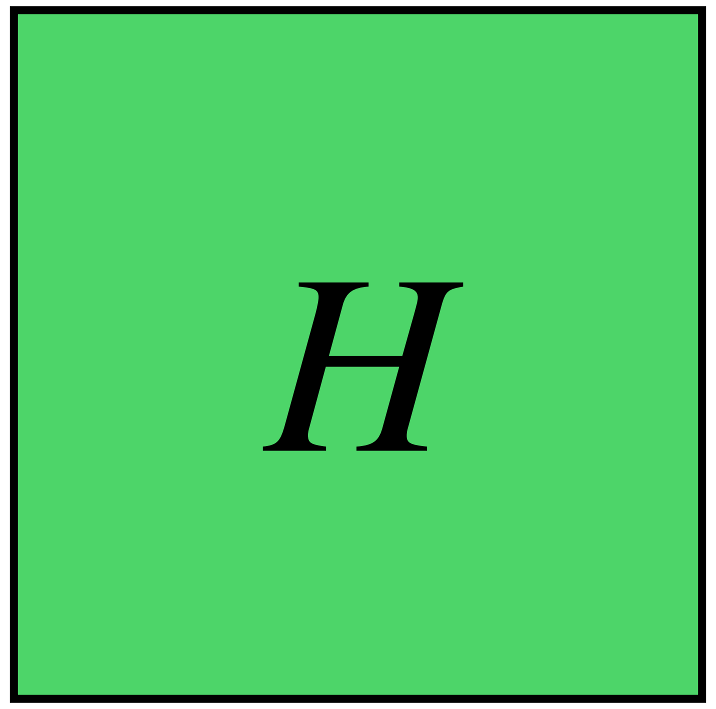
Composable

Non-interactive

Zero-knowledge Proofs

in the Random Oracle Model

Random Oracle Model



H

$$H : \{0,1\}^* \mapsto \{0,1\}^\ell$$

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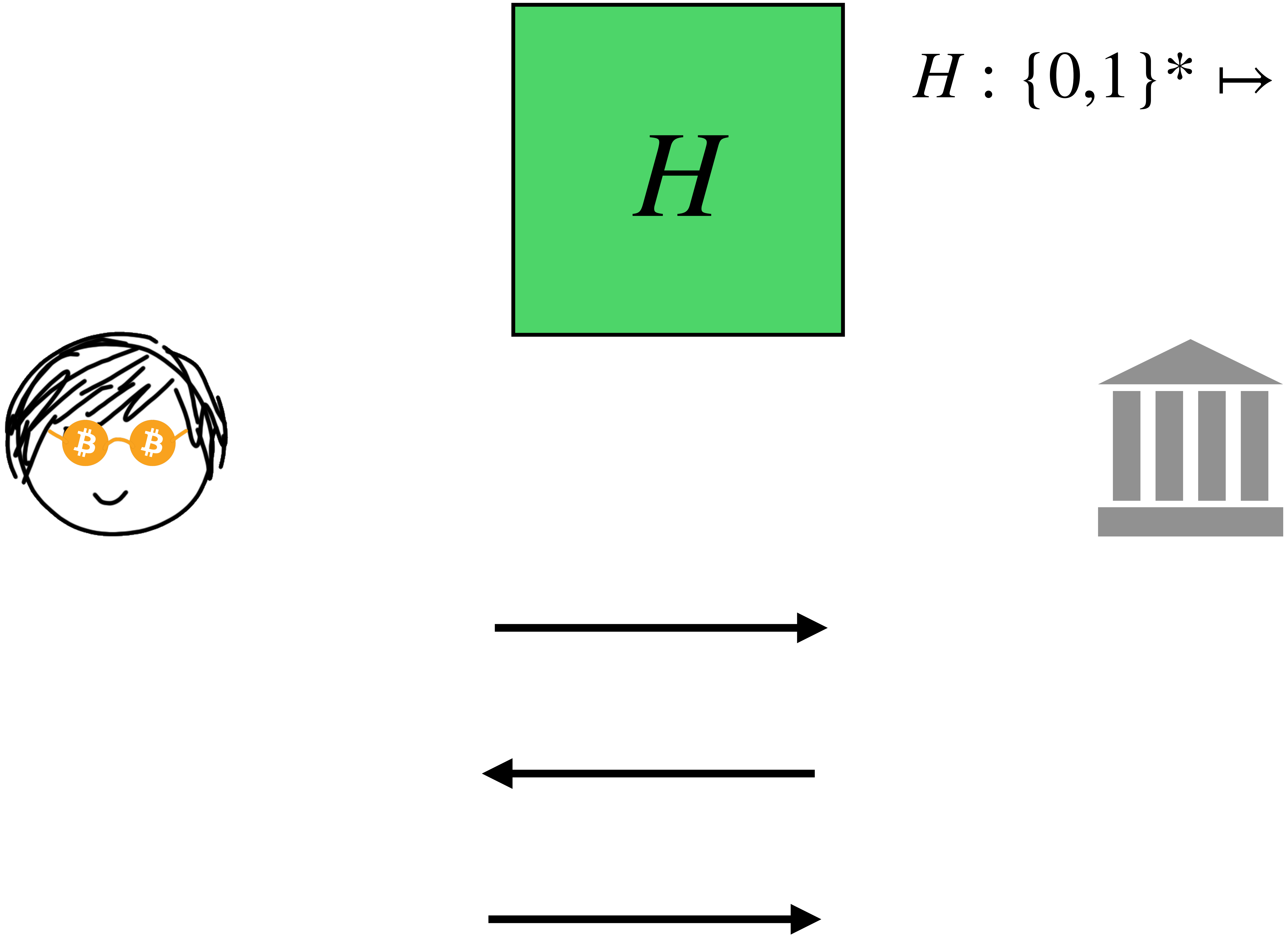


$H : \{0,1\}^* \mapsto \{0,1\}^\ell$

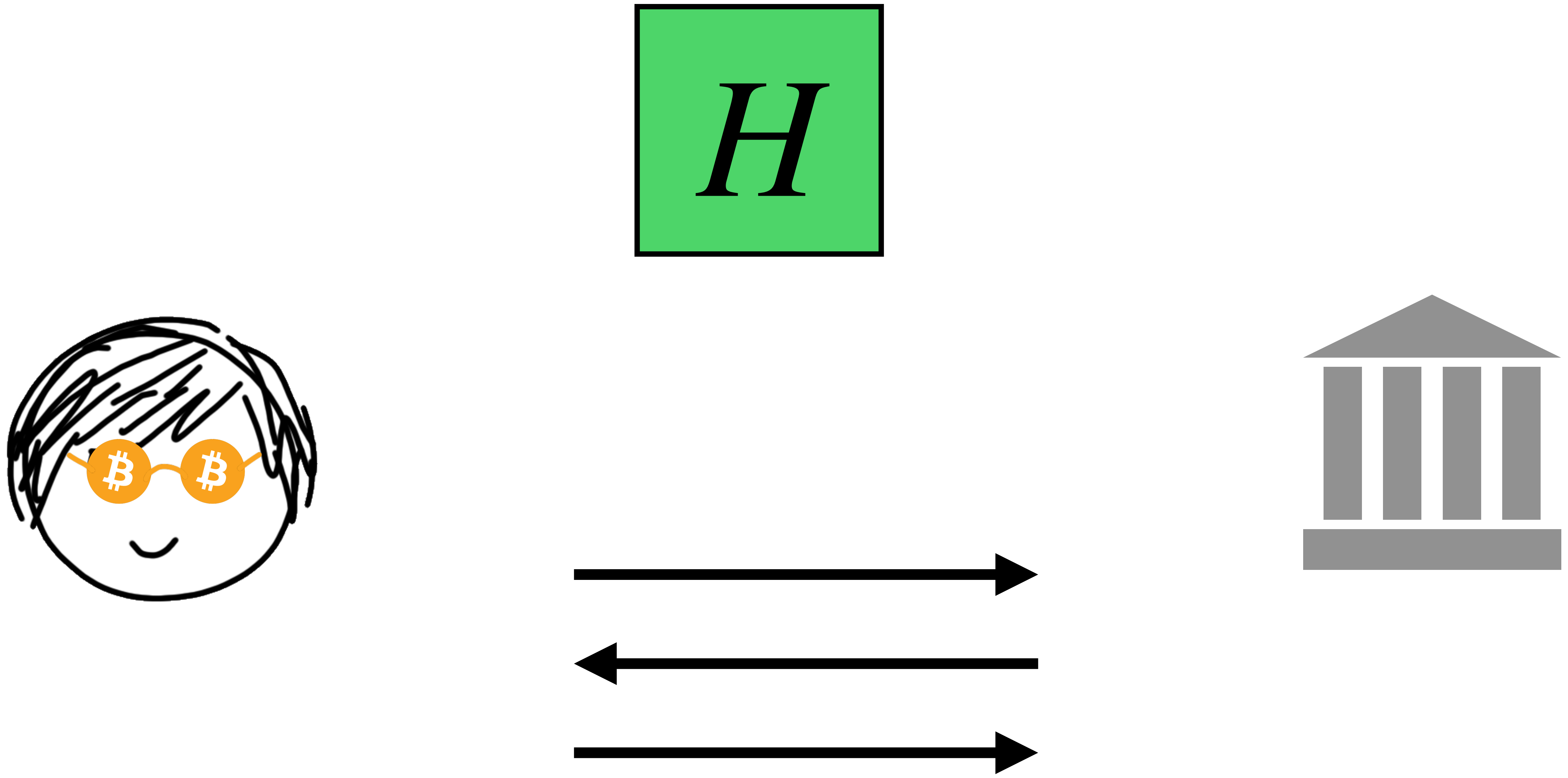
Random Oracle Model

- Began as a heuristic to analyze protocols that use cryptographic hash functions

- Developed as a methodology to design efficient protocols with meaningful provable guarantees

- <u>Intuition</u>:
  - Cryptographic hashes are complex and highly unstructured
  - Unless you evaluate $H(x)$ from scratch, it looks random

Random Oracles as Ext Privilege



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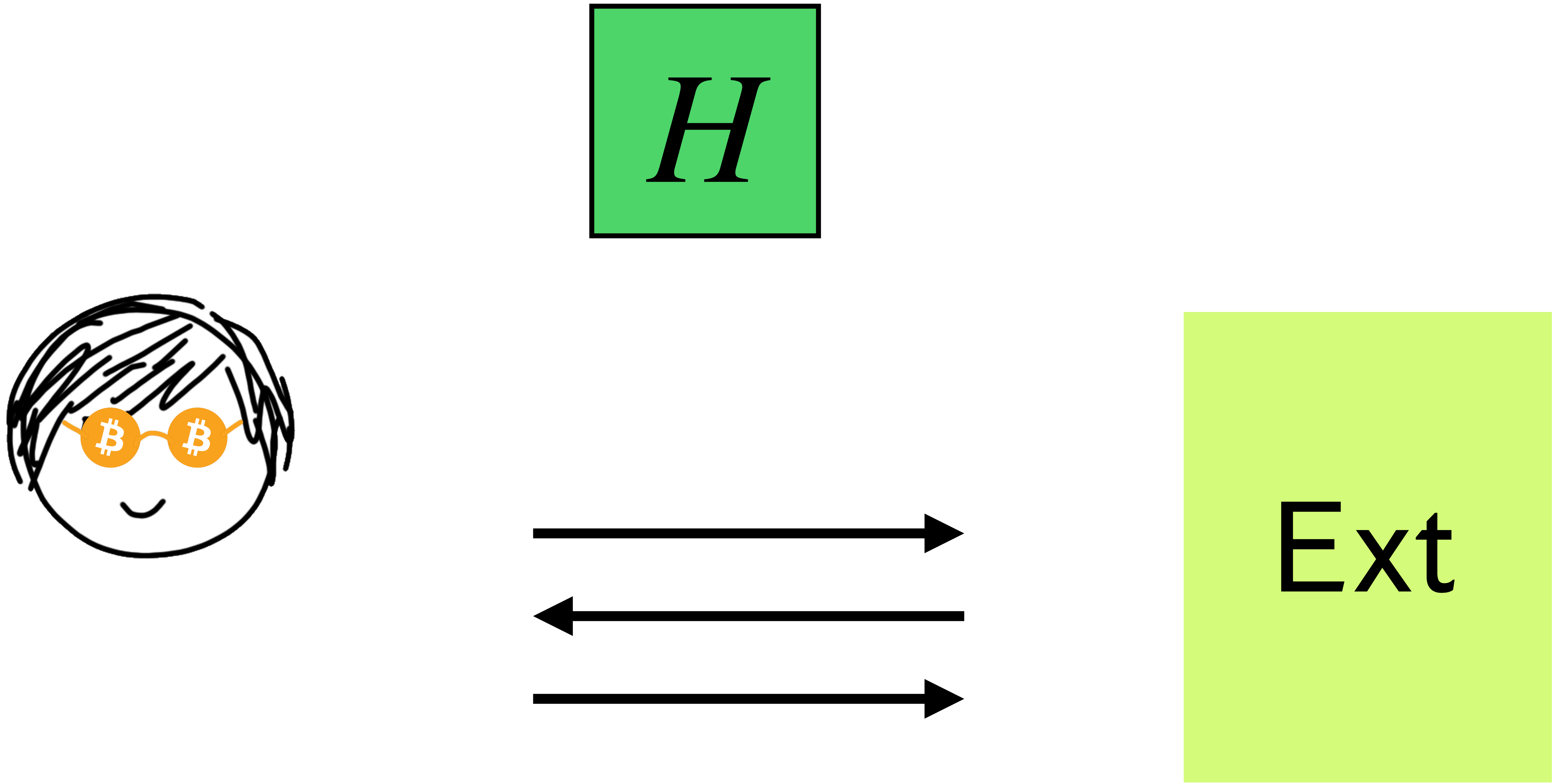
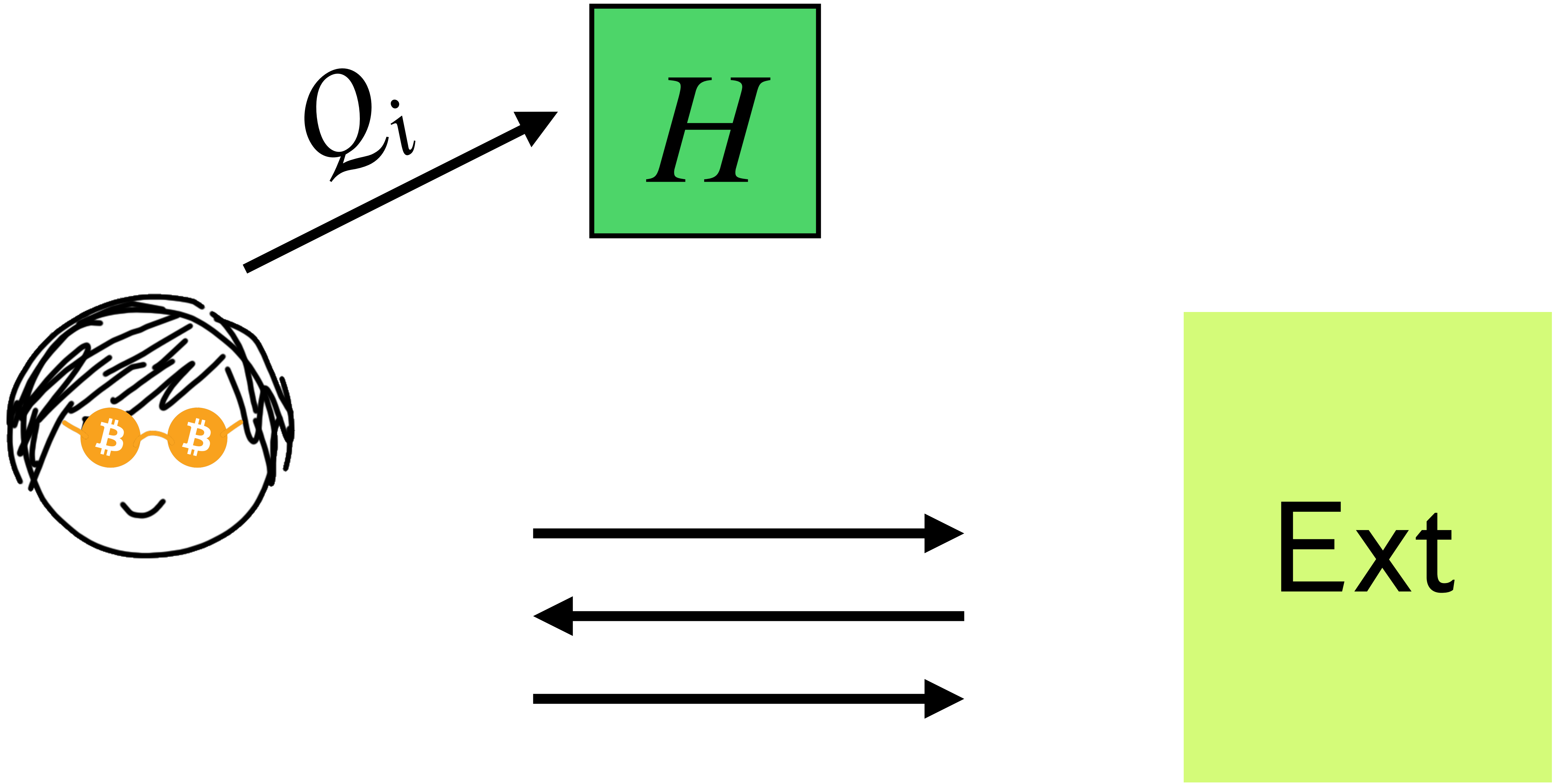


H

Ext

Random Oracles as Ext Privilege



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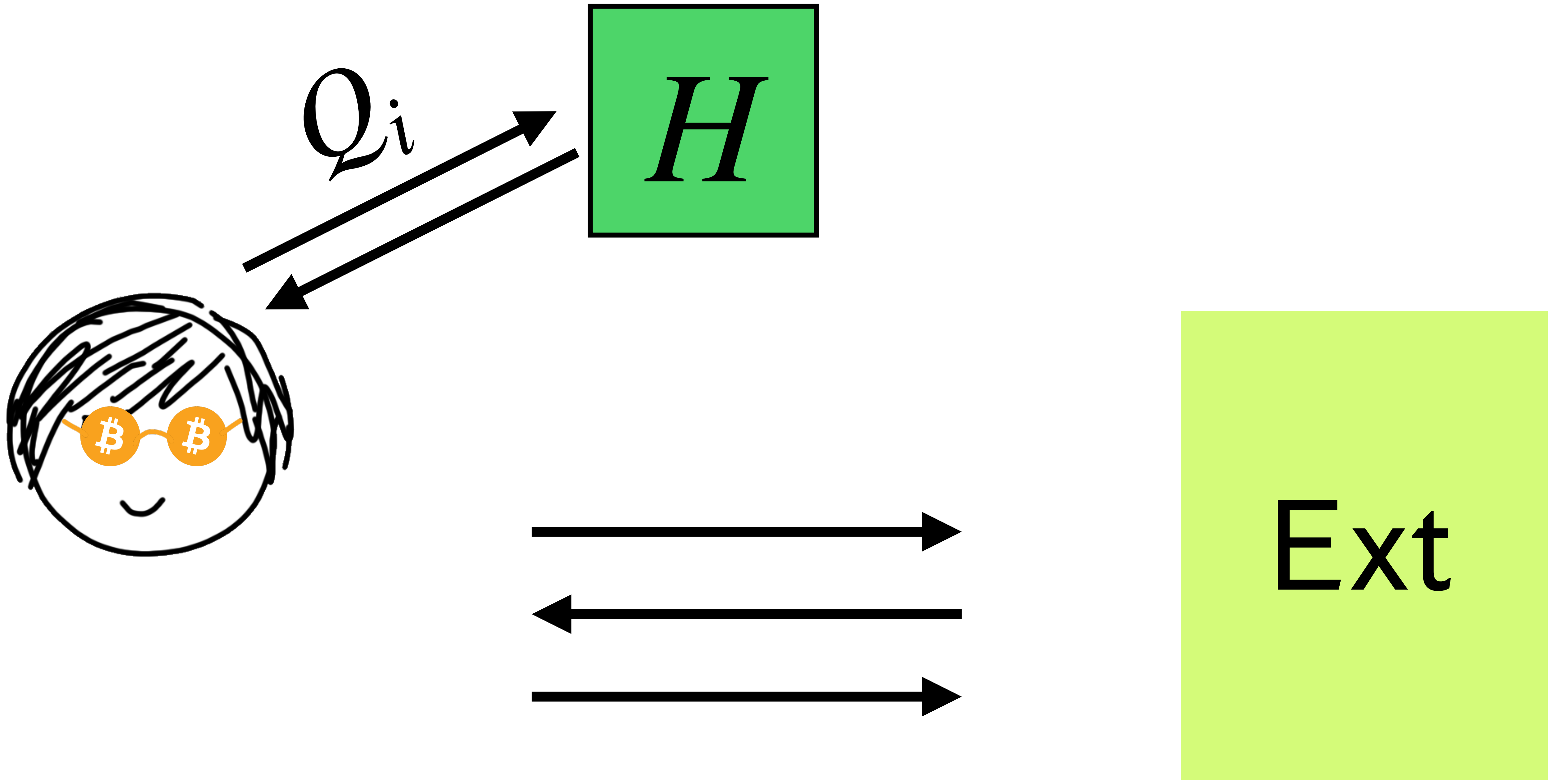

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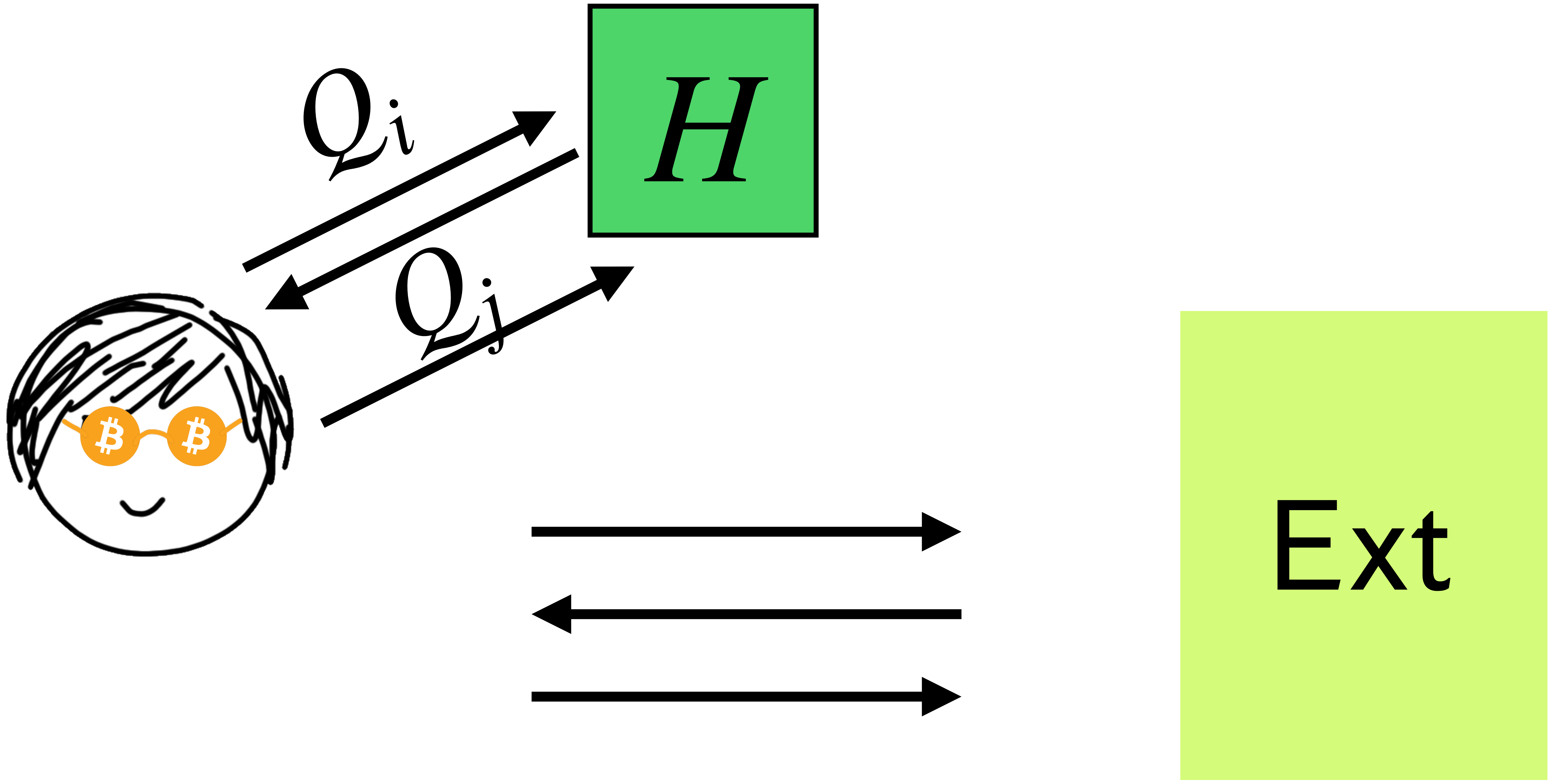

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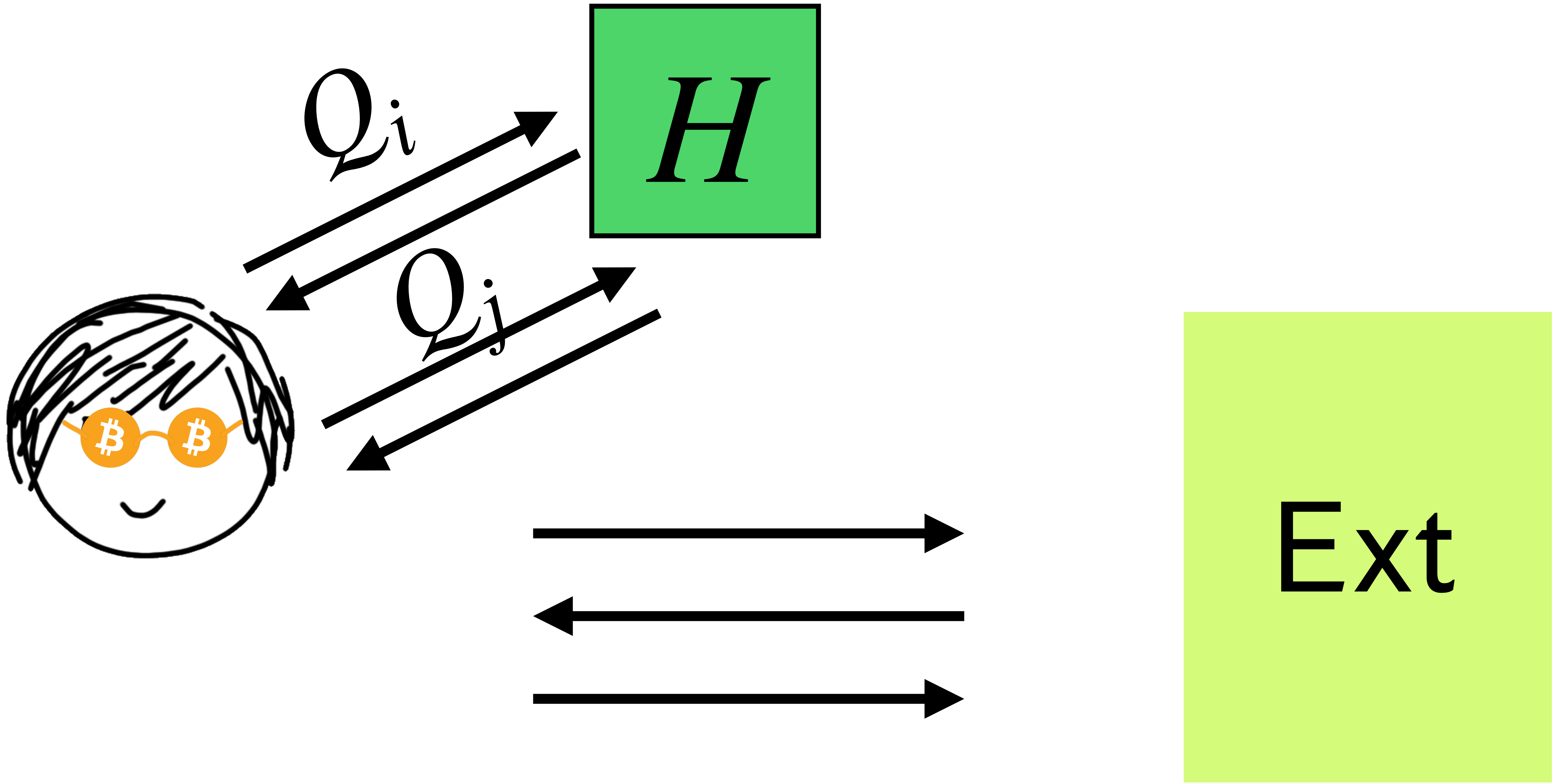

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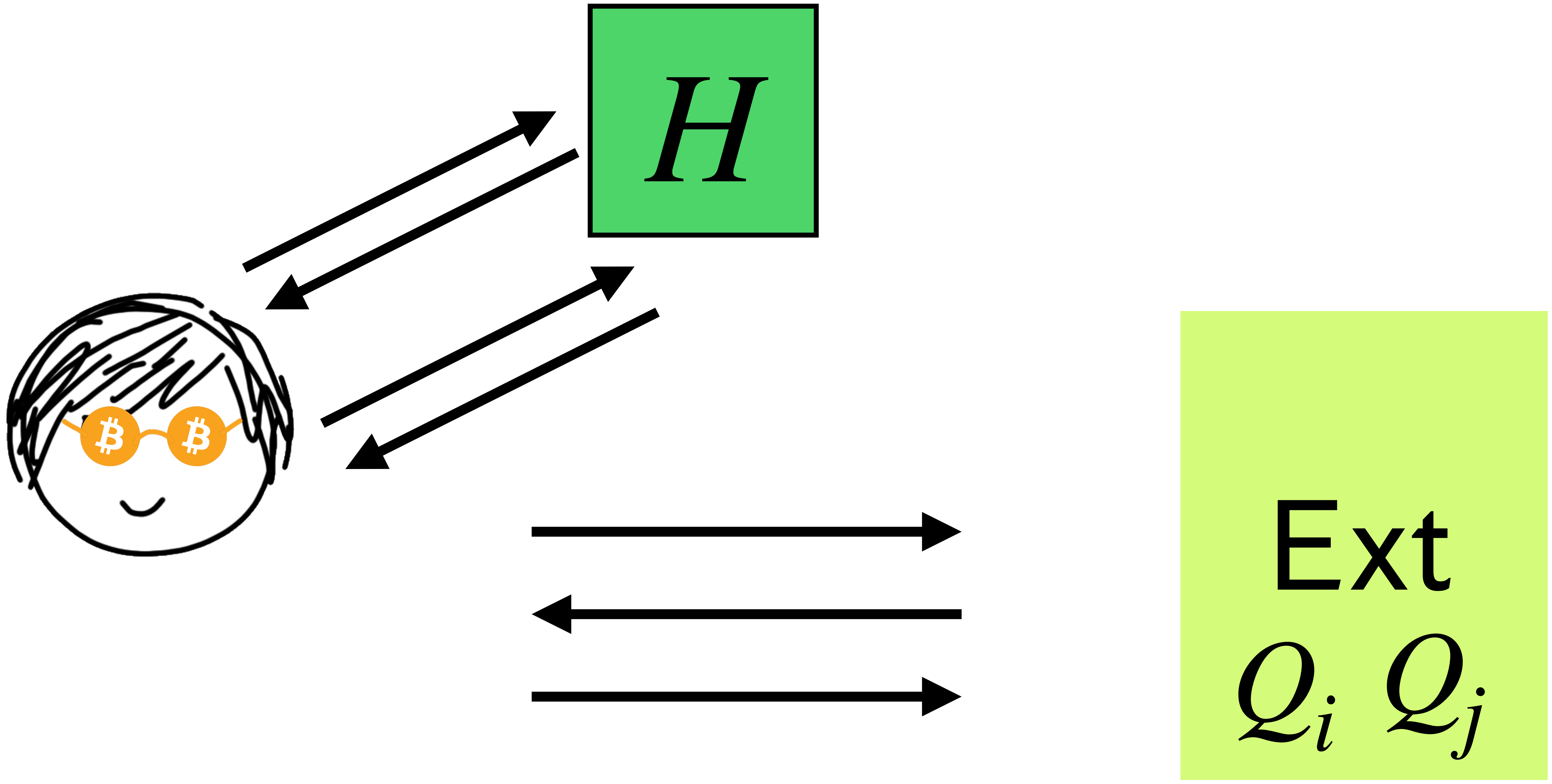
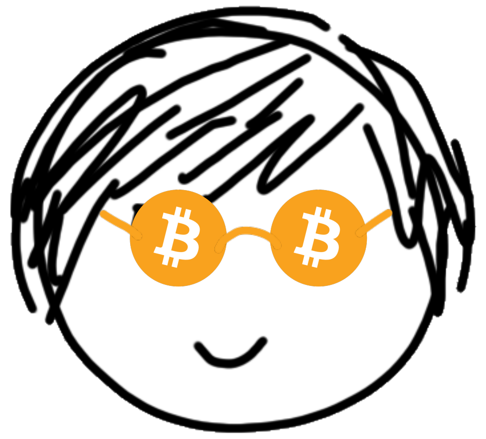
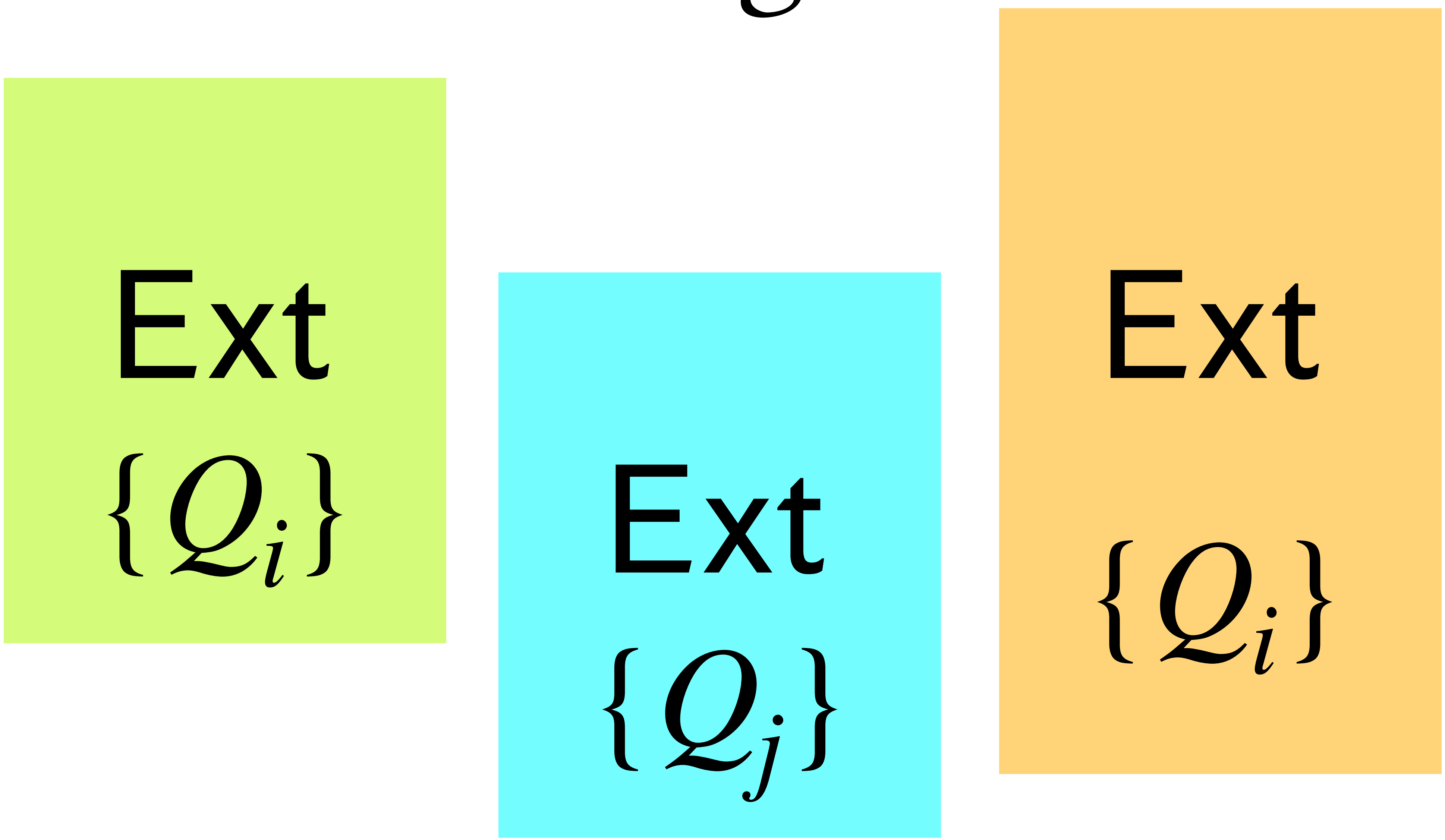


Random Oracles as Ext Privilege

- Bob "knows" all of the $\{Q_i\}$ values queried to H
- Ext could obtain useful information from $\{Q_i\}$
- $\{Q_i\}$ can be obtained without rewinding





Ext $\{Q_i\}$

Ext $\{Q_j\}$

Ext $\{Q_i\}$

Composable

Non-interactive

Zero-knowledge Proofs

in the Random Oracle Model

Non-interactive

- As the name suggests, a non-interactive proof is a single message protocol

- Useful communication pattern for many applications

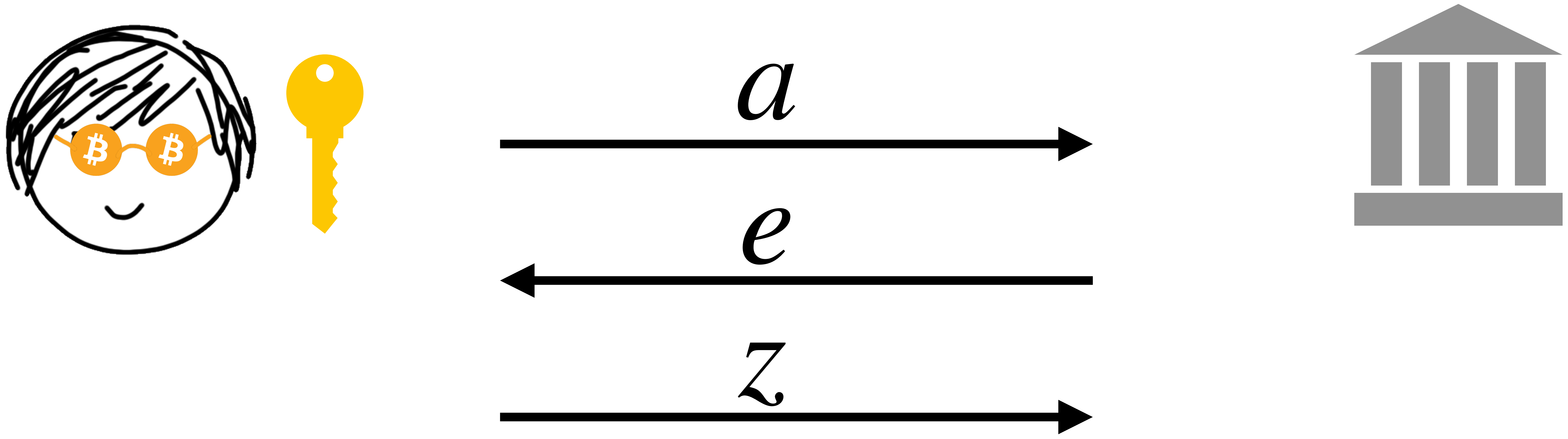
- Common methodology: compile Σ protocol

- [Pass 03] gave a simple straight-line extractable compiler in the random oracle model

Fischlin’s Compiler

- [Fischlin 05] gave a much more efficient compiler in the same model as [Pass 03]

- More interesting to analyze, and has remained the state of the art for $\Sigma \mapsto \mathsf{NIZK}$ compilers



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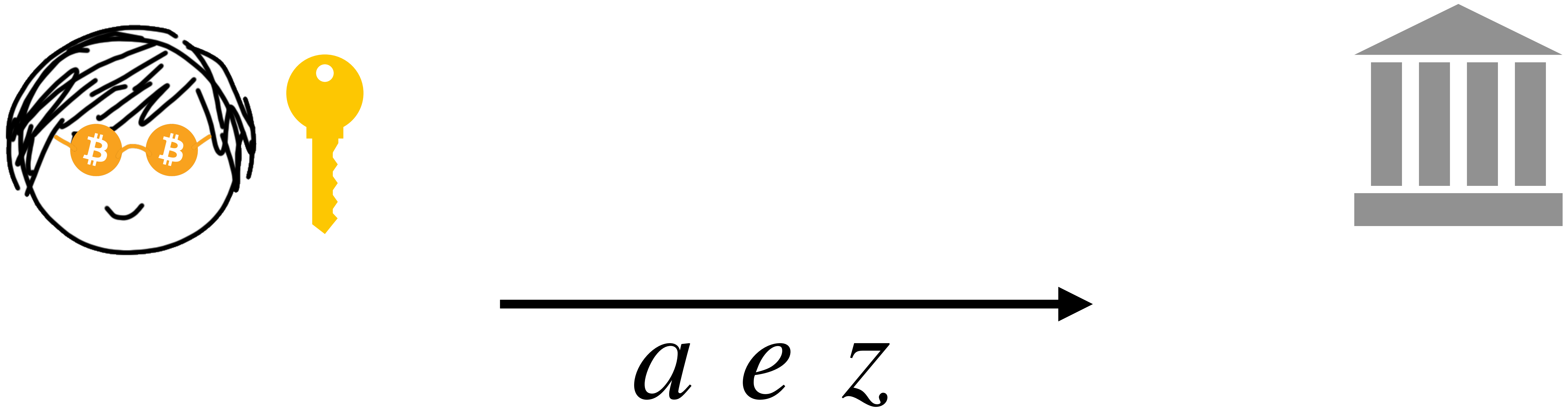

$a \quad e \quad z$

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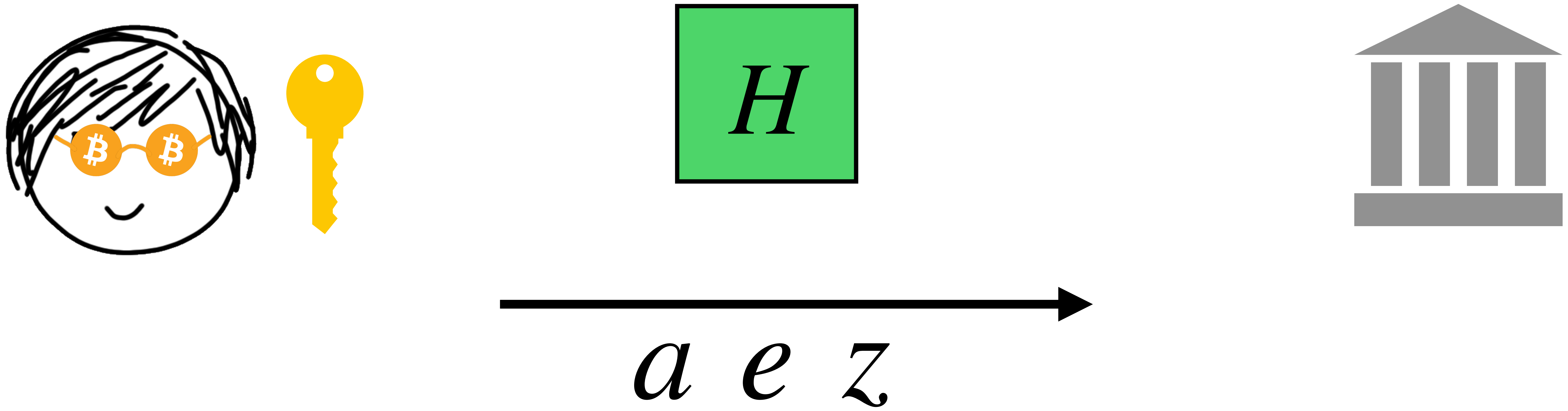

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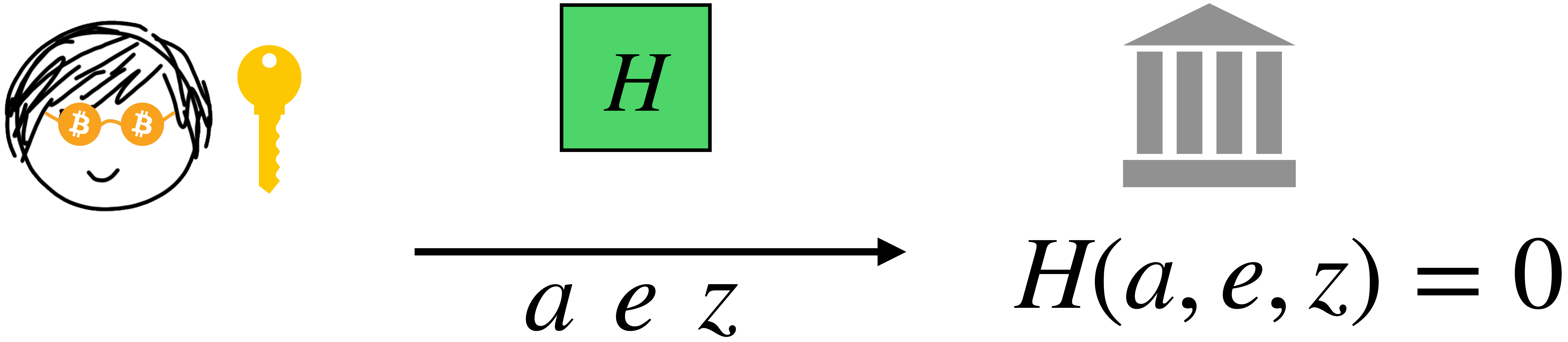
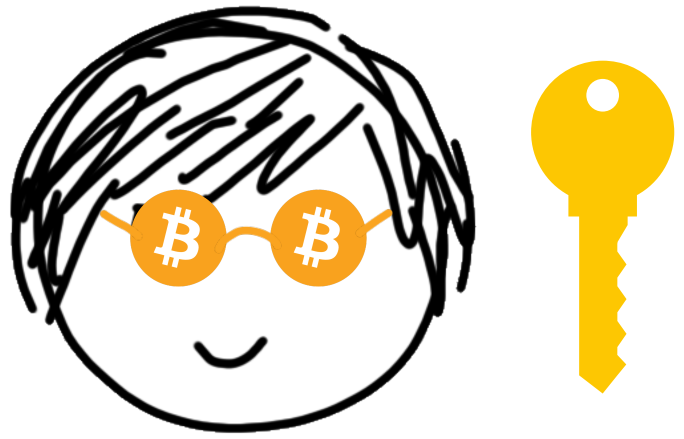
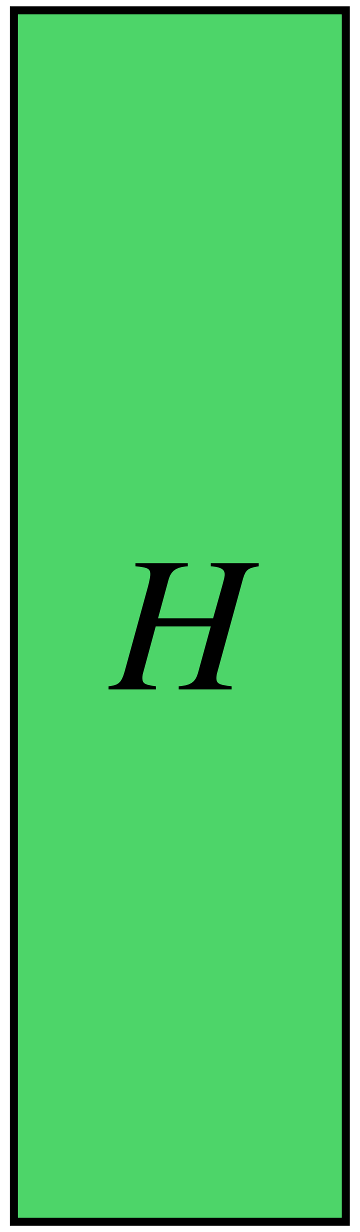


$$H(a, e, z) = 0$$

Fischlin's Transformation

- Let $\boxed{H}: \{0,1\}^* \mapsto \{0,1\}^\ell$ be a random oracle





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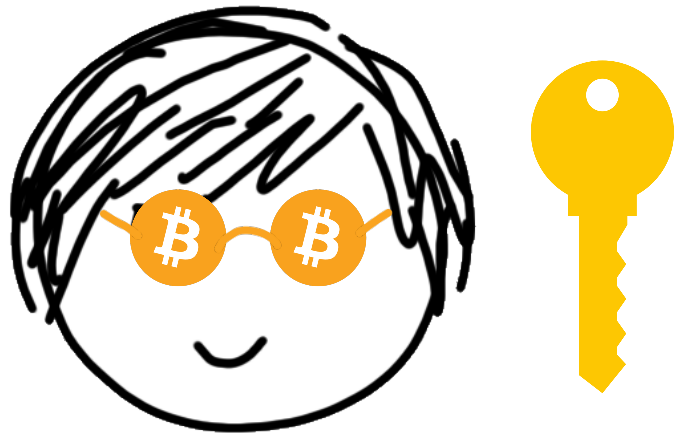
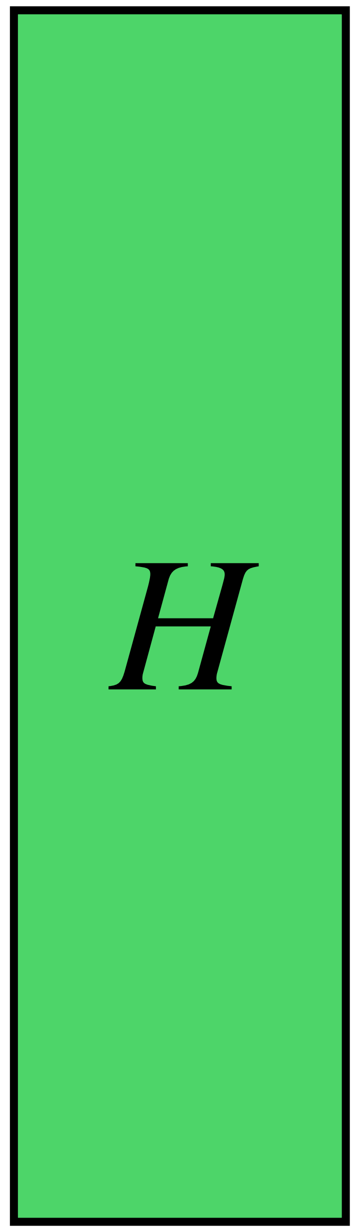


Sample Σ-protocol first message 'a'



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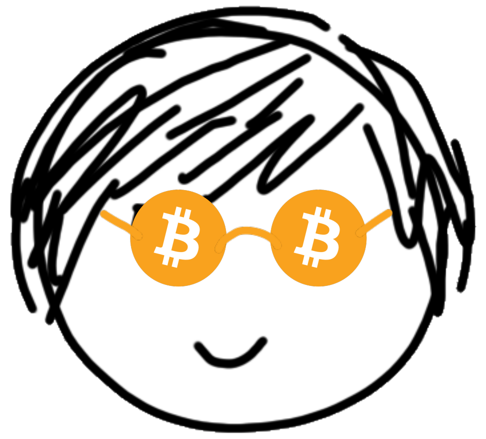

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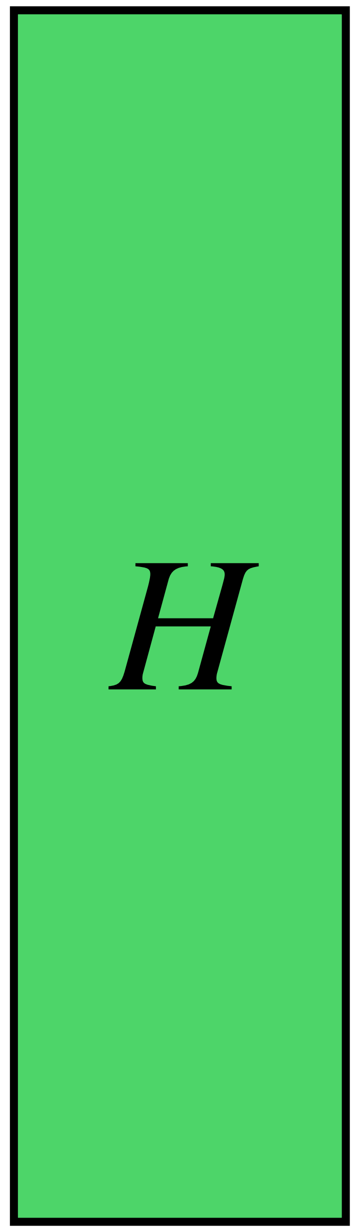
$(a,0,z_0)$



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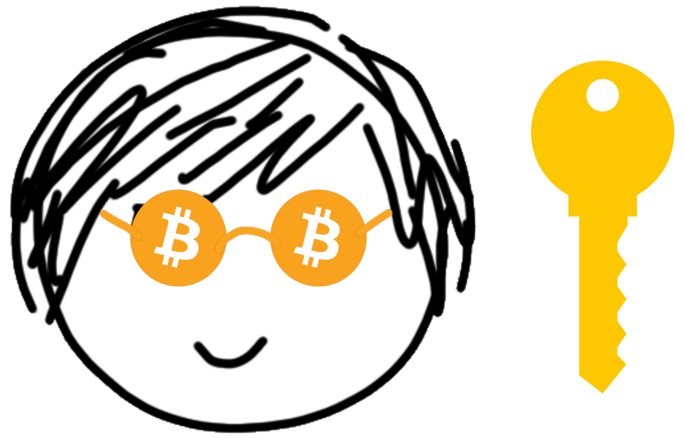

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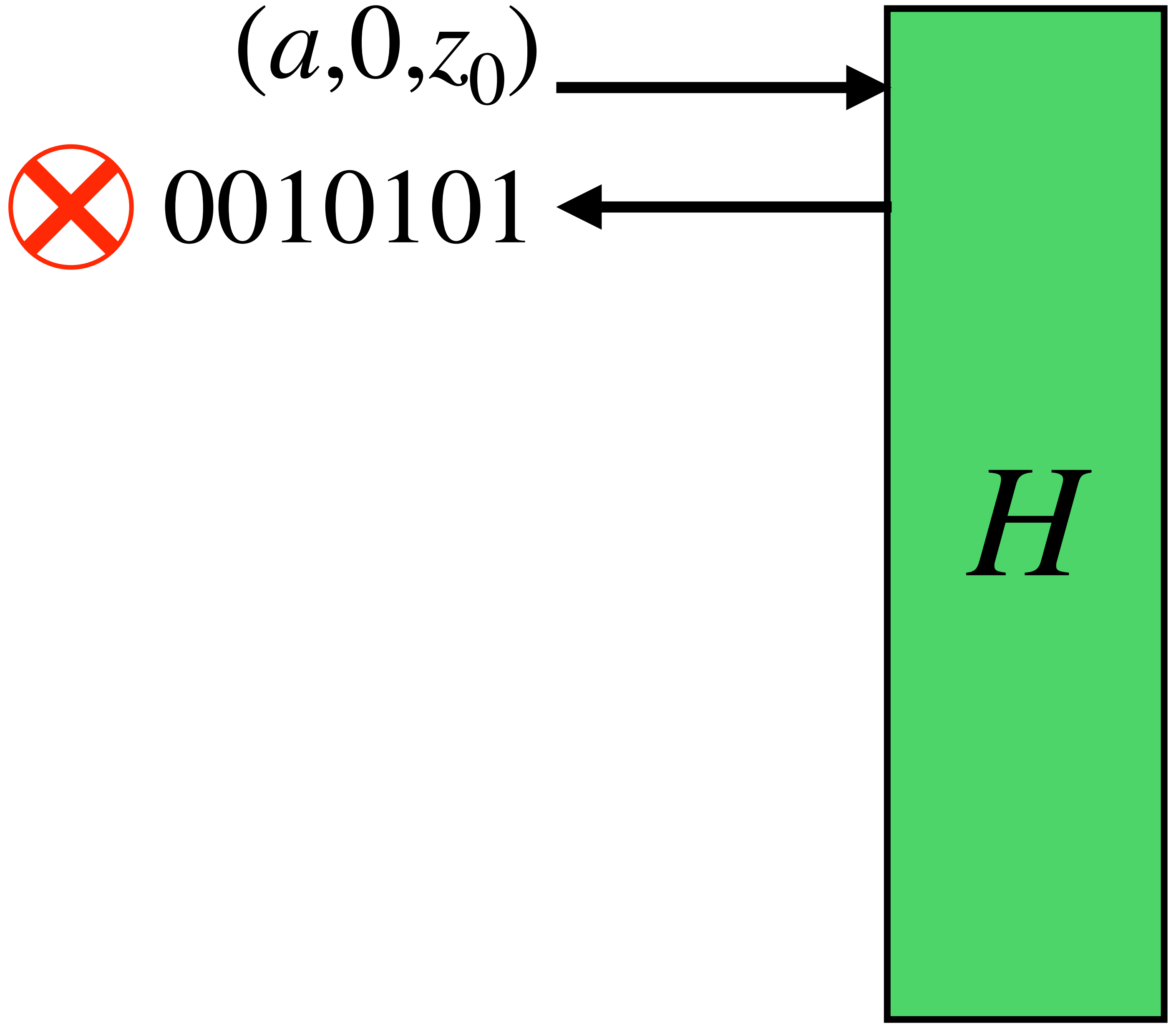

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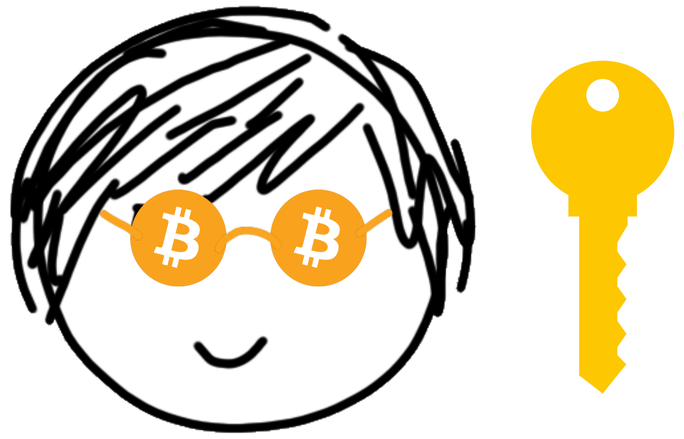

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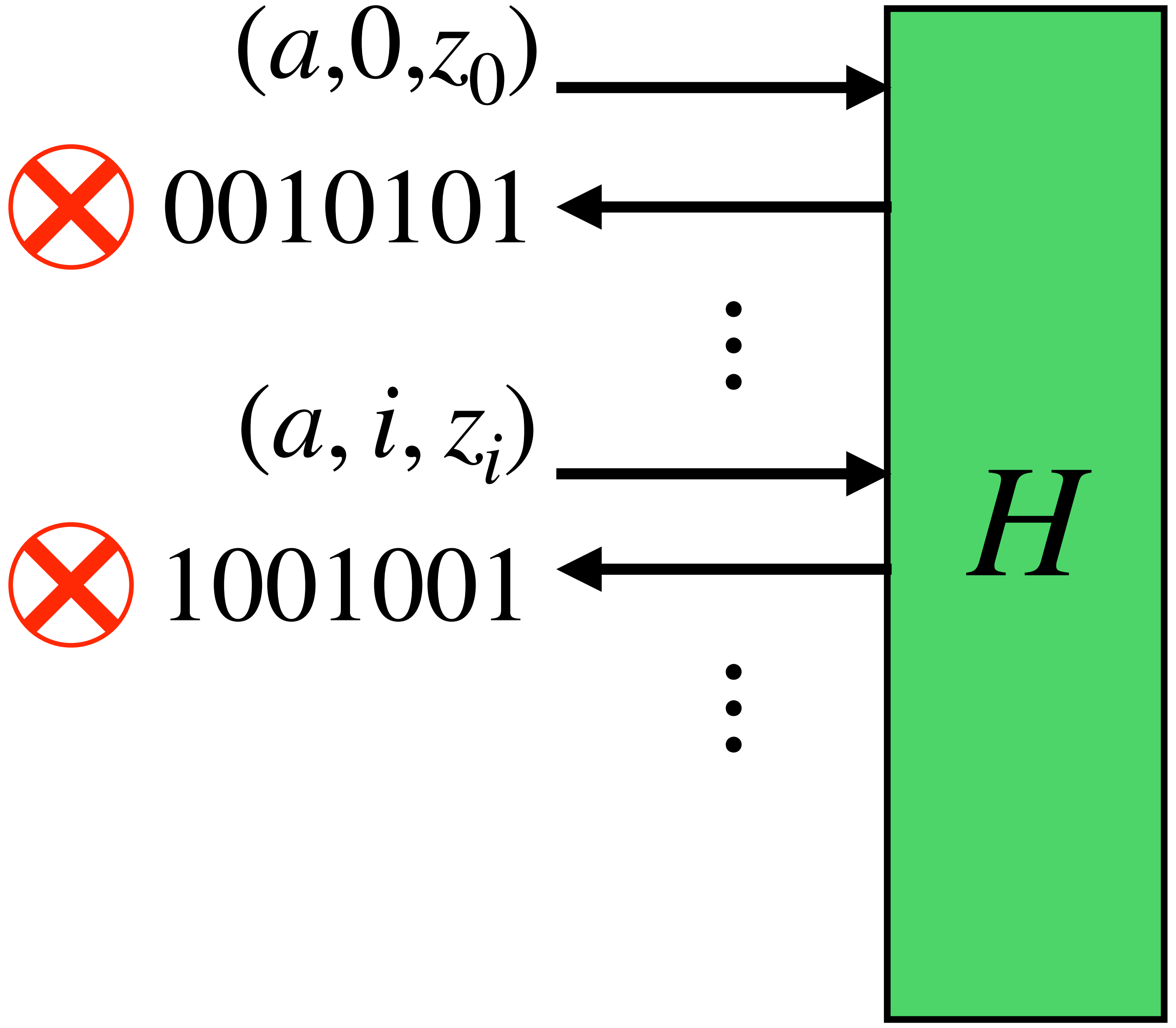

$(a,0,z_0)$

☒ 0010101

⋮

(a, i, z_i)

☒ 1001001

⋮

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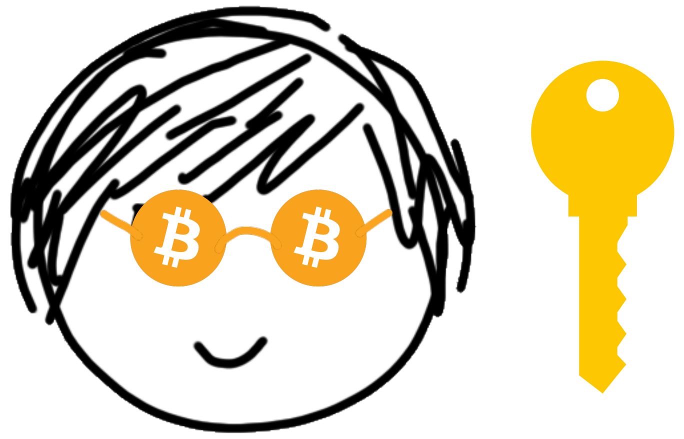

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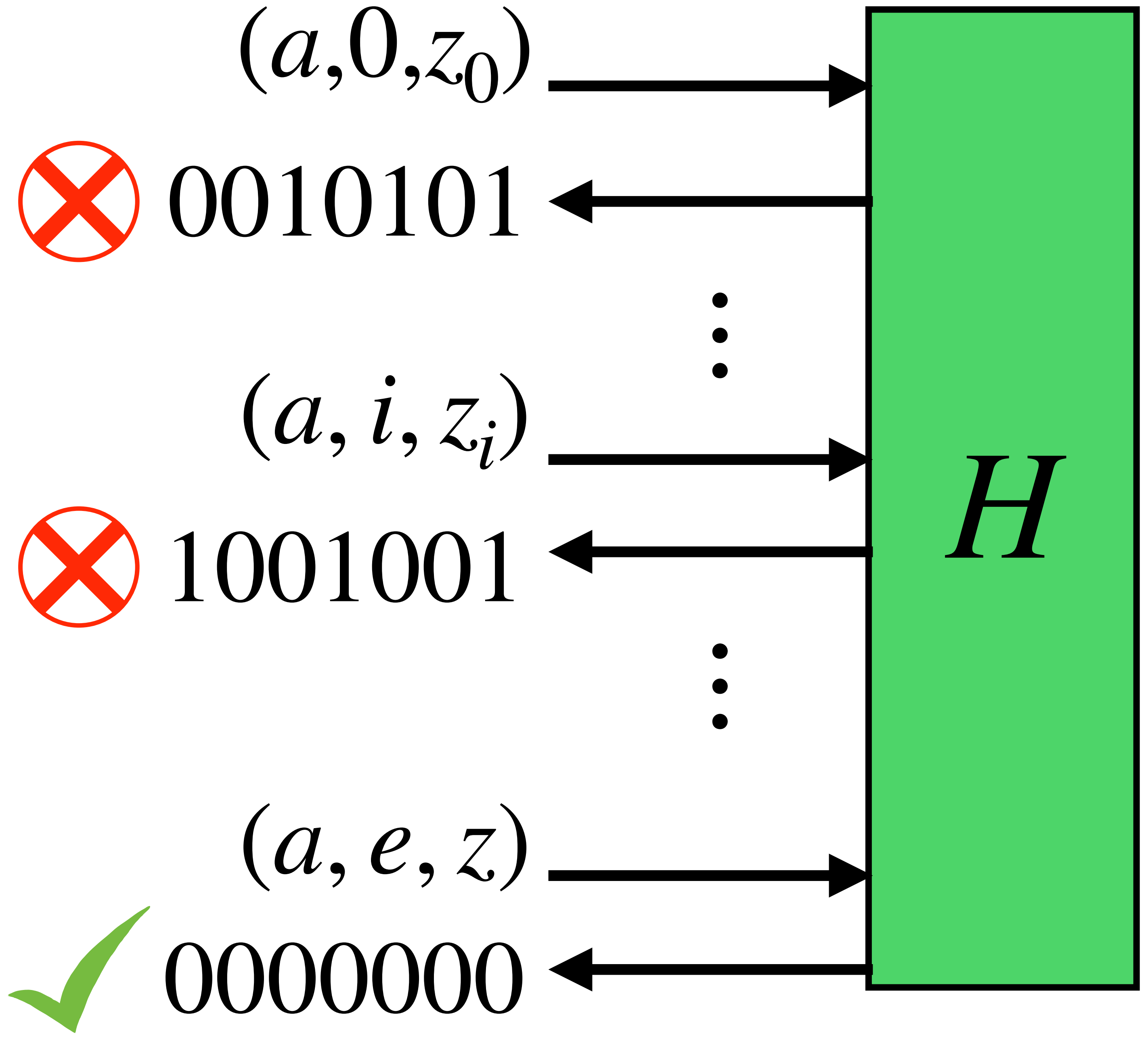

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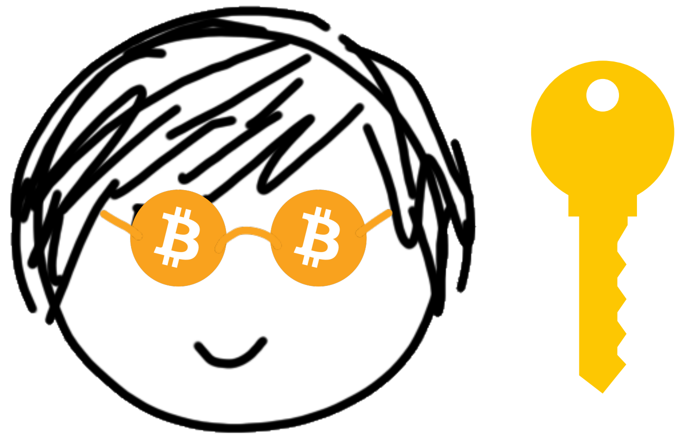

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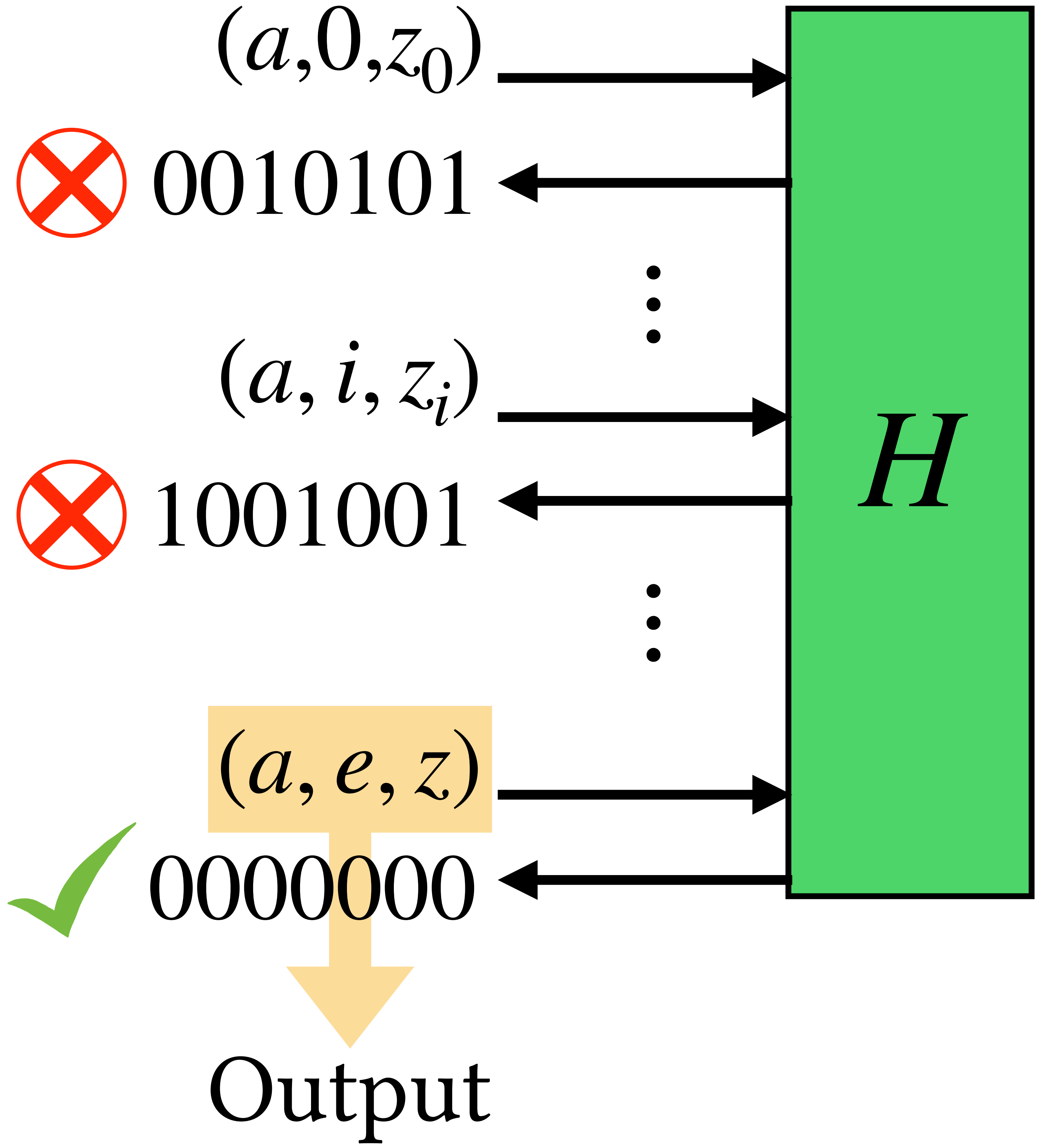

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Output

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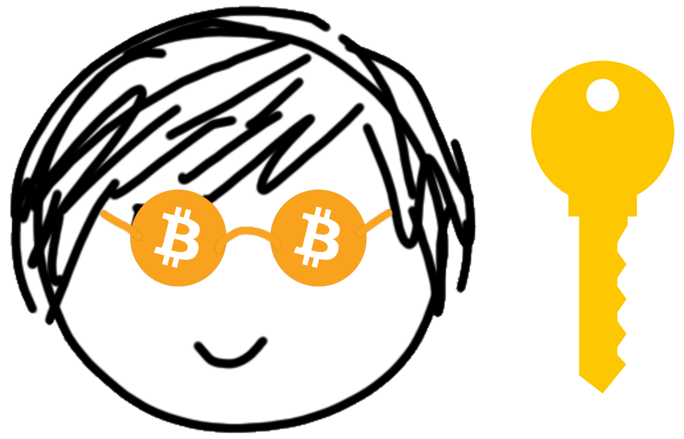

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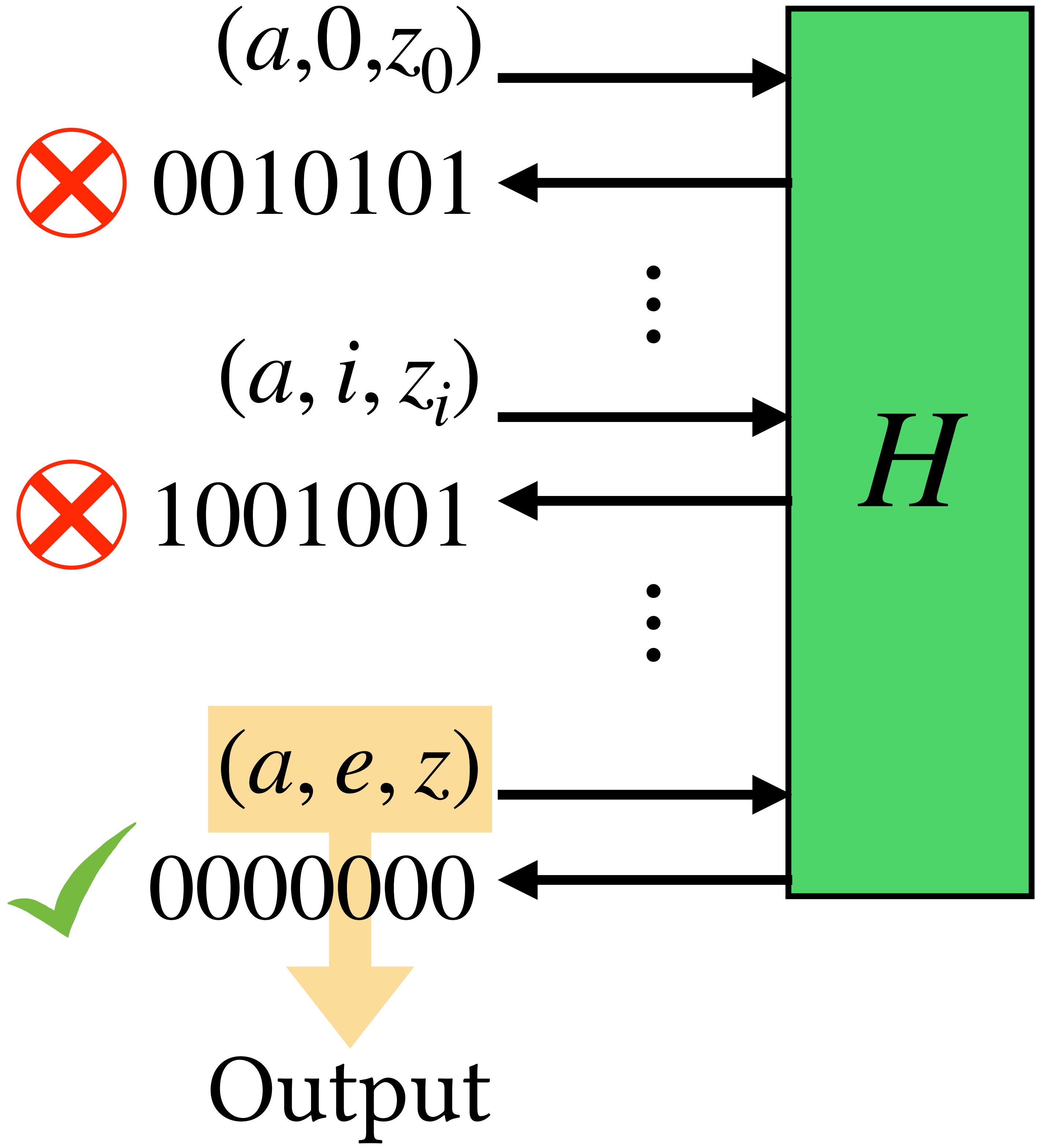

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Soundness: Except with Pr=$2^{-\ell}$, P is forced to query more than one accepting transcript to $\boxed{H}$

Completeness: P terminates in poly time when ℓ is small, i.e. $O(\log \kappa)$

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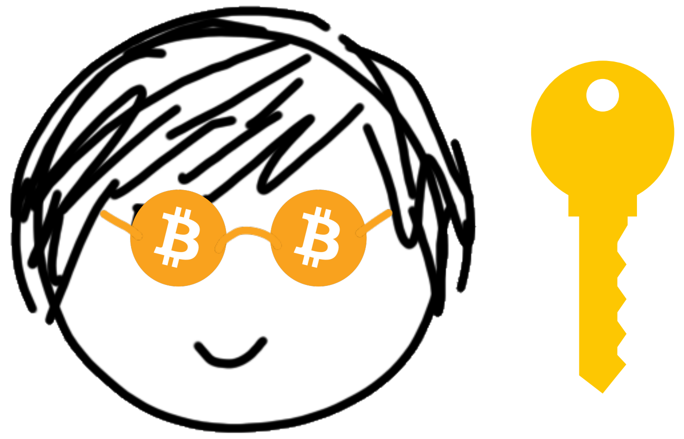

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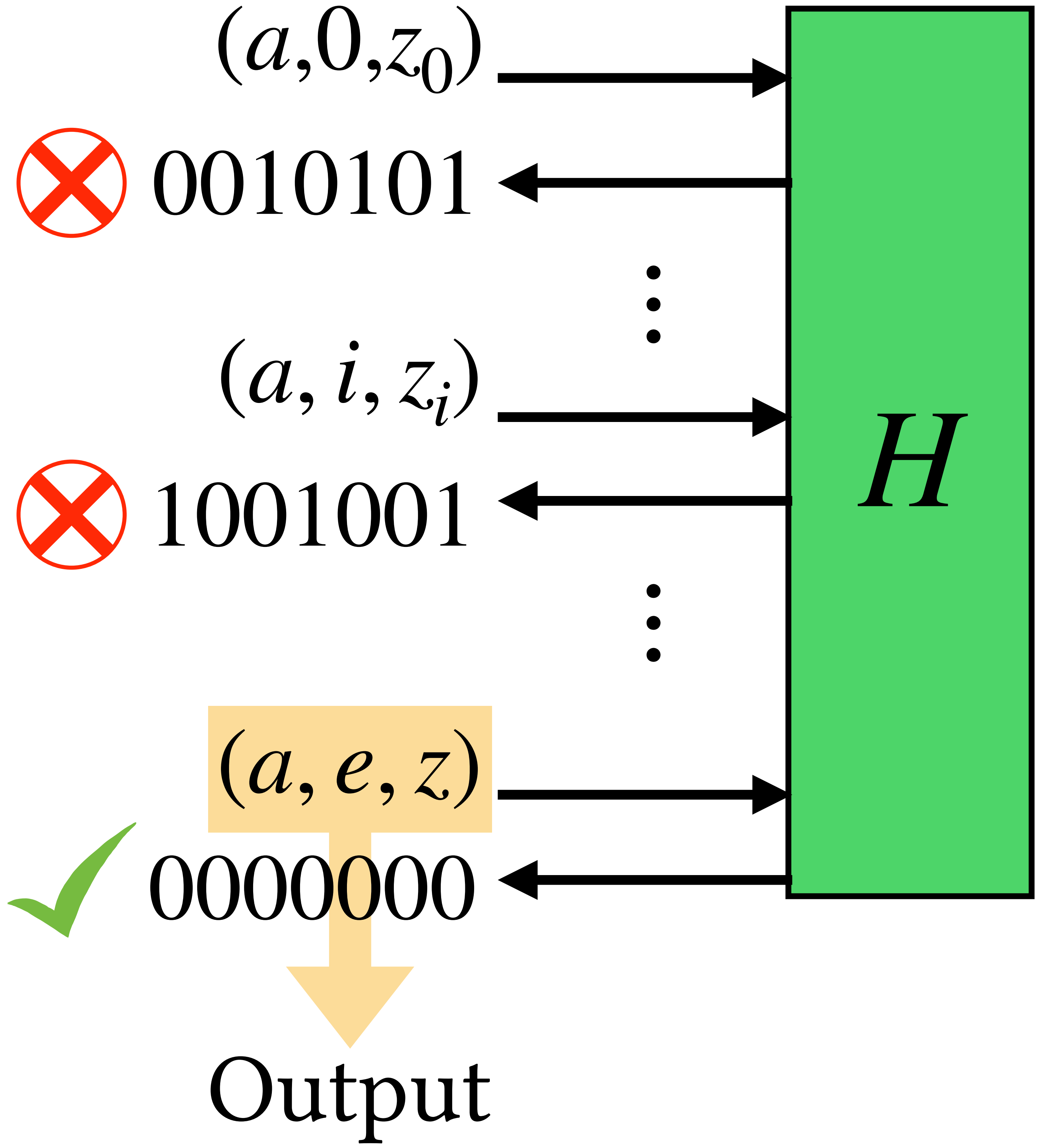

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⋮

(a,i,z_i) ✗ 1001001

⋮

(a,e,z) ✓ 0000000

H

Output

This gives Ext the values (e,z) and (e',z') as needed, by looking at queries made to H

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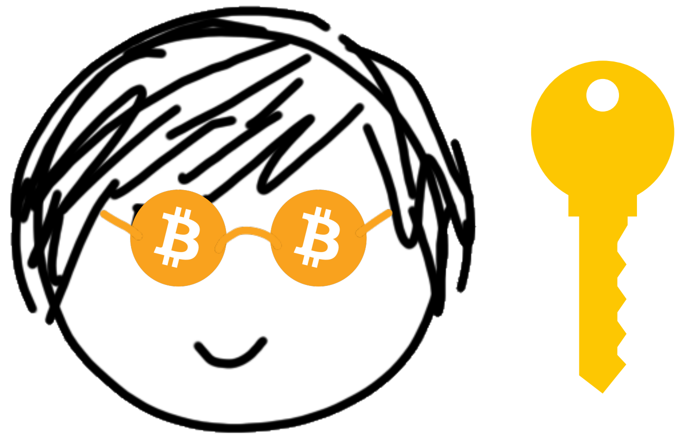

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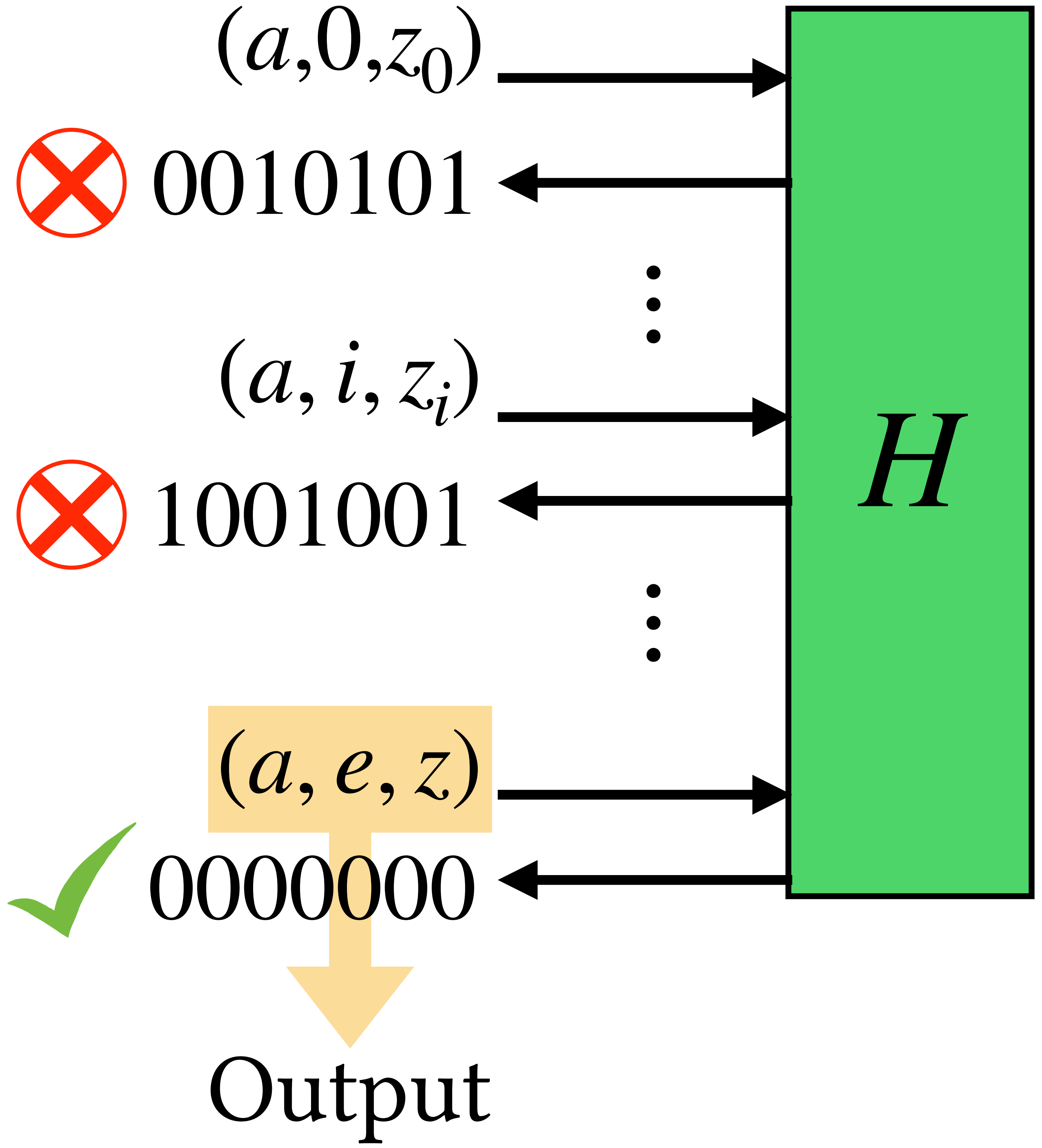

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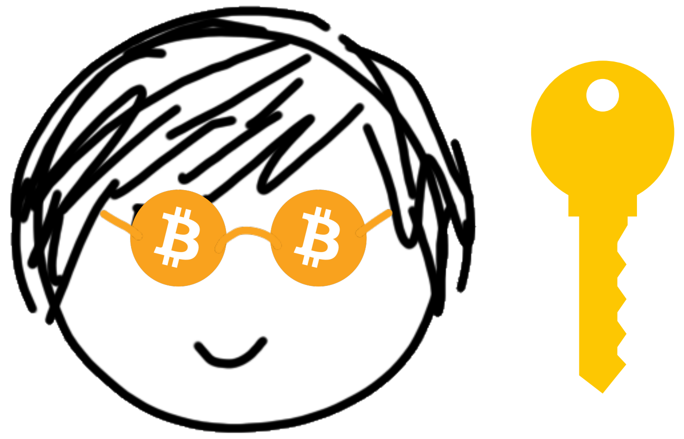

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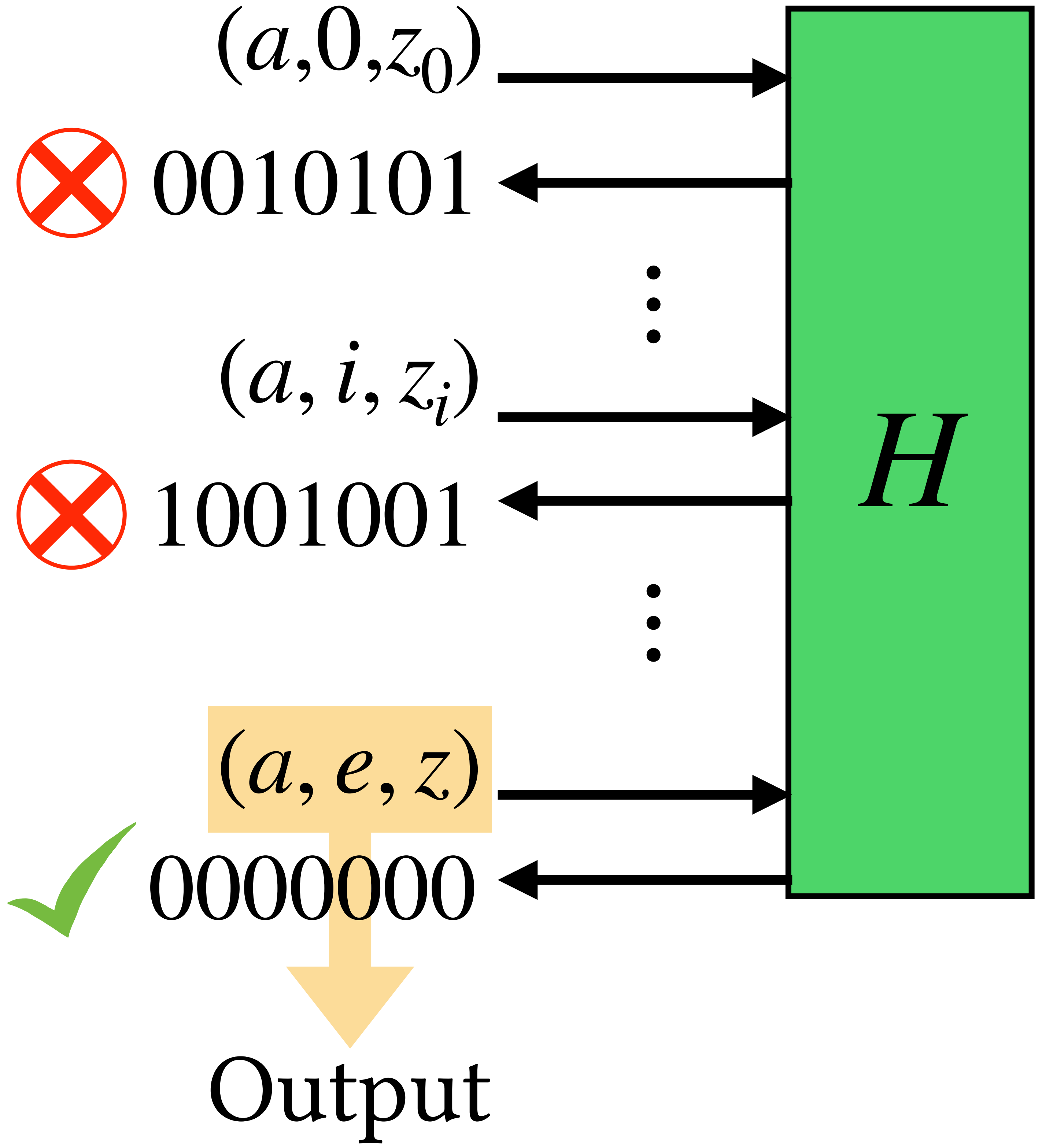

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Problem!

Full Soundness: Repeat r times

Fischlin vs Pass: Qualitative

- Pass' compiler works for *any* Sigma protocol
- Fischlin's compiler works for a restricted class of Sigma protocols with 'quasi-unique responses'
- Supported by many standard Sigma protocols (eg. DLog), but many *may* not—especially if a statement can have multiple witnesses (eg. Pedersen Commitment opening, 1-of-2 witnesses, etc.)

Quasi-unique Responses [Fischlin 05]

Hard: $(a, e, z, z') \leftarrow \mathscr{A}(\mathsf{pp})$ such that $V(a, e, z) = V(a, e, z') = 1$

Fixing (a, e) fixes z

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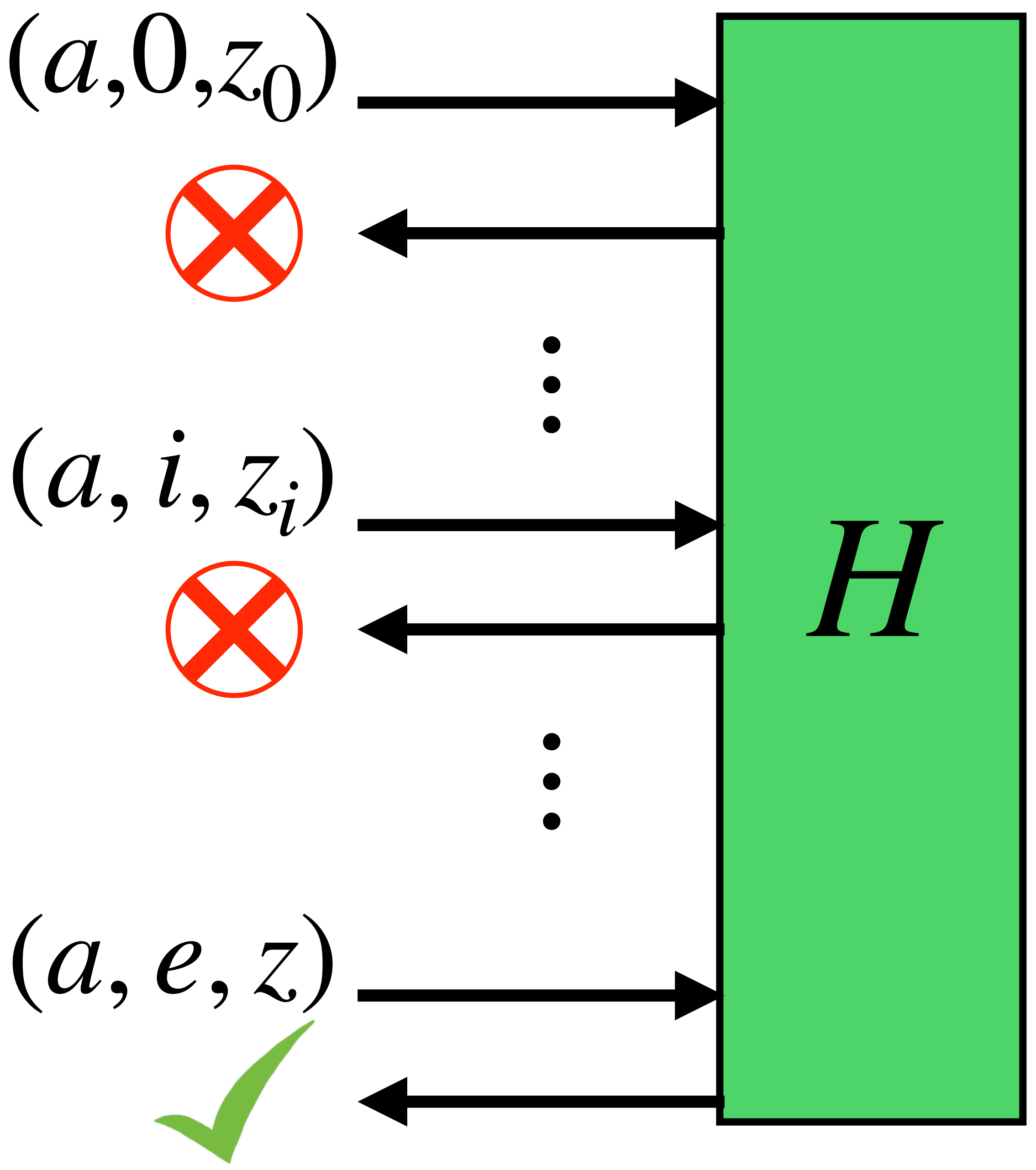

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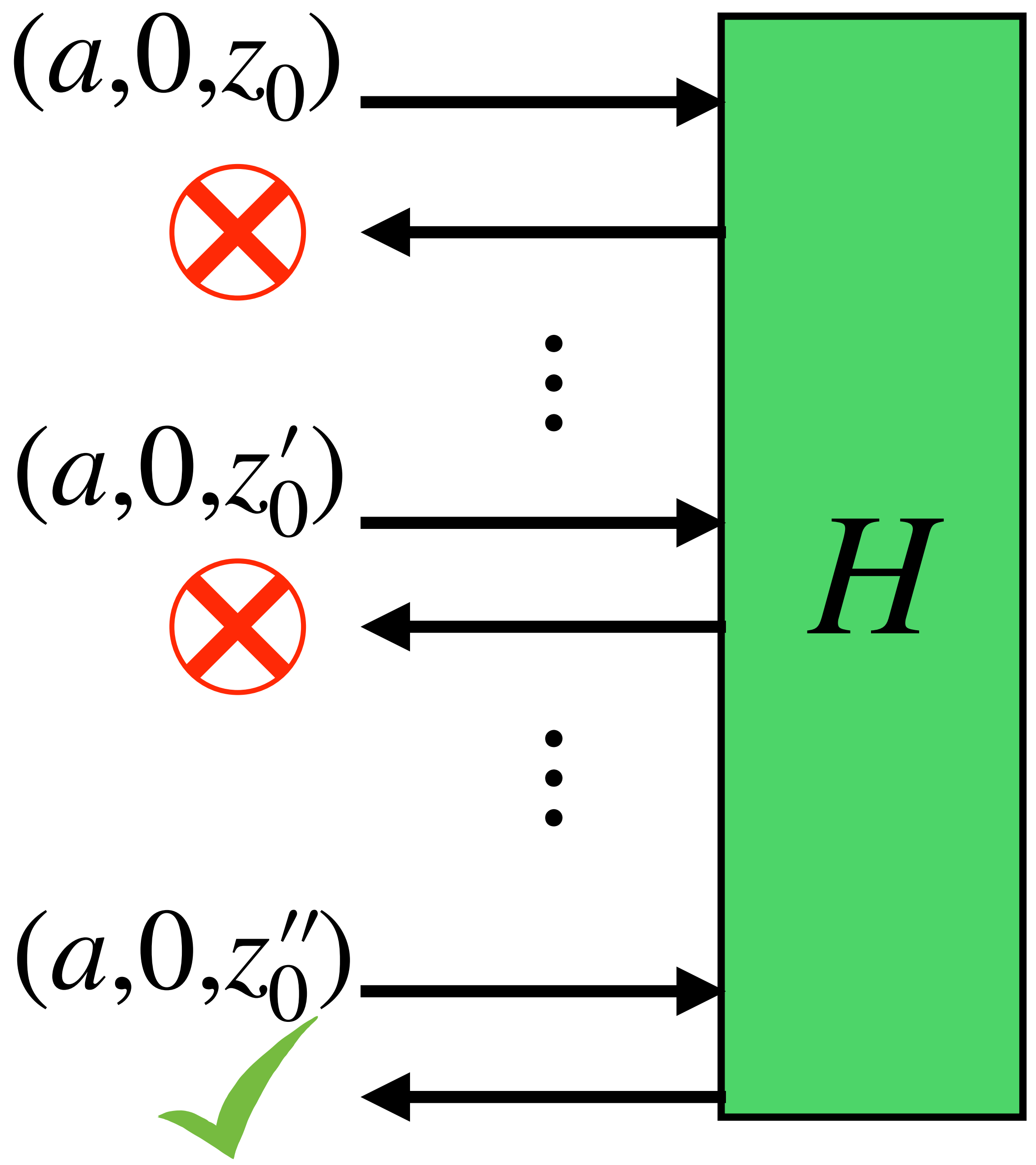

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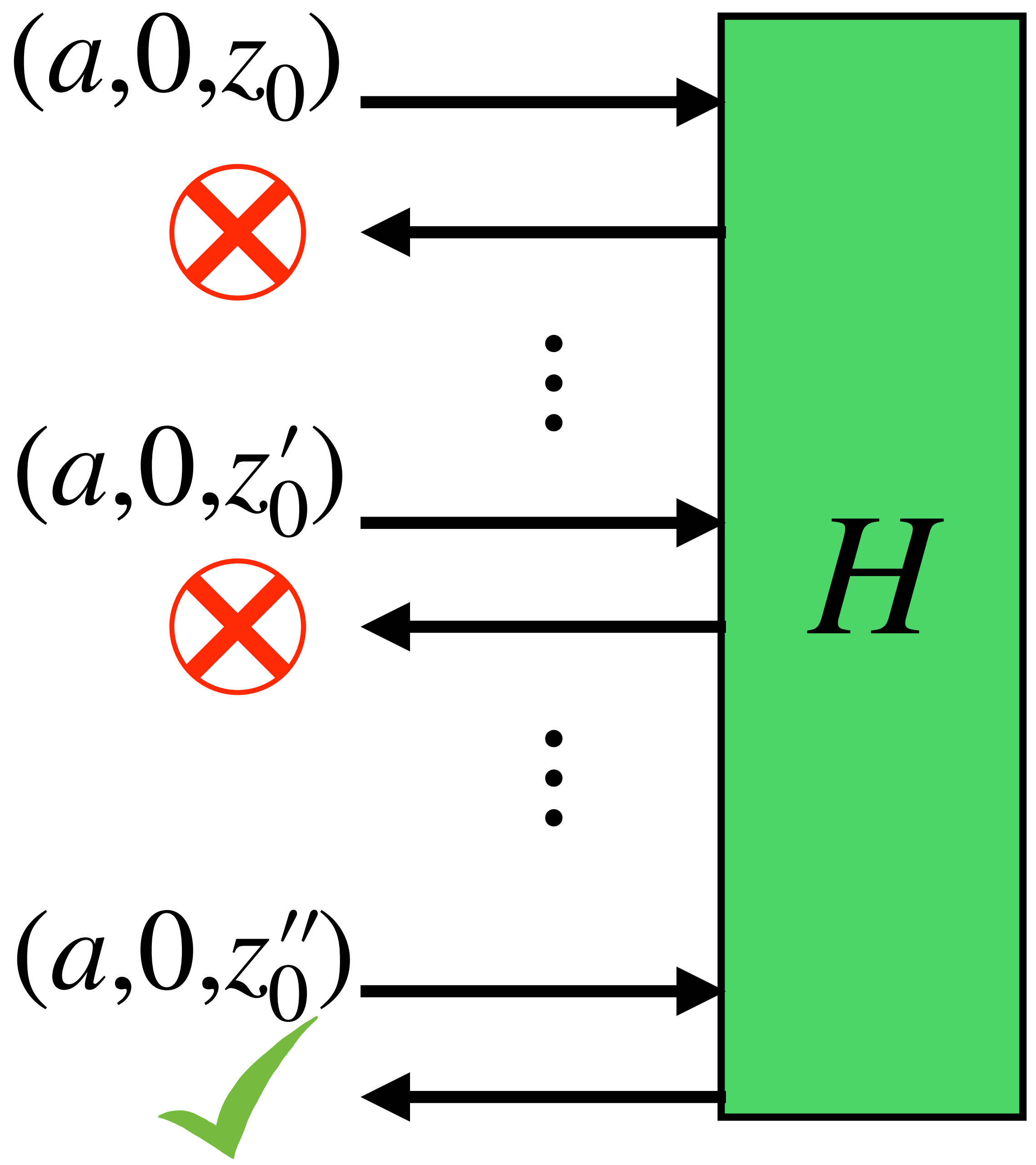

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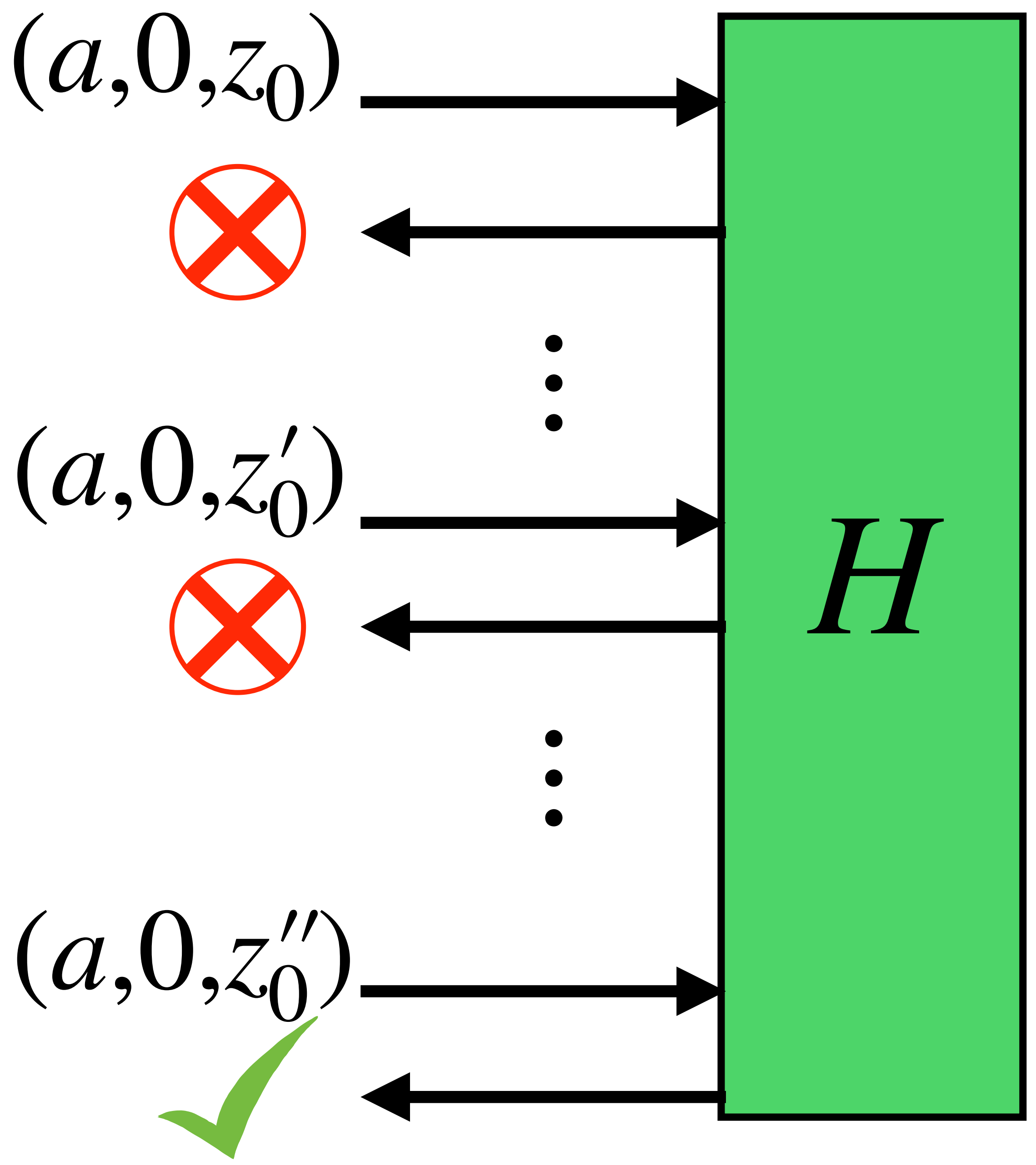


Prover can produce a proof without ever having to try more than one challenge

Recall:
Extractor needs transcripts with different challenges

Is it *really* necessary, though?

- Folklore: breaking Sigma protocol abstraction, and simply ‘adjusting syntax’ of the extractor is usually sufficient to preserve Proof of Knowledge

- This is demonstrated by the Sigma protocol to prove knowledge of one-out-of-two witnesses [Cramer Damgård Schoenmakers 94]

- In [K shelat 22] we formalize this folklore

What about Zero-knowledge?

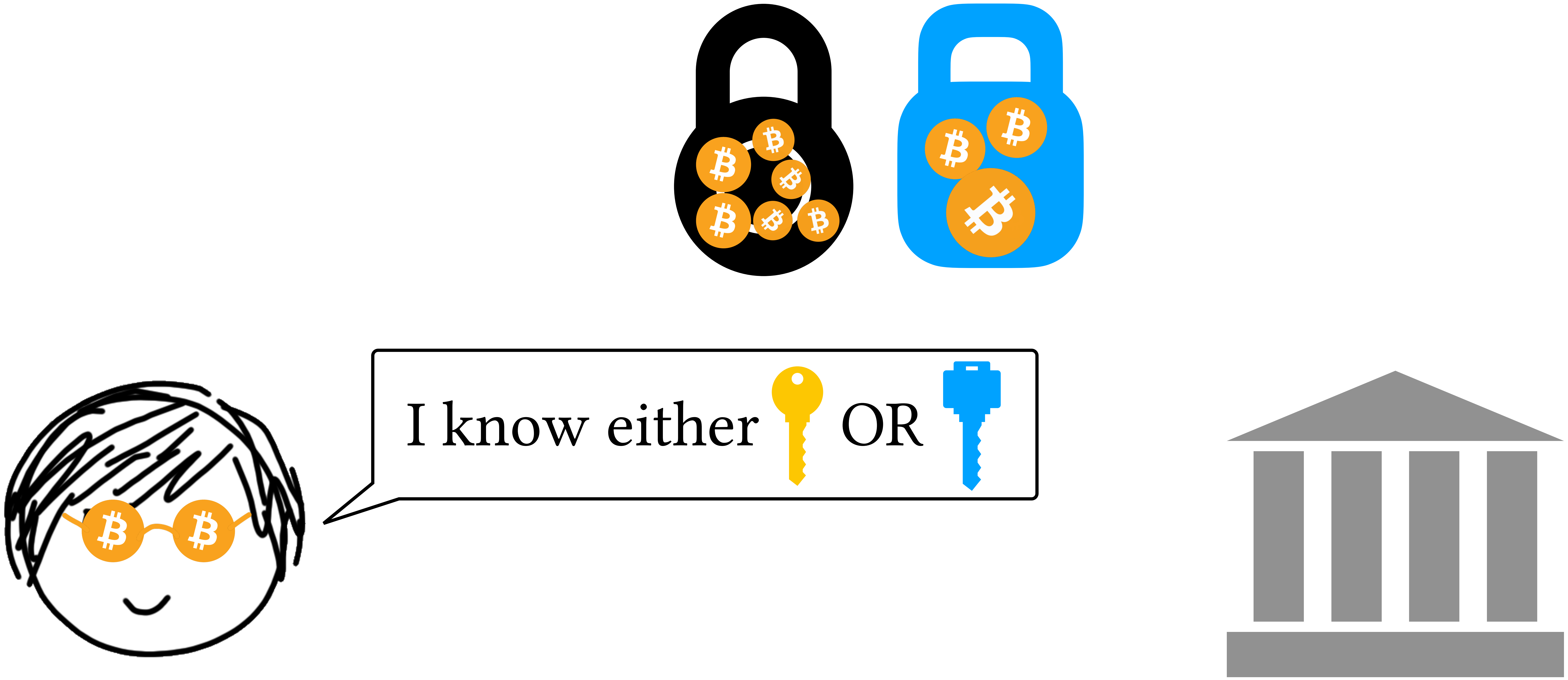
- Interestingly, Fischlin's proof of Zero-knowledge also depends on quasi-unique responses

- Unlike extraction, it is not intuitive as to why (or whether it's even necessary)

- [K shelat 22]: In the absence of unique responses, an explicit attack on *Witness Indistinguishability* (WI)

Witness Indistinguishability

- The following kind of statement finds many applications:



I know either 🔑 OR 🔑

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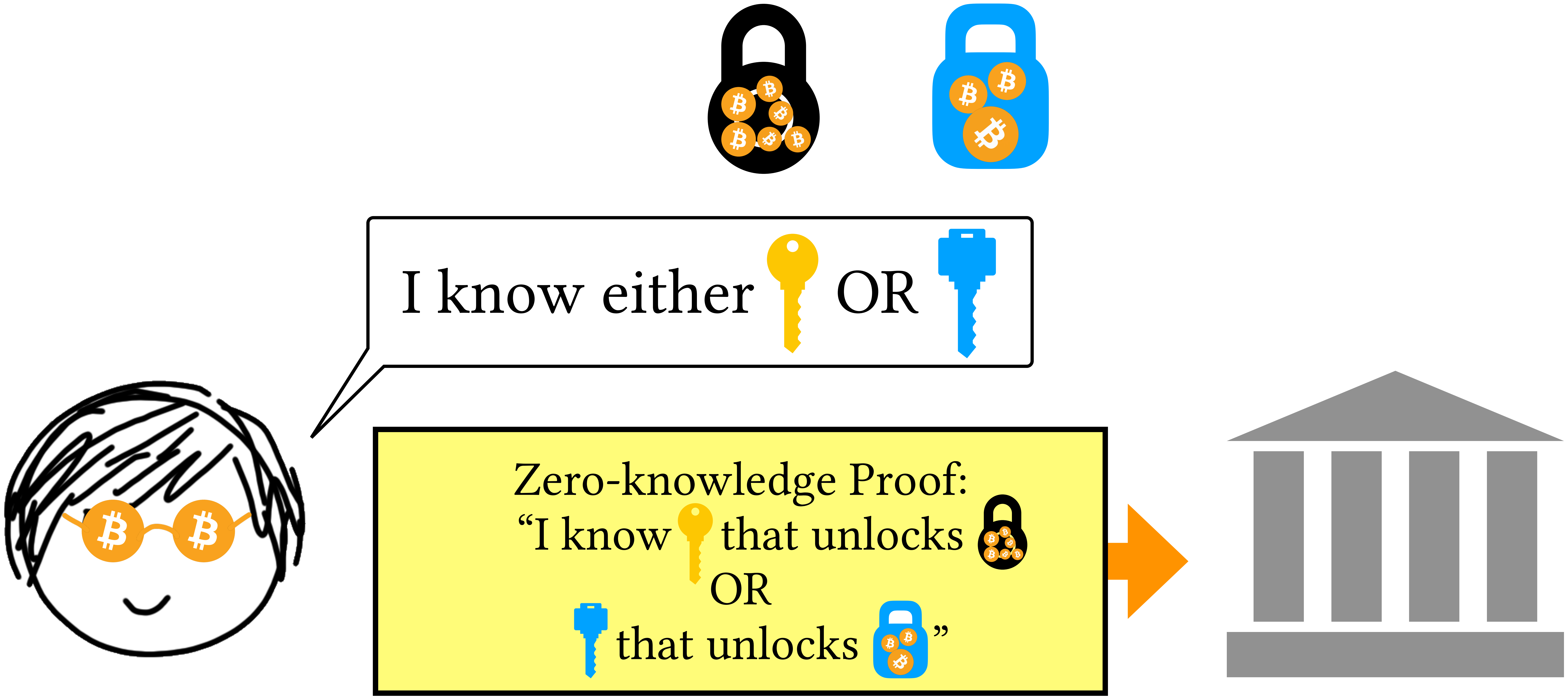

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Zero-knowledge Proof:
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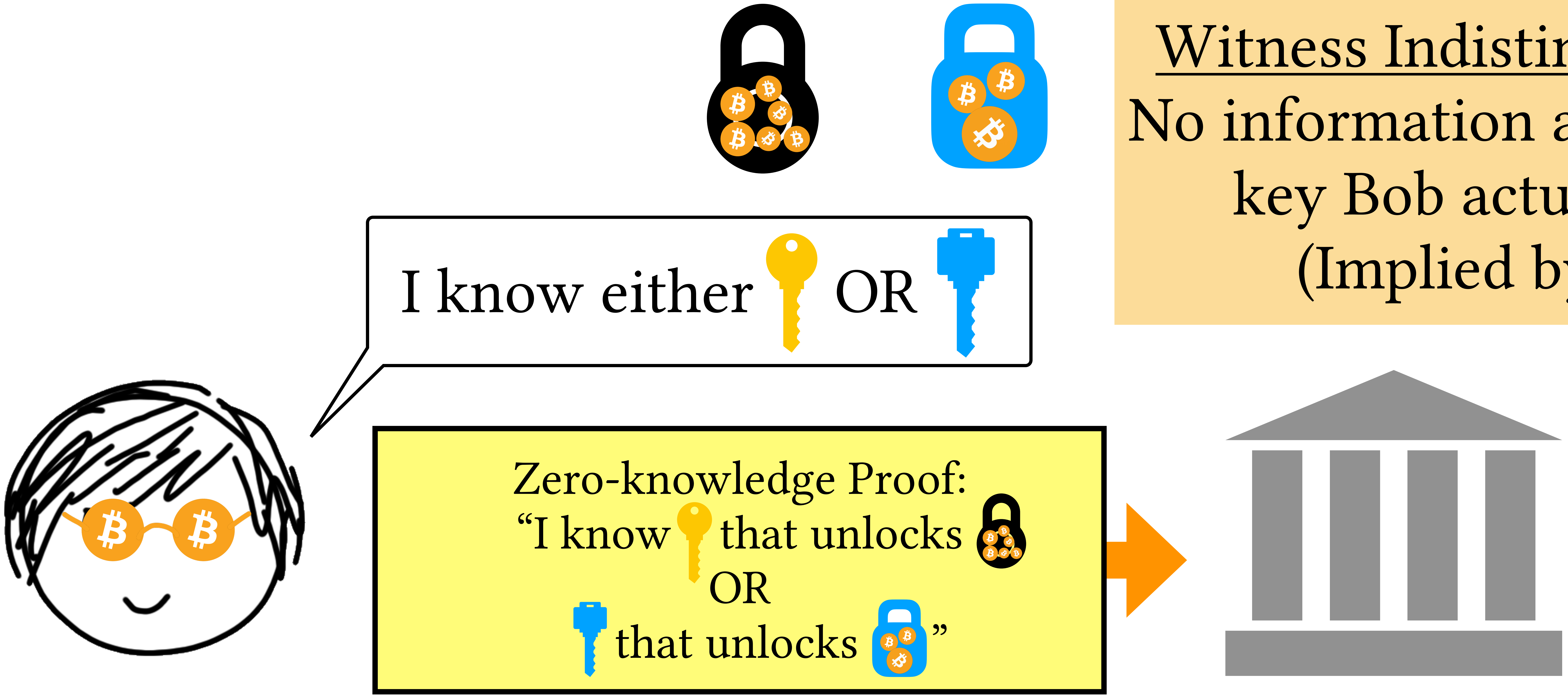

I know either OR

Zero-knowledge Proof:
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Witness Indistinguishable:
No information about which key Bob actually has
(Implied by ZK)

Witness Indistinguishability

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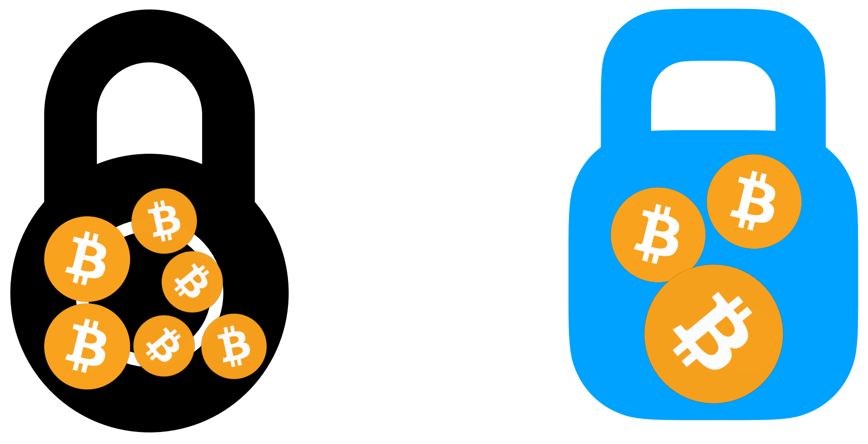

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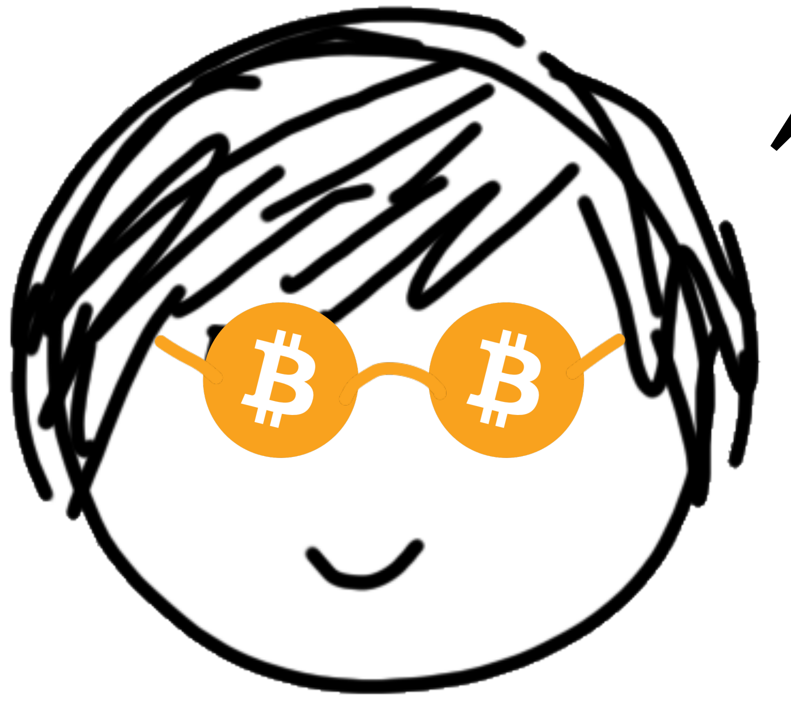

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"I know 🔑 that unlocks 🔒
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<u>Witness Indistinguishable:</u>
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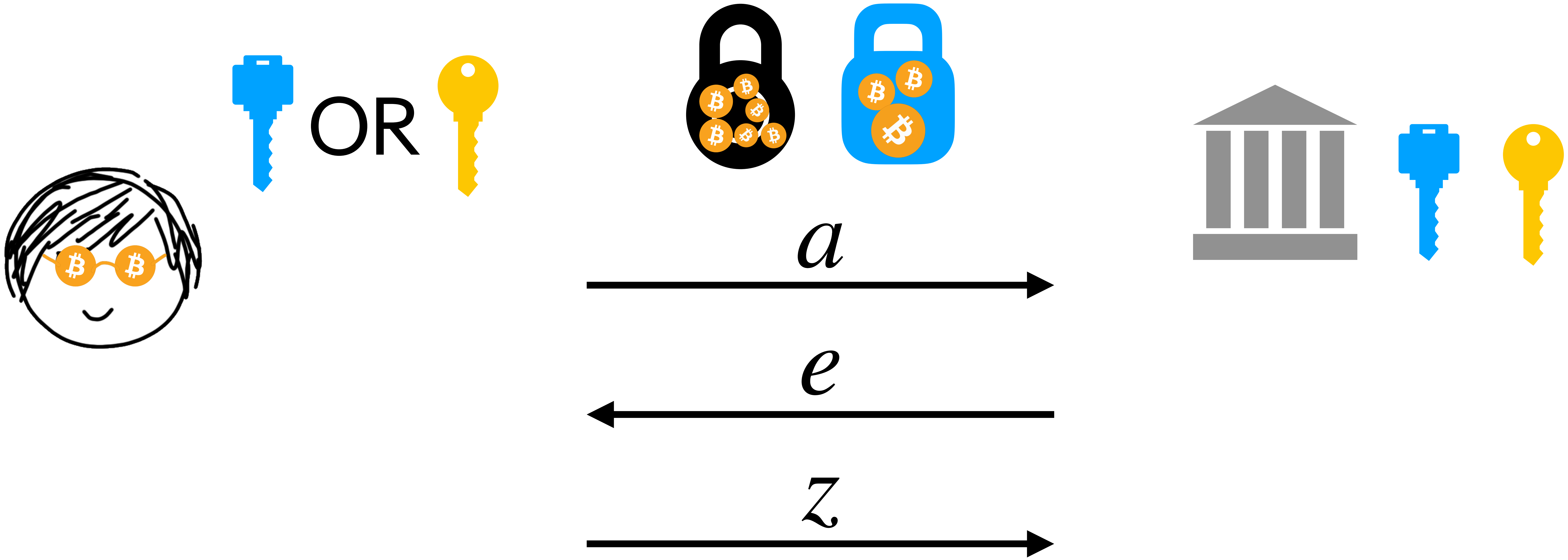
<u>Important note:</u>
This holds even if both keys are actually known to bank
(like known plaintext security)

Useful Fact

- Some Σ protocols have the following property: (including some multi-witness ones)



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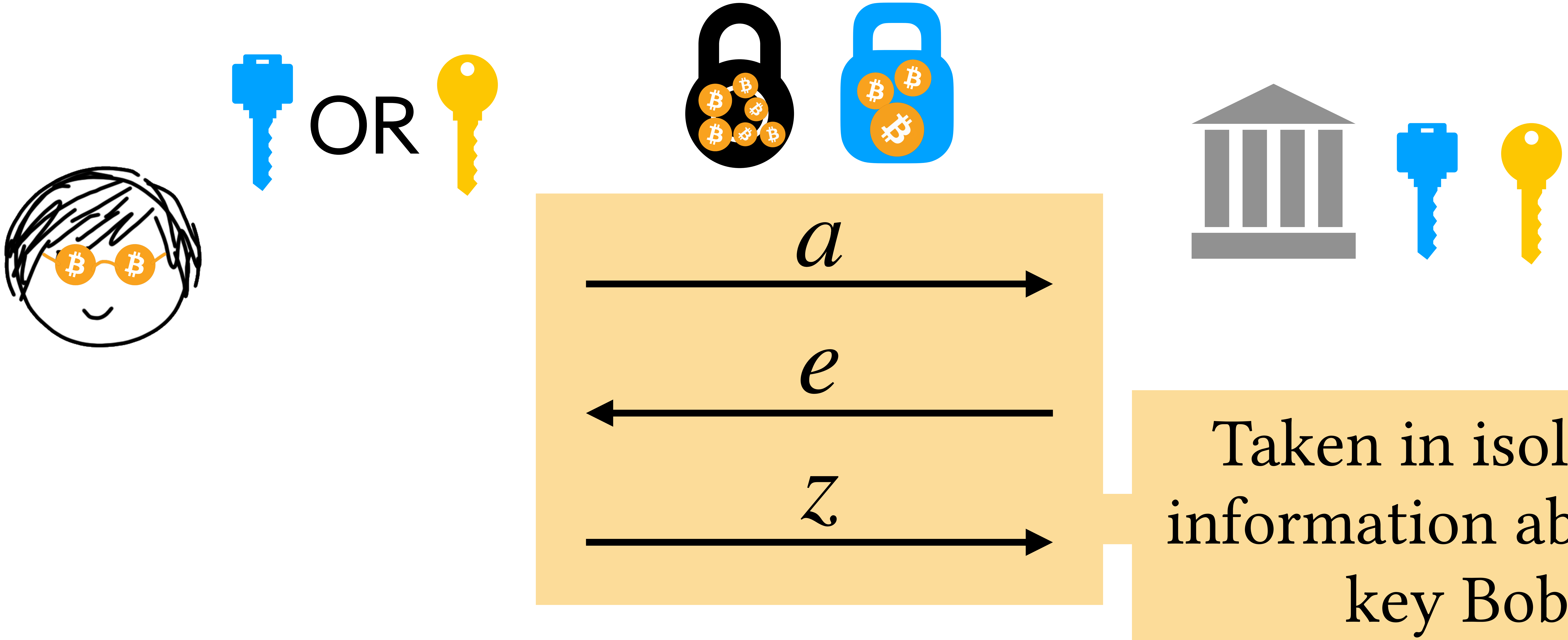

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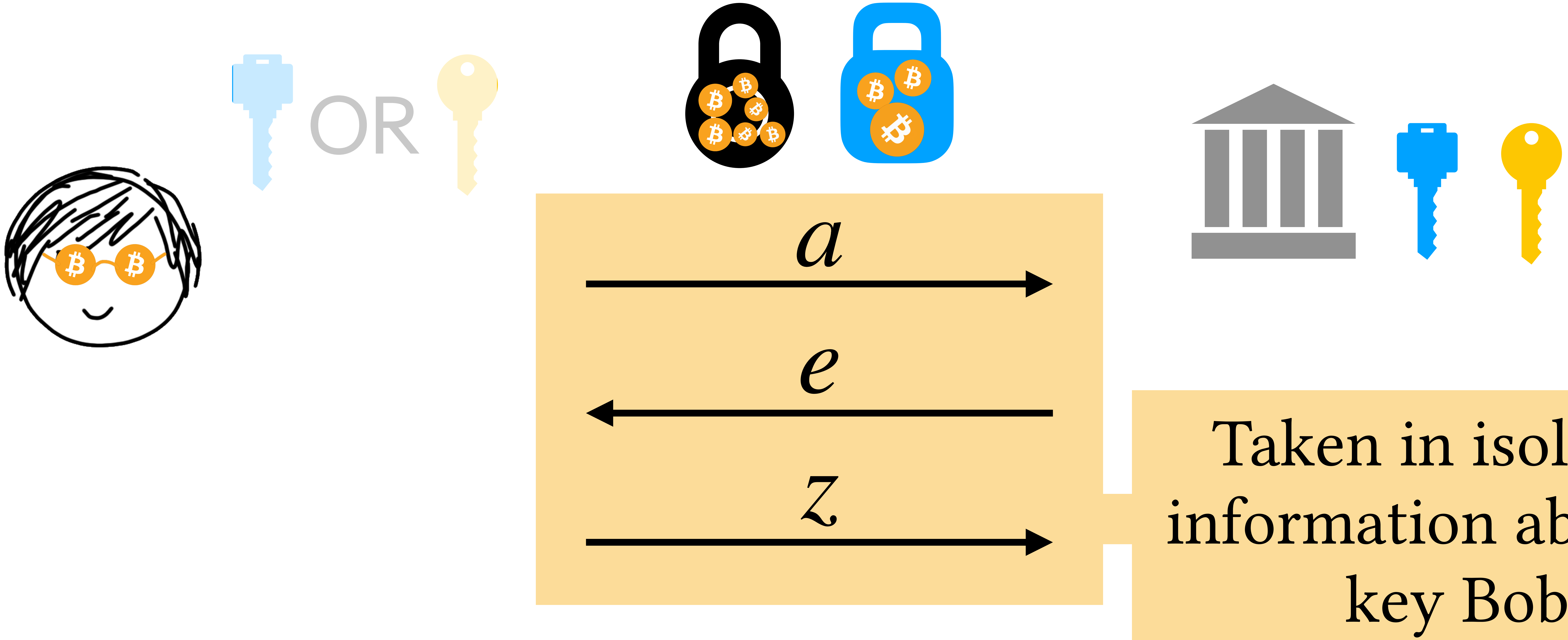

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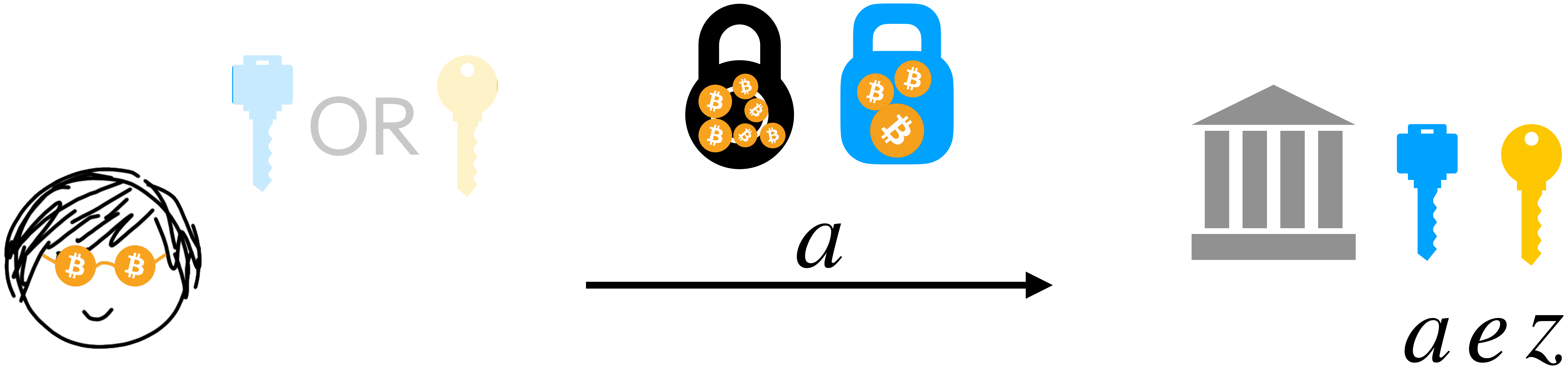
Useful Fact

- Some Σ protocols have the following property:
  (including some multi-witness ones)



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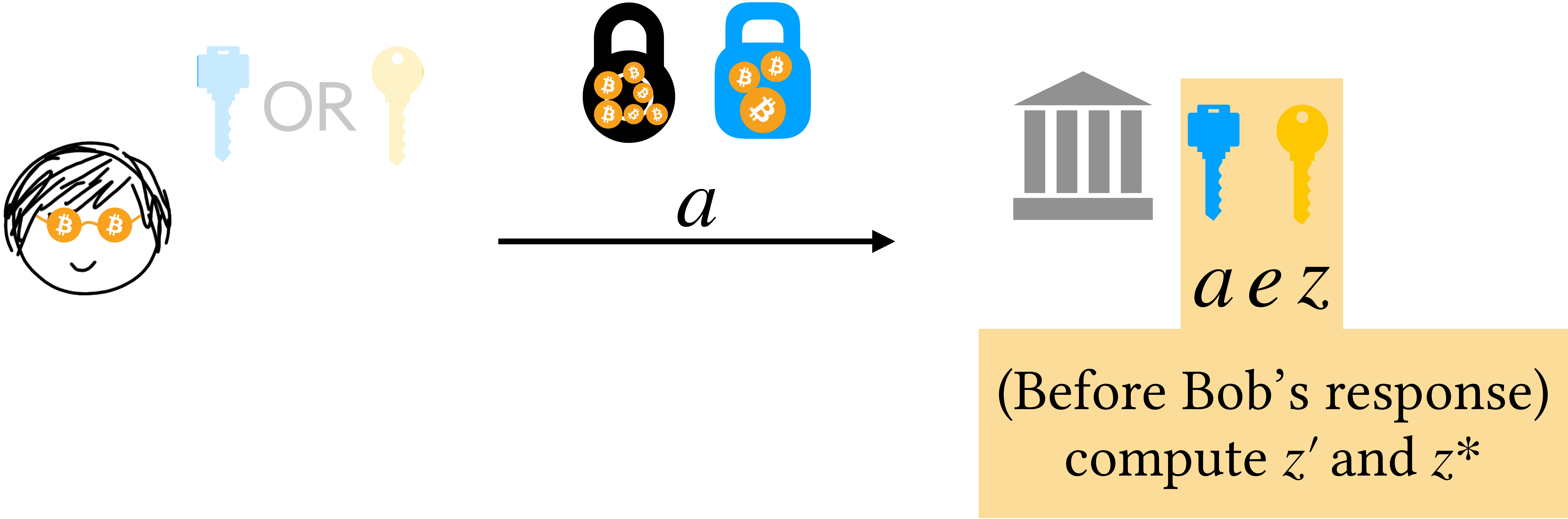

OR

a

$a\ e\ z$

(Before Bob's response) compute z' and z^*

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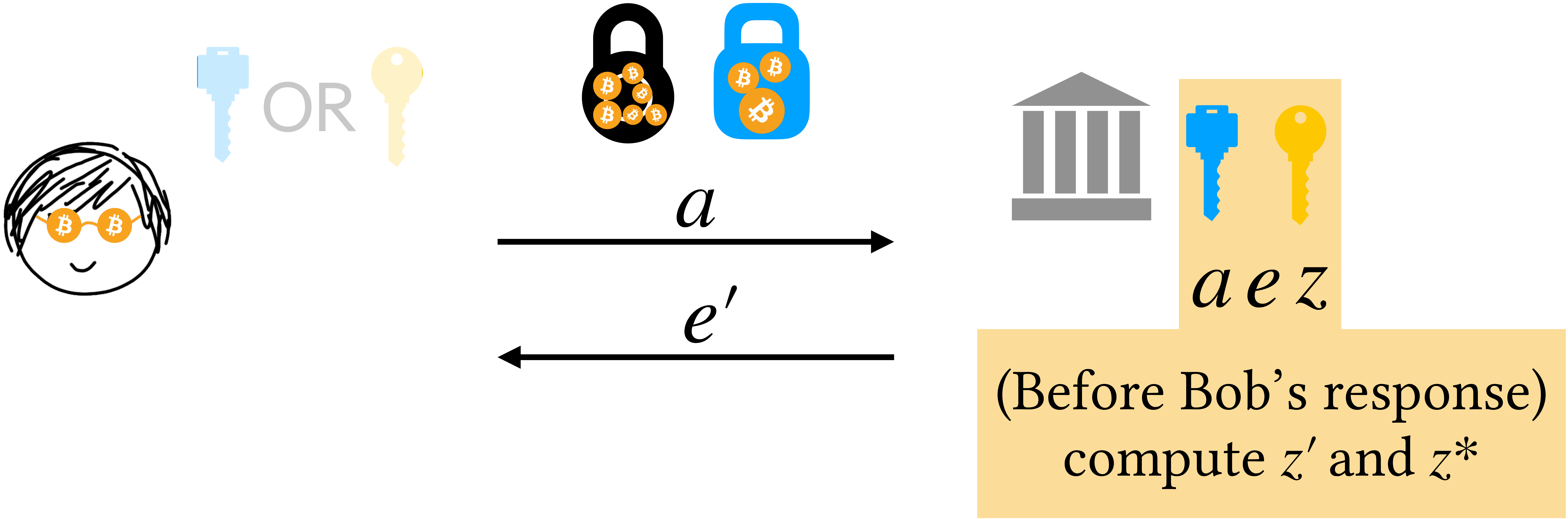

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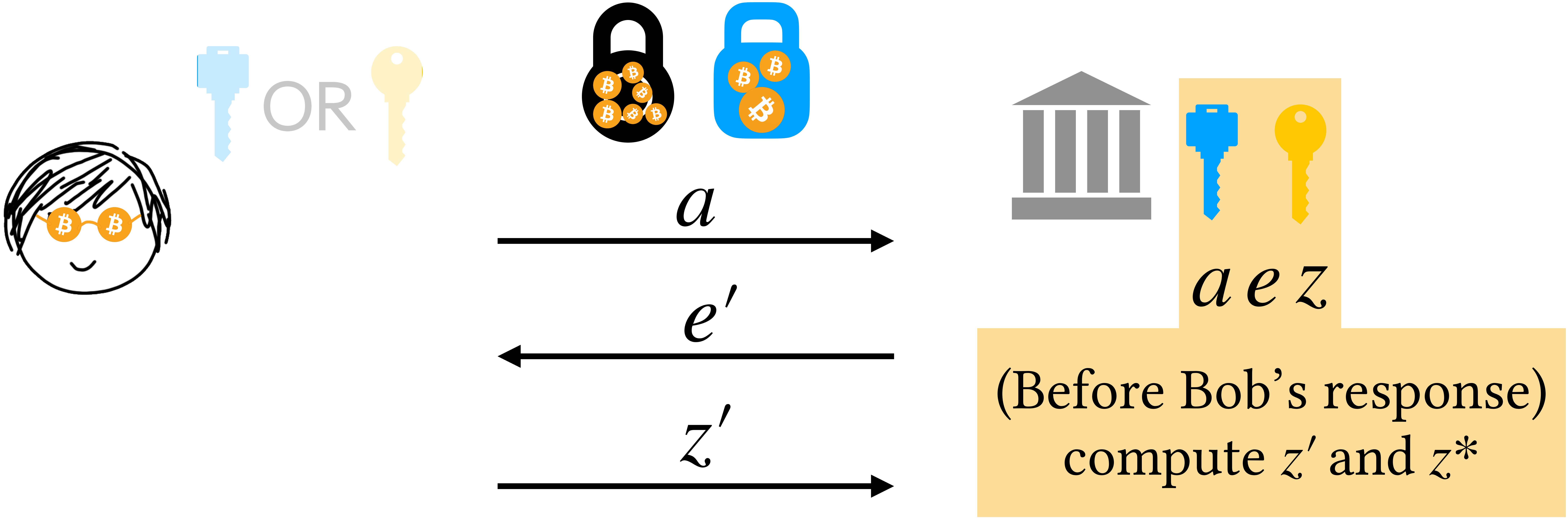

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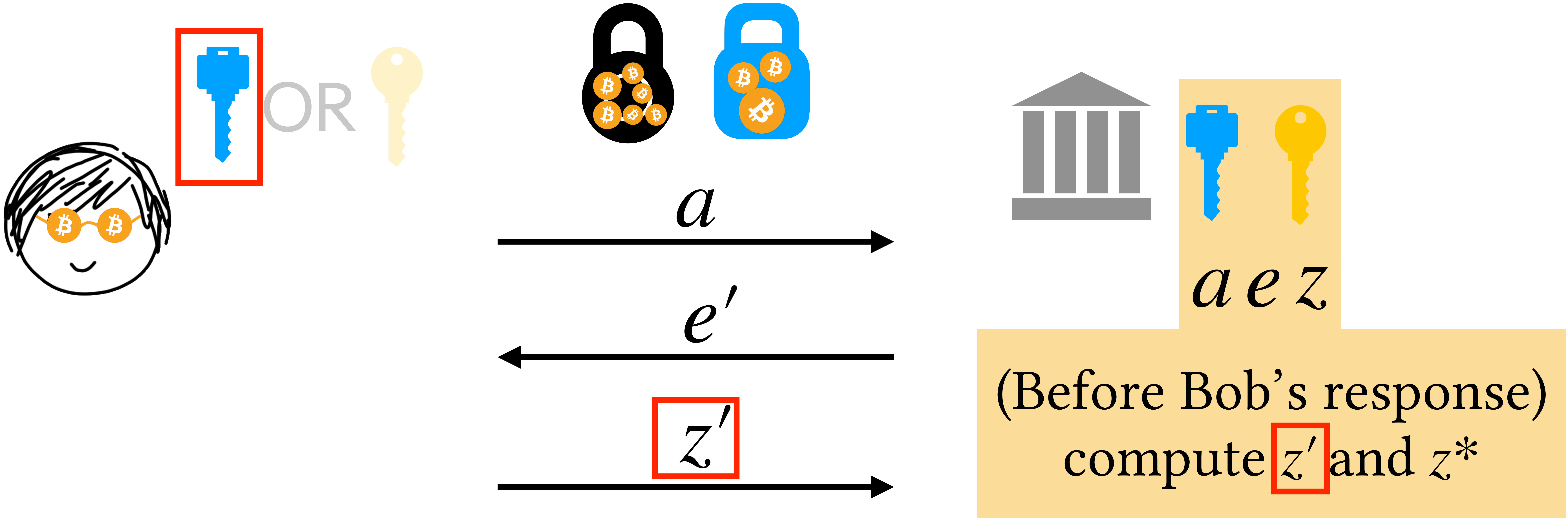

(Before Bob's response) compute $\boxed{z'}$ and z^*

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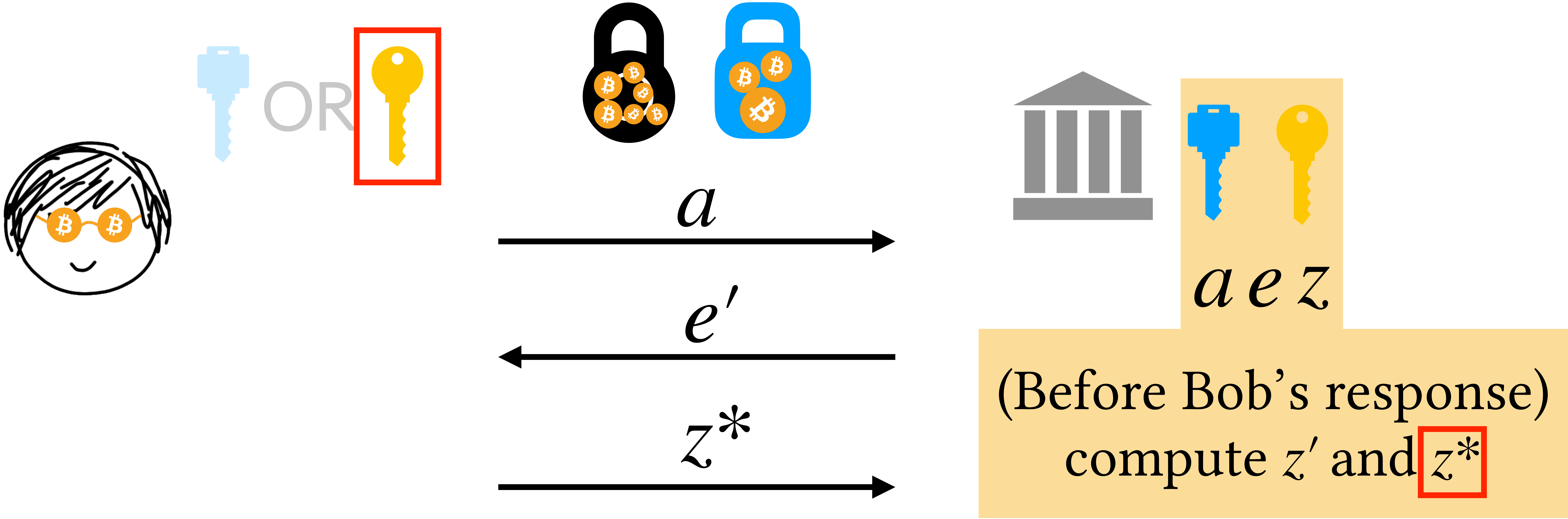

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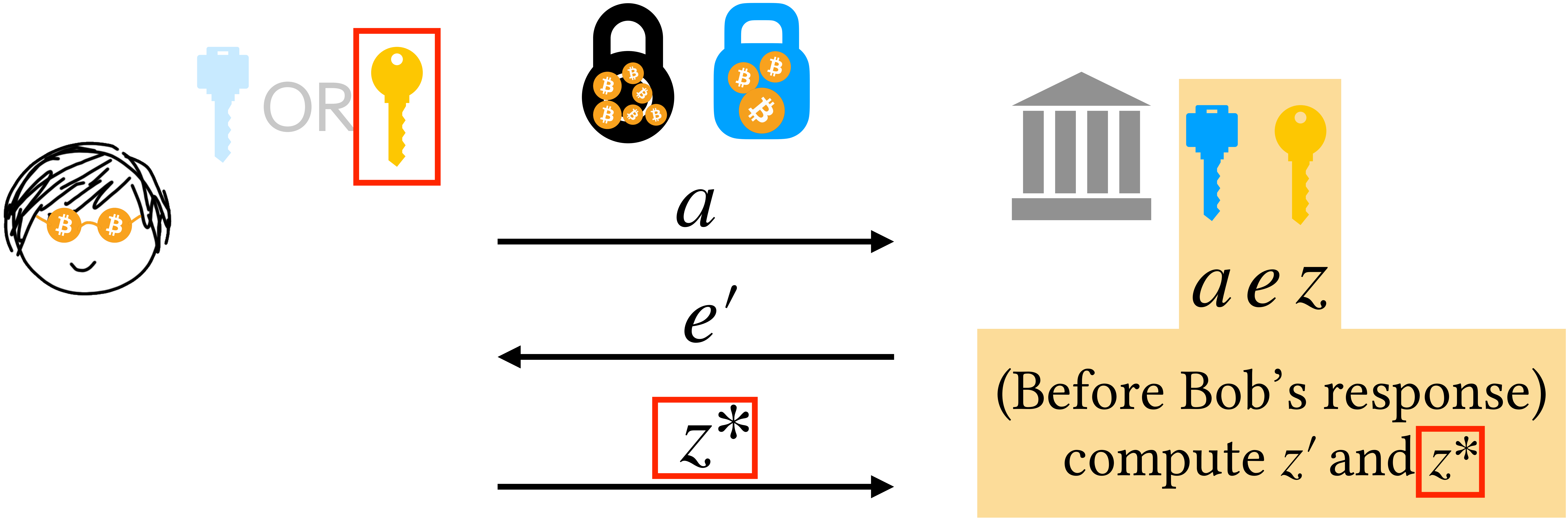

a

e'

$\boxed{z^*}$

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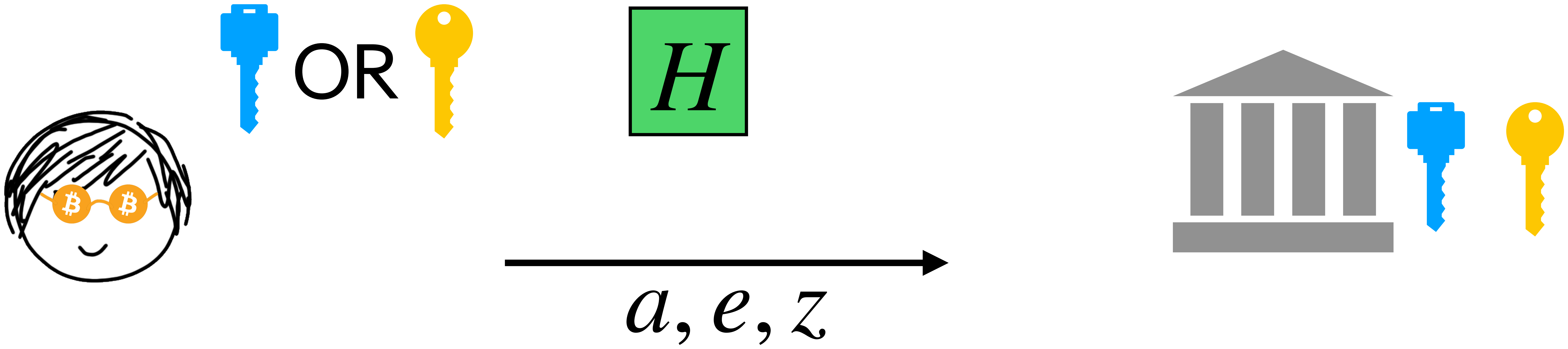
(Before Bob's response) compute z' and $\boxed{z^*}$

Attack Strategy



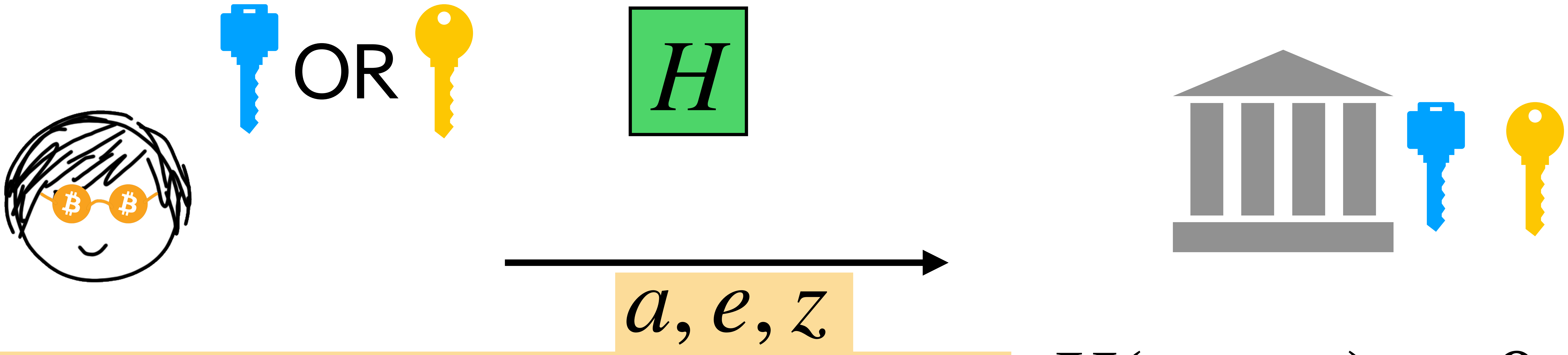
OR

H

a, e, z

$$H(a, e, z) = 0$$

Attack Strategy



Reveals nothing about Bob's key in isolation

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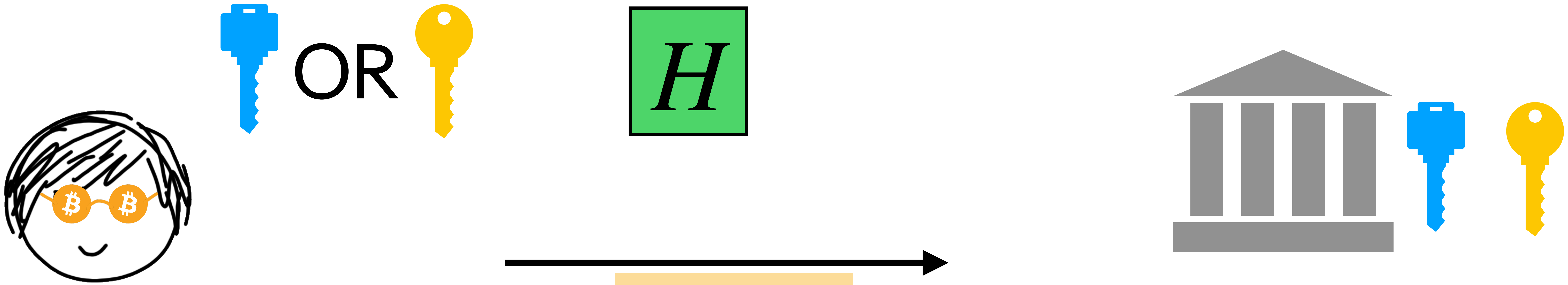


a, e, z

Reveals nothing about Bob’s key in isolation

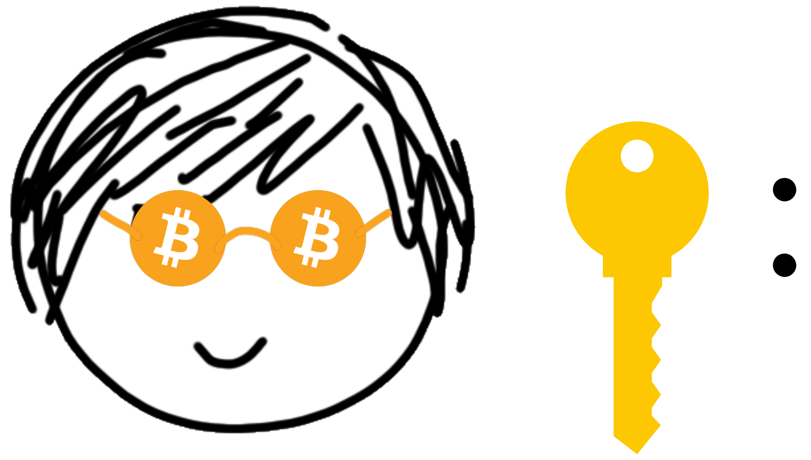
$H(a, e, z) = 0$

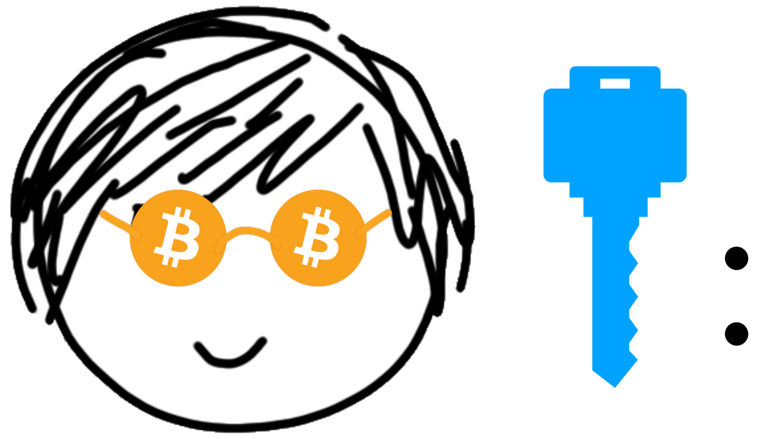
- Imagine we could ask Bob to answer challenge e' …his answer (z' or z^*) would determine which key he has

- Turns out we can achieve this effect by probing H (with no special privileges)

Probing Strategy

Common a

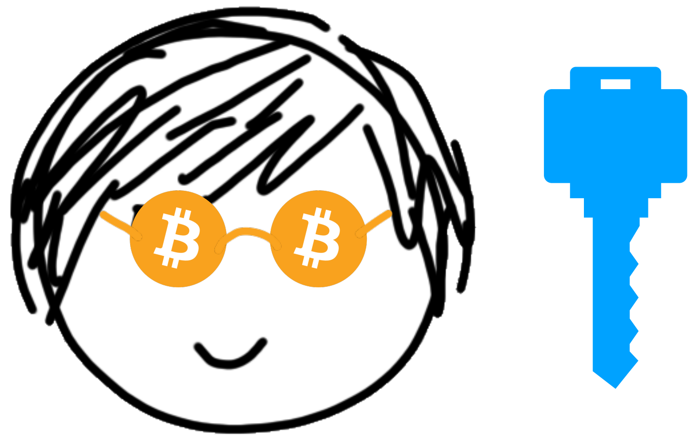




(e, z)

If both possibilities "agree" at e, then they "disagree" at any $e' \neq e$

Probing Strategy



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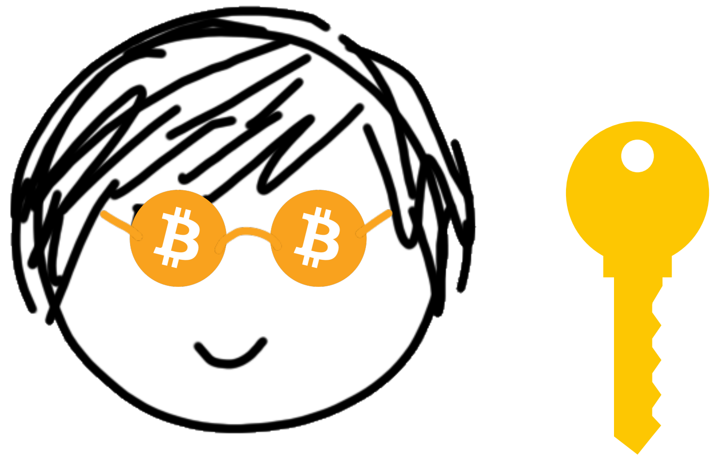

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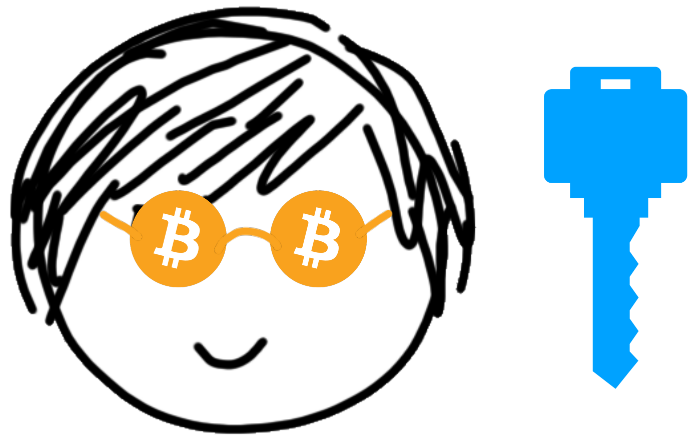
(e, z)

Probing Strategy



:

$(0,\ z_0')$

Common a

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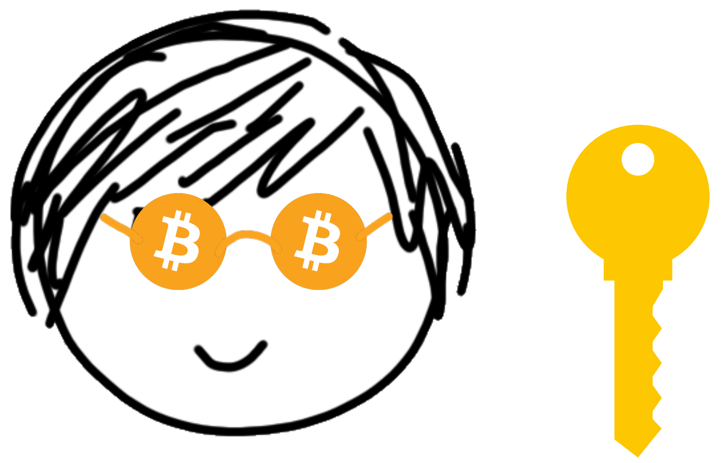
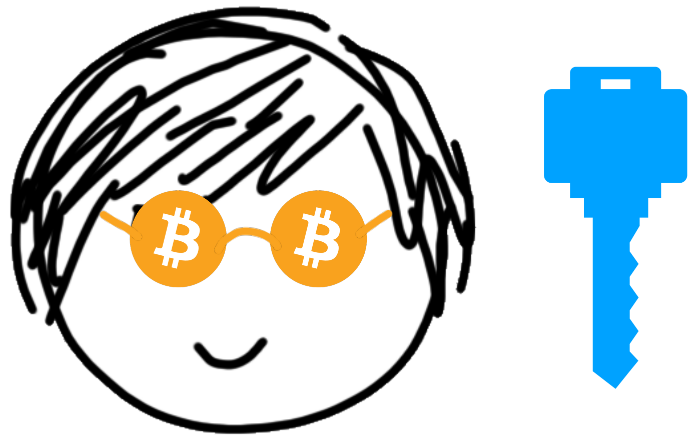


: $(0,\ z_0^*)$

$(e\,,z)$

Probing Strategy



:

$(0,\ z'_0)$

$(1,\ z'_1)$

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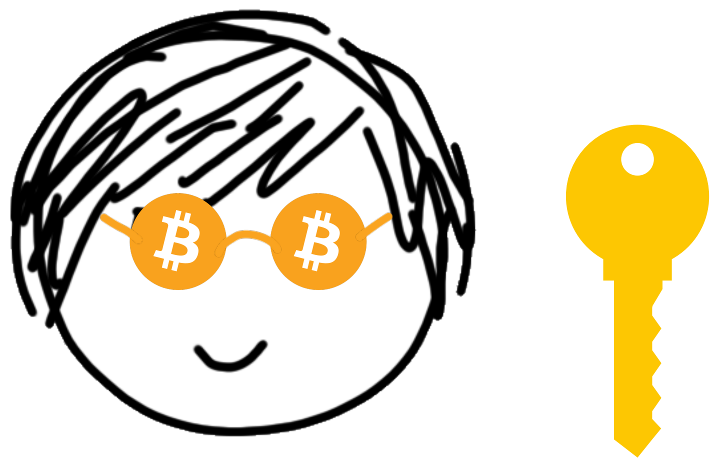
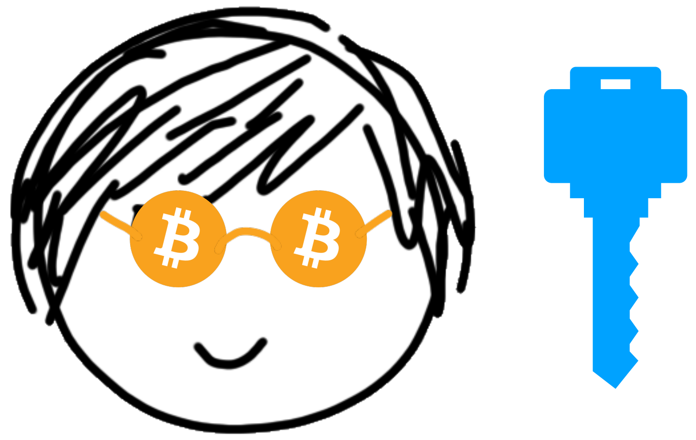


: $(0,\ z_0^*)$ $(1,\ z_1^*)$ $(e\,,z)$

Probing Strategy



:

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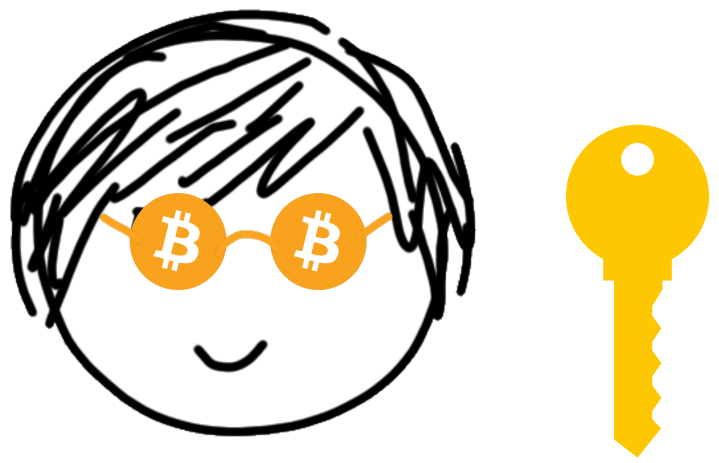
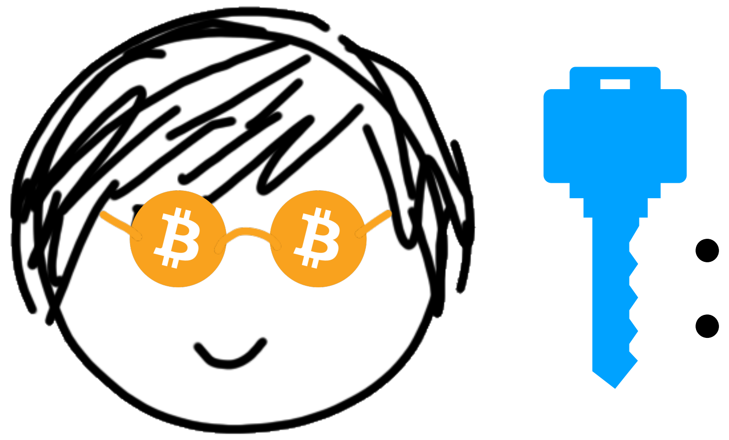


: $(0,\ z^*_0)$ $(1,\ z^*_1)$ $\cdots$ $(e\ ,z\ )$

Probing Strategy

Given (a, e, z) produced by Fischlin's compiler, we can test which path is "plausible"



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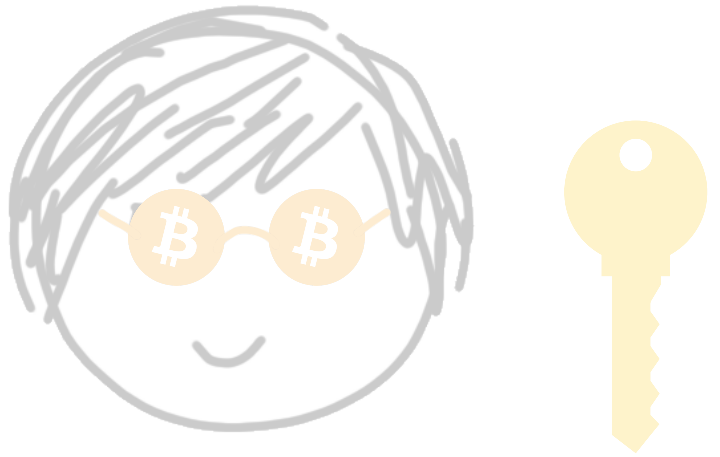
H



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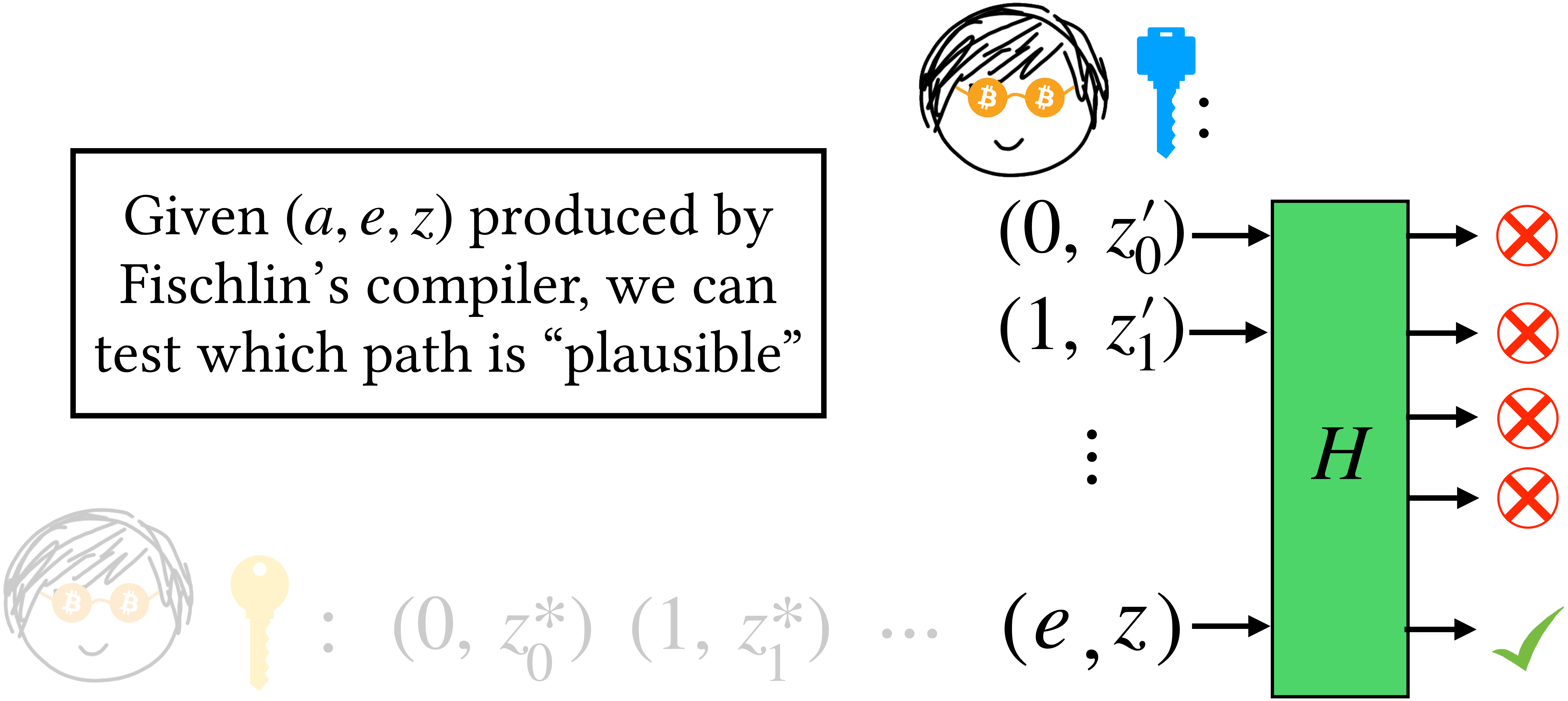

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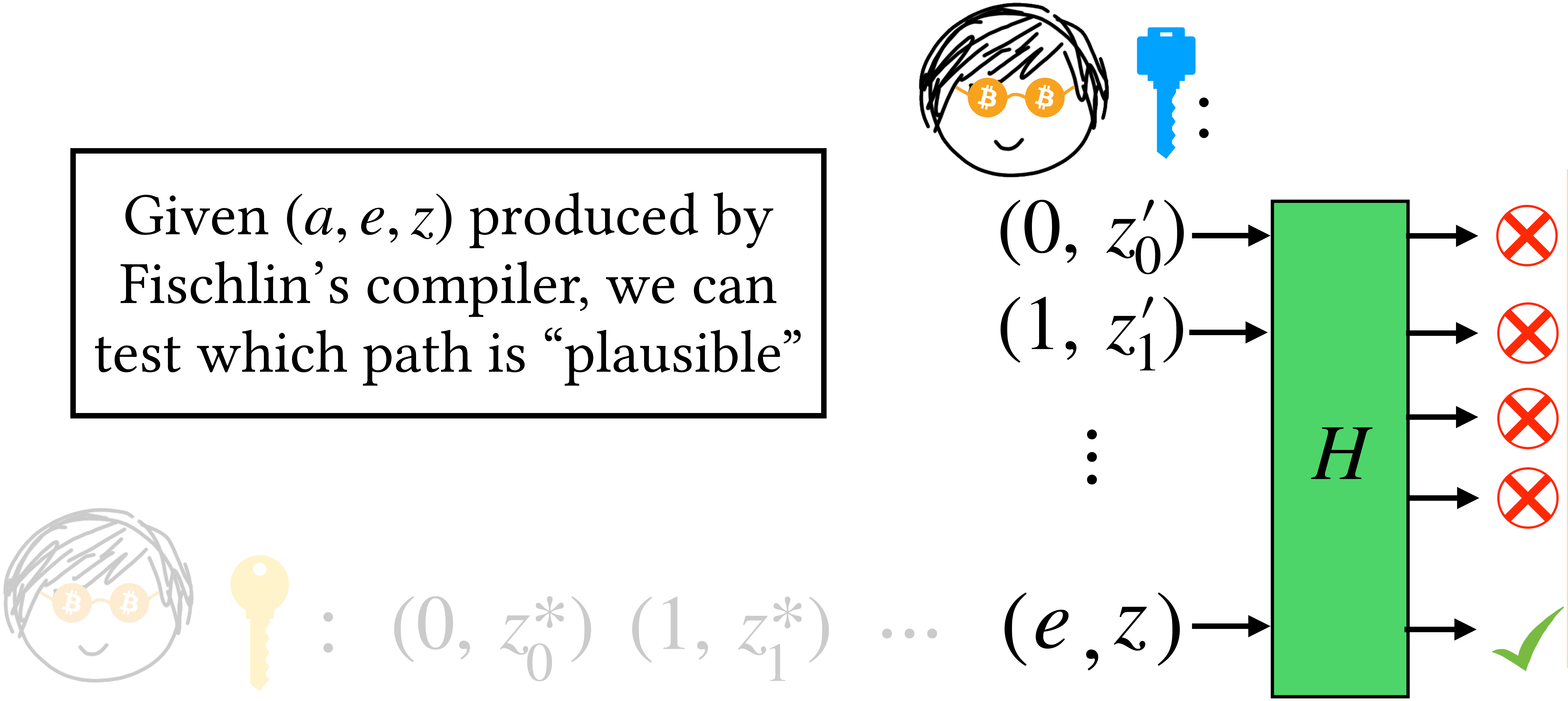

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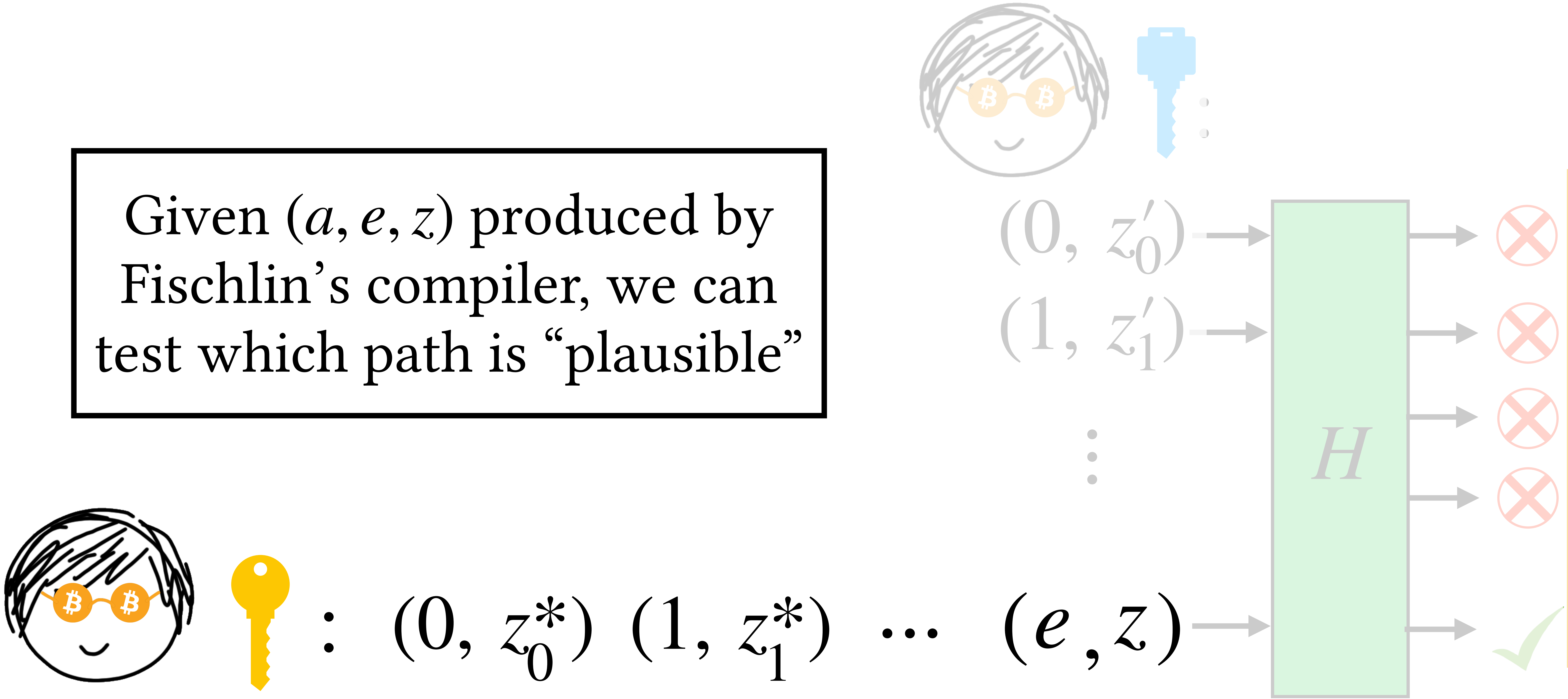

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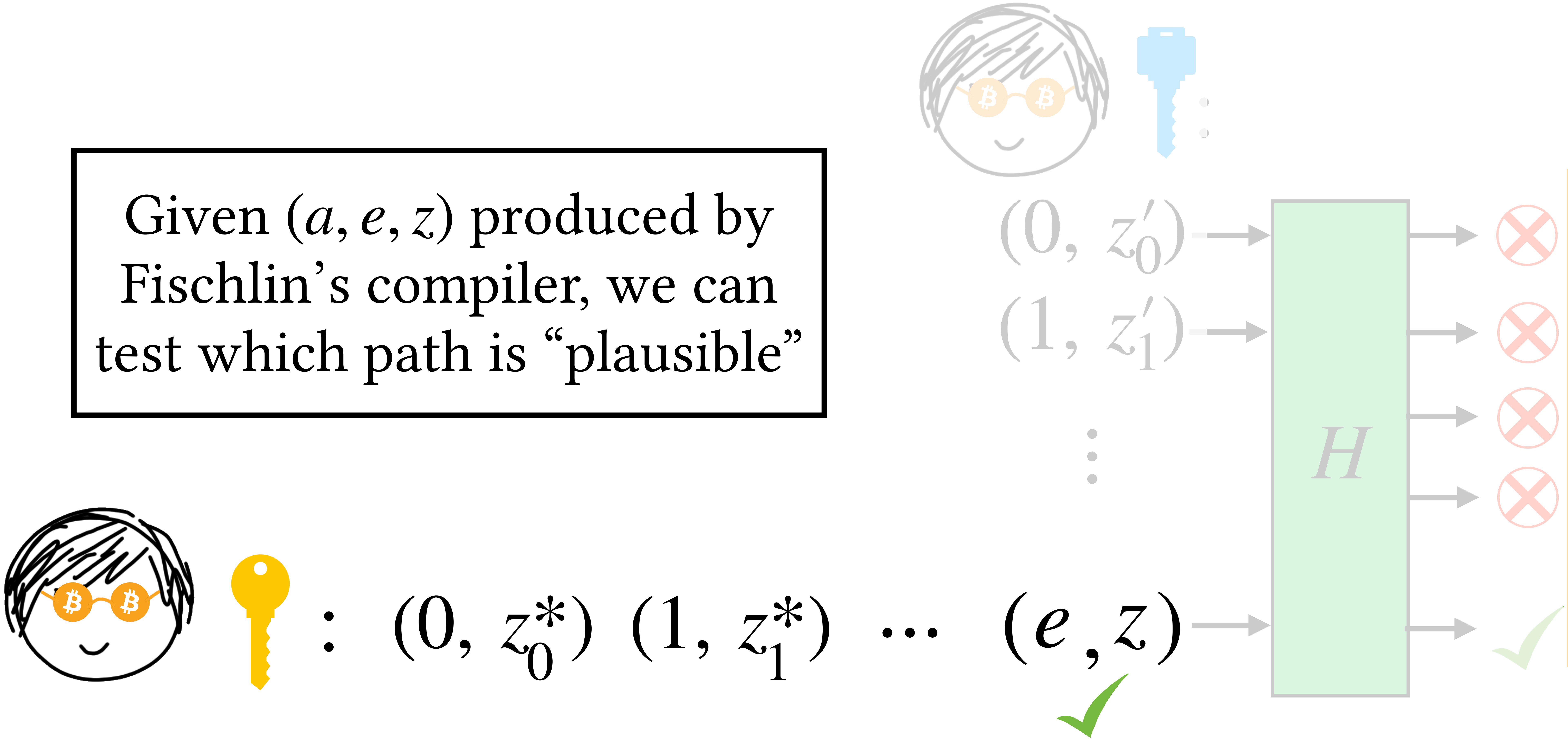

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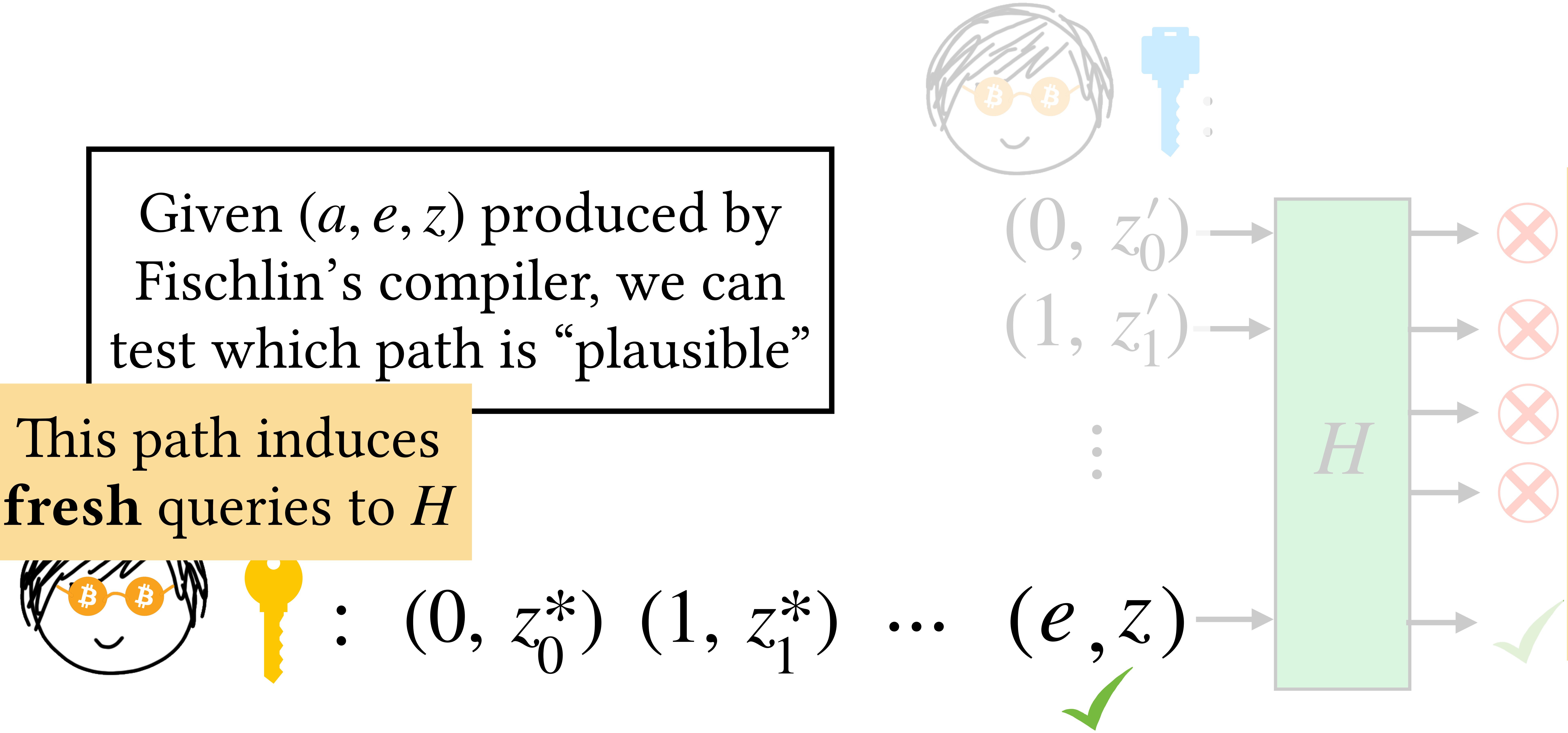
This path induces **fresh** queries to H



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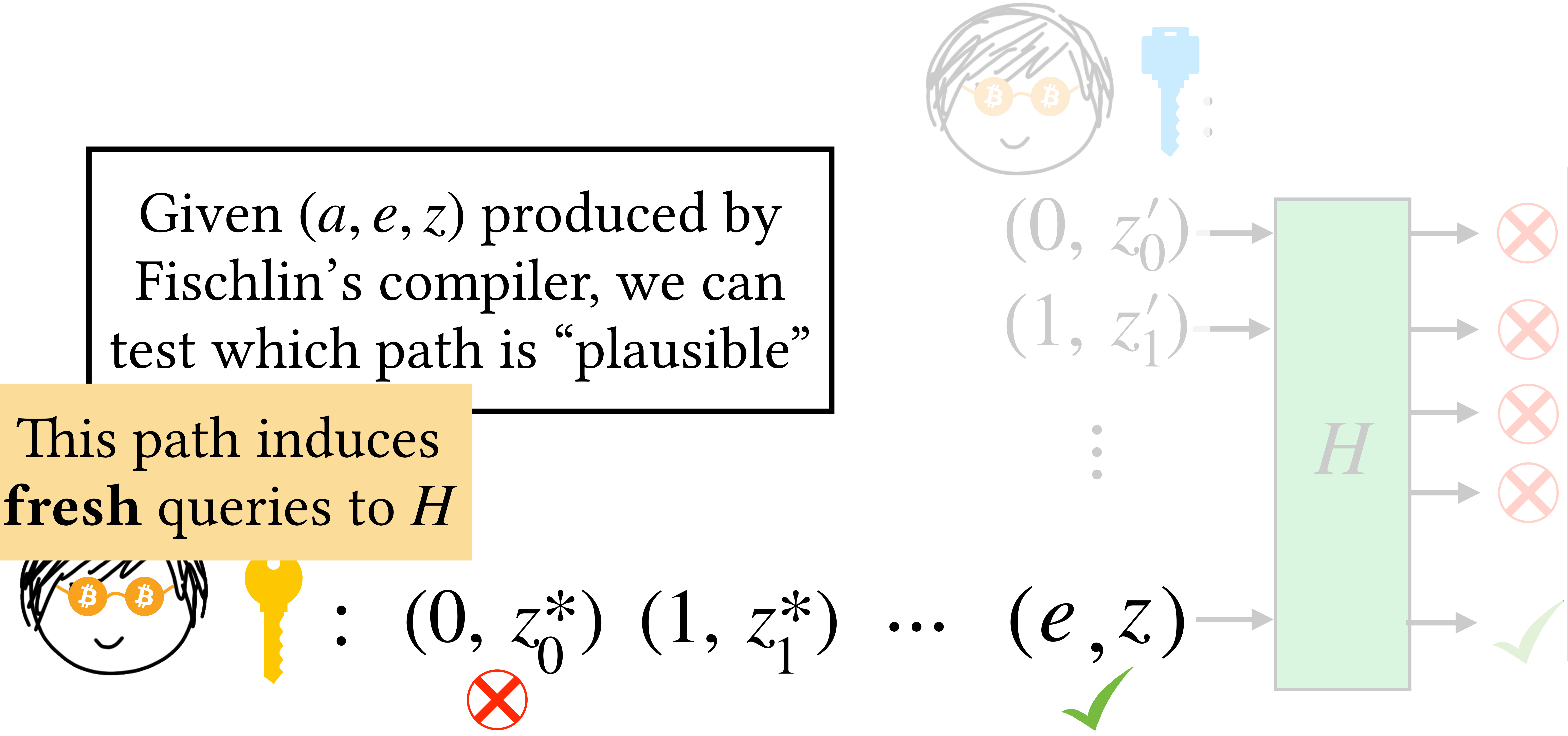

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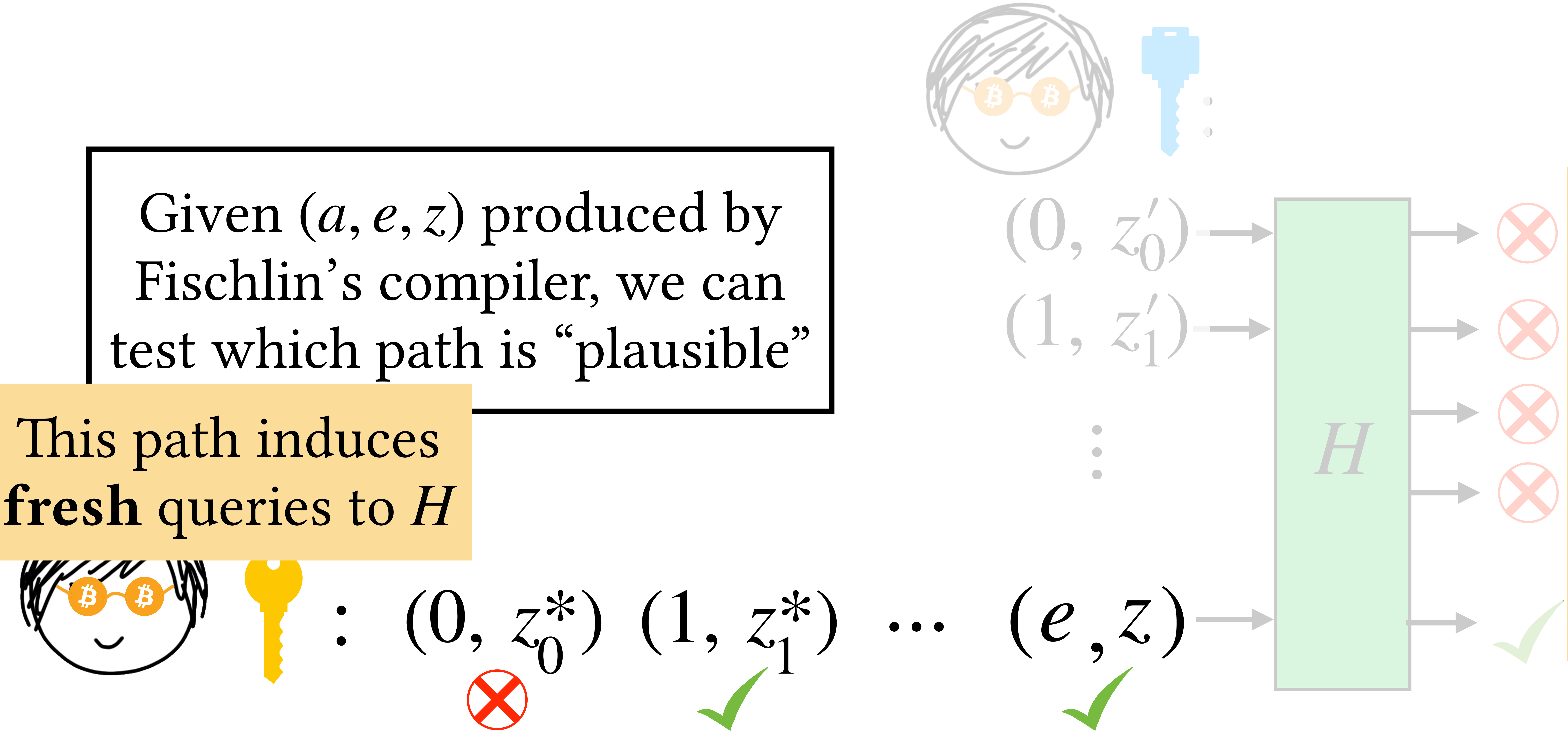

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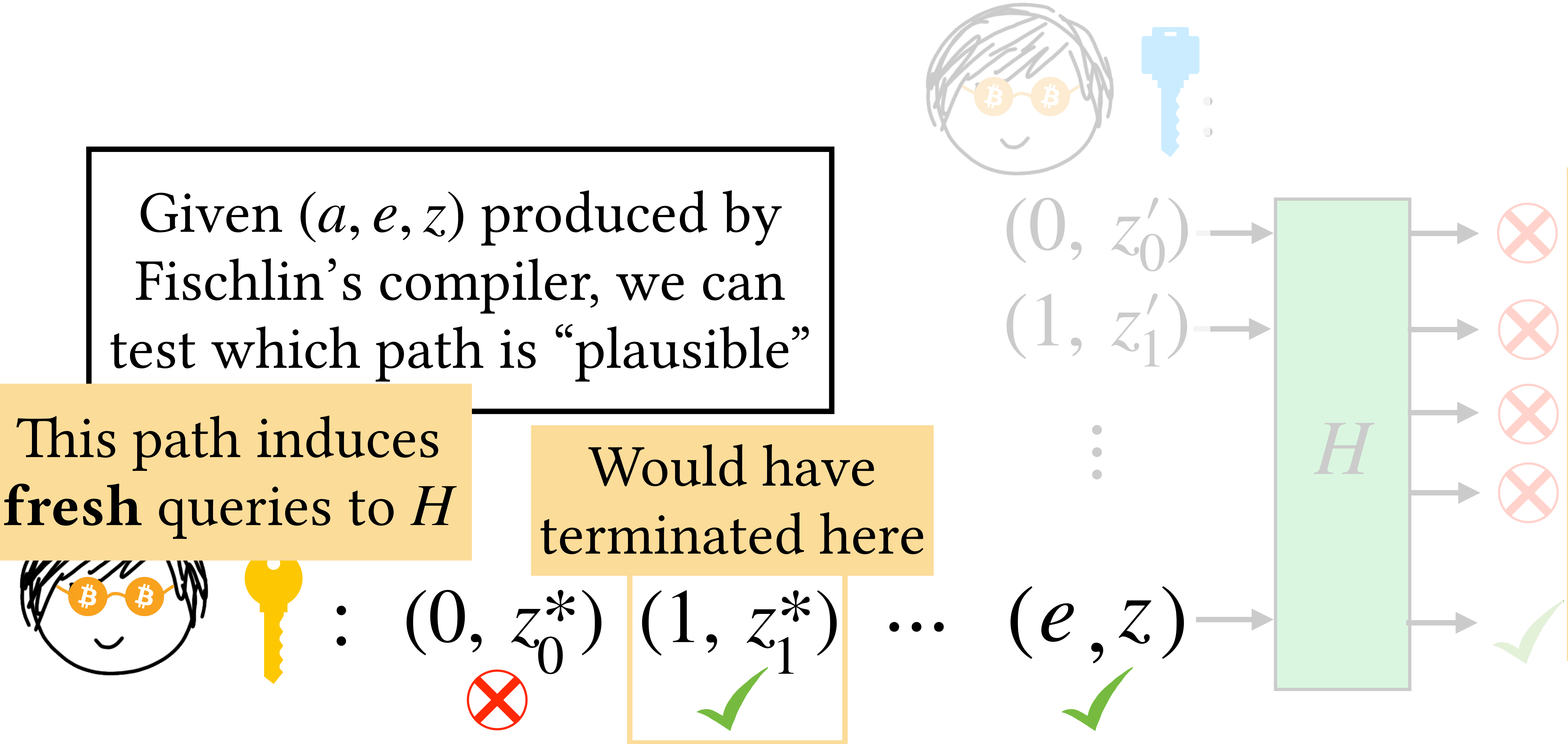


This path induces **fresh** queries to H

Would have terminated here

W.h.p., only one path—induced by one of the two keys—is plausible

How to Fix it? [Ks 22]

- The probing strategy very strongly depends on being able to "re-trace" the Prover's steps
  - This is enabled by the deterministic nature of Fischlin's compiler

- We showed that randomizing the order in which the Prover tries challenges will fix the problem

- We strengthen Fischlin's technique to be good enough to apply to most useful Sigma protocols

In Summary

- We saw what non-interactive zero-knowledge proofs of knowledge are, how they can be used

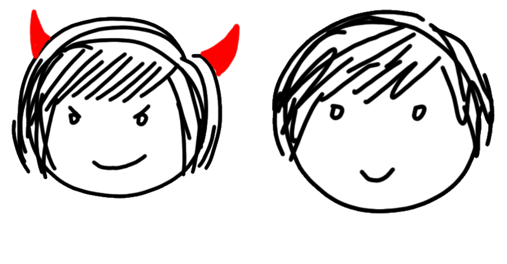
- We got a taste for how they are designed and analysed, and how to understand security guarantees like concurrent composition and ROM

- We uncovered a gap in the literature that was glossed over as folklore, and saw how it turned out to be a vulnerability
  (and briefly discussed how it's now fixed)

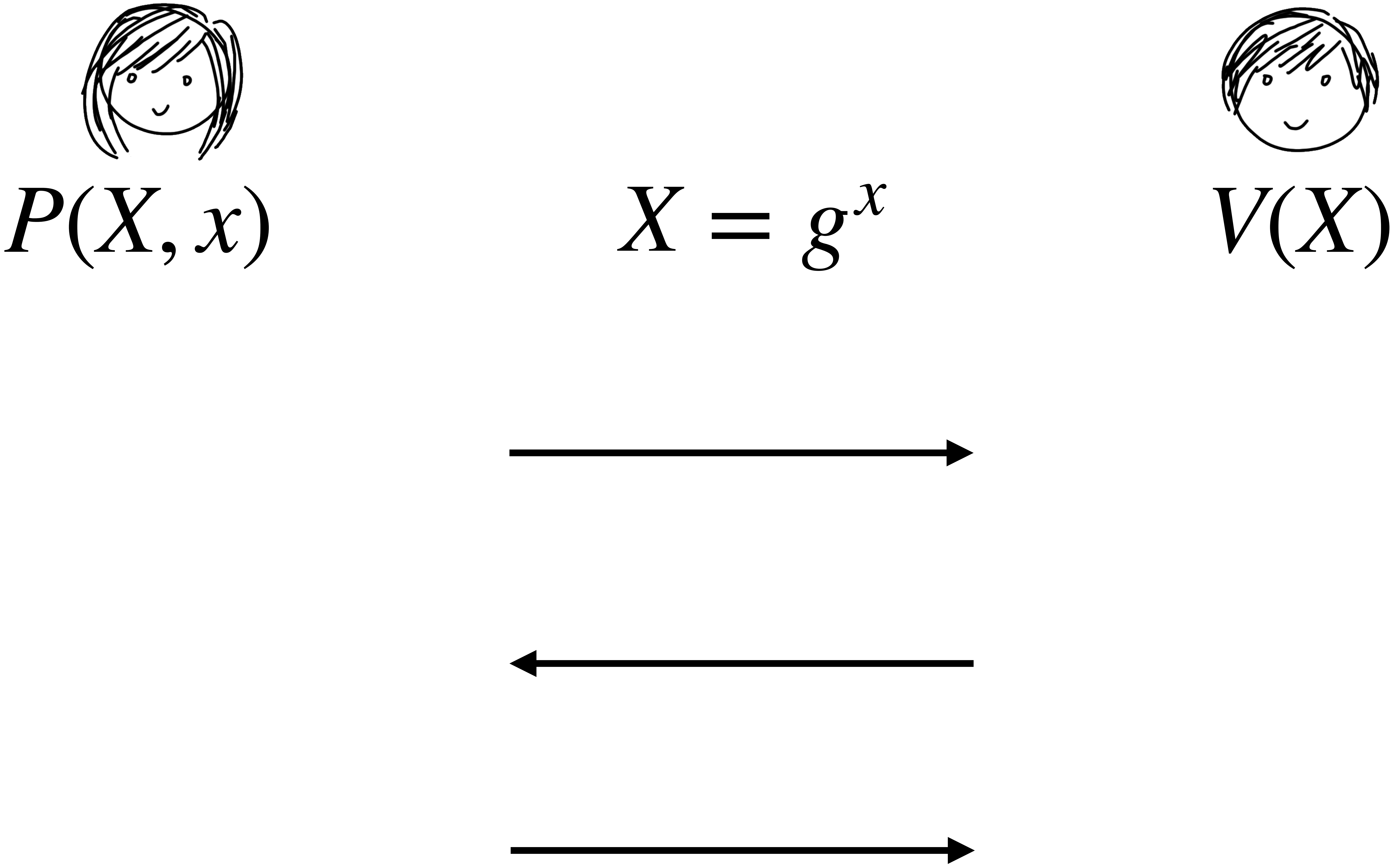
Questions?

eprint.iacr.org/2022/393

Thanks **Eysa Lee** for



Example: Schnorr PoK of Discrete Logarithm



$P(X, x)$

$X = g^x$

$V(X)$

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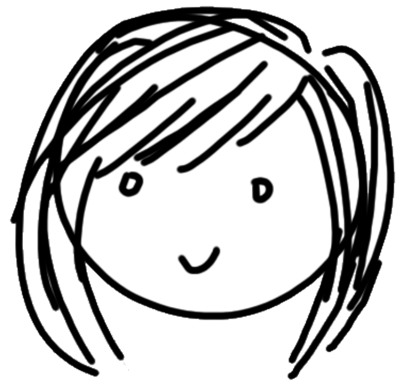
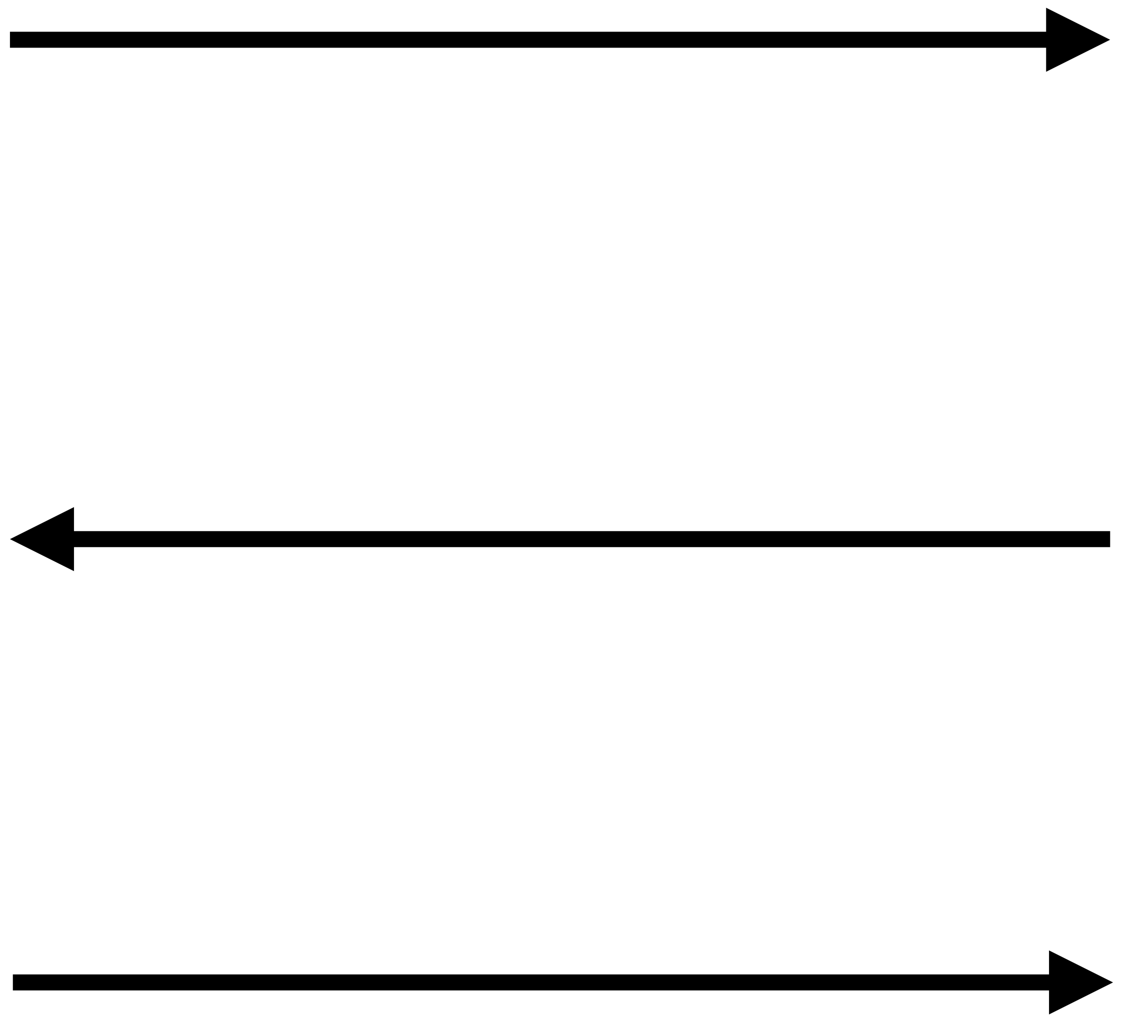


$P(X, x)$ $\quad\quad$ $X = g^x$ $\quad\quad$ $V(X)$

$r \leftarrow \mathbb{Z}_q$



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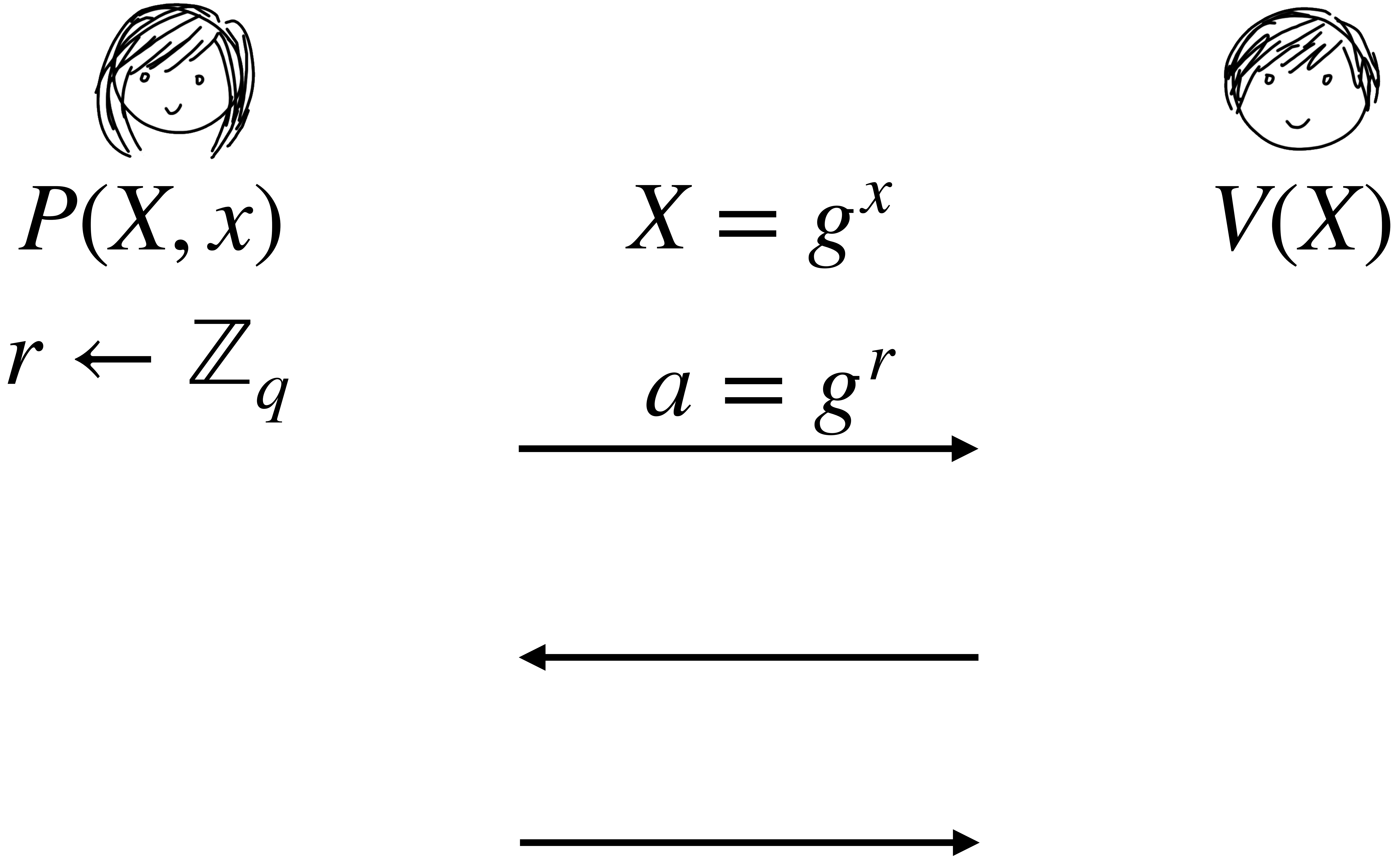

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$a = g^r$

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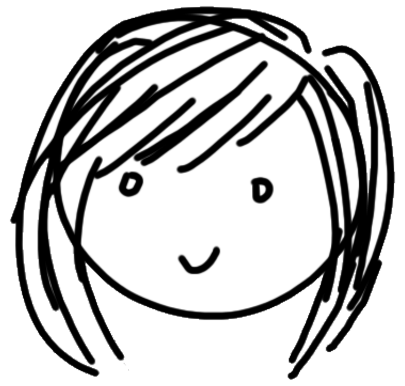

$P(X, x)$ $\qquad$ $X = g^x$ $\qquad$ $V(X)$

$r \leftarrow \mathbb{Z}_q$

$$\xrightarrow{\qquad a = g^r \qquad}$$

$$\xleftarrow{\qquad e \in \mathbb{Z}_q \qquad}$$

$$\xrightarrow{\qquad\qquad\qquad}$$

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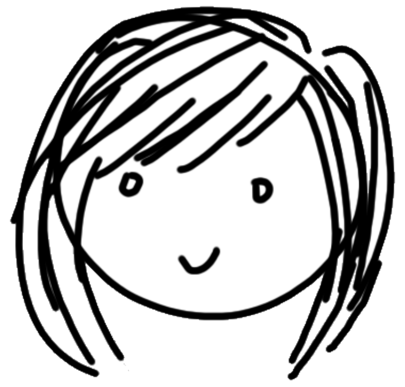

$P(X, x)$ $\quad\quad X = g^x \quad\quad$ $V(X)$

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$$\xrightarrow{\quad a = g^r \quad}$$

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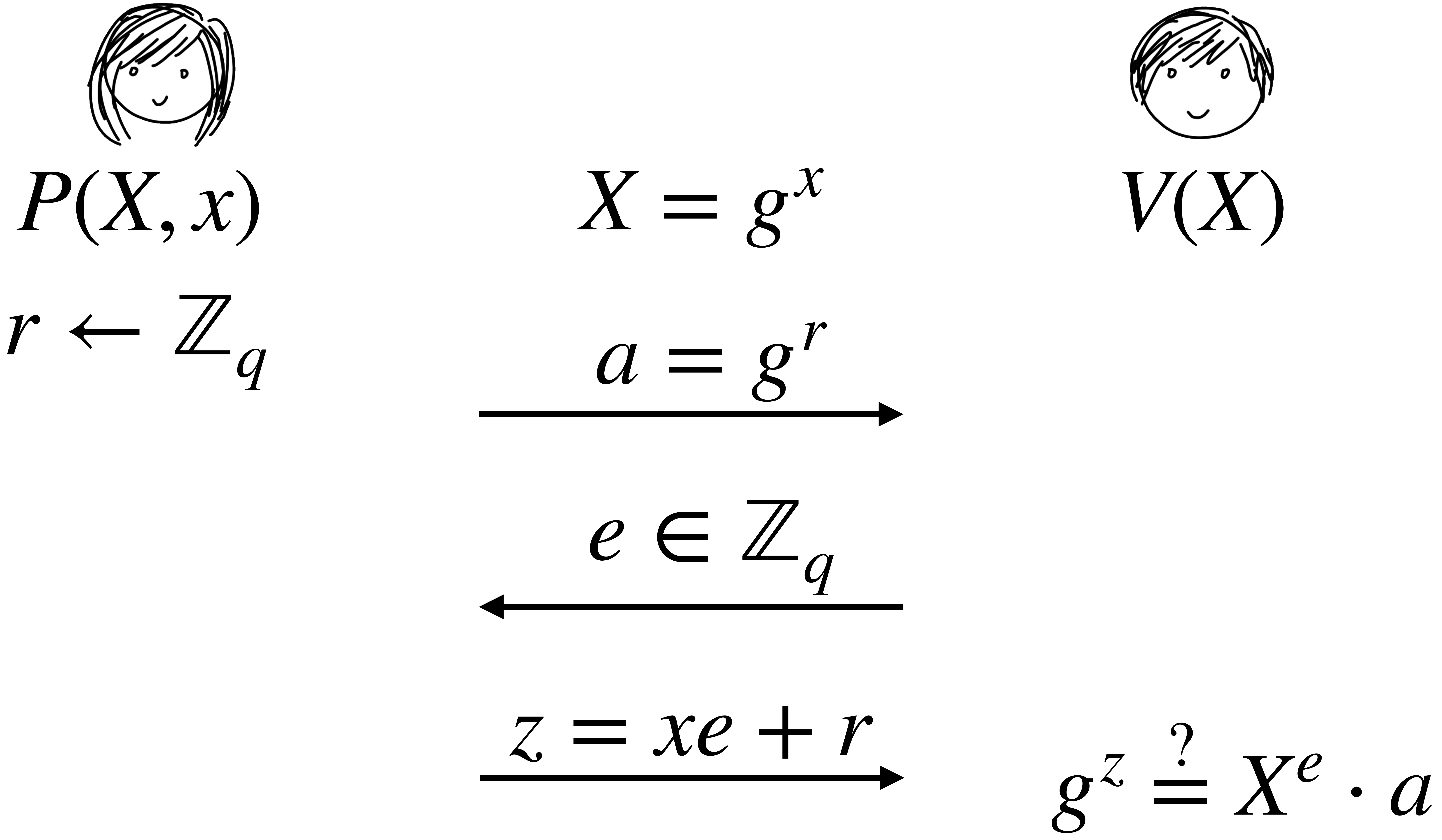

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$$g^z \overset{?}{=} X^e \cdot a$$

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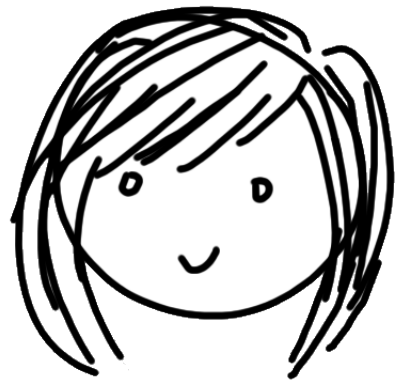

$P(X, x)$ $\qquad X = g^x \qquad$ $V(X)$

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$$\xrightarrow{a = g^r}$$

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$$\xrightarrow{z = xe + r}$$

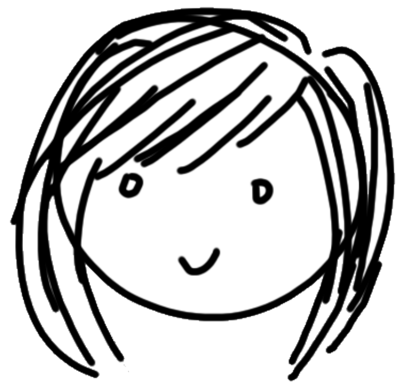
$$g^z \stackrel{?}{=} X^e \cdot a$$

Ext$(a, (e, z), (e', z'))$:

$x = (z' - z)/(e' - e)$

Output x

Example: Schnorr PoK of Discrete Logarithm



$P(X, x)$ $\quad X = g^x \quad$ $V(X)$

$r \leftarrow \mathbb{Z}_q$

$$\xrightarrow{\quad a = g^r \quad}$$

<u>HVZK $\mathcal{S}(e)$:</u>

$z \leftarrow \mathbb{Z}_q$

$a = g^z / X^e$

Output (a, z)

<u>Ext$(a, (e, z), (e', z'))$:</u>

$x = (z' - z)/(e' - e)$

Output x

$$\xleftarrow{\quad e \in \mathbb{Z}_q \quad}$$

$$\xrightarrow{\quad z = xe + r \quad}$$

$$g^z \overset{?}{=} X^e \cdot a$$

The Fiat-Shamir Transform

- [Fiat Shamir 87] provides a simple method to compile any public-coin protocol to a non-interactive proof, given a suitably chosen hash function

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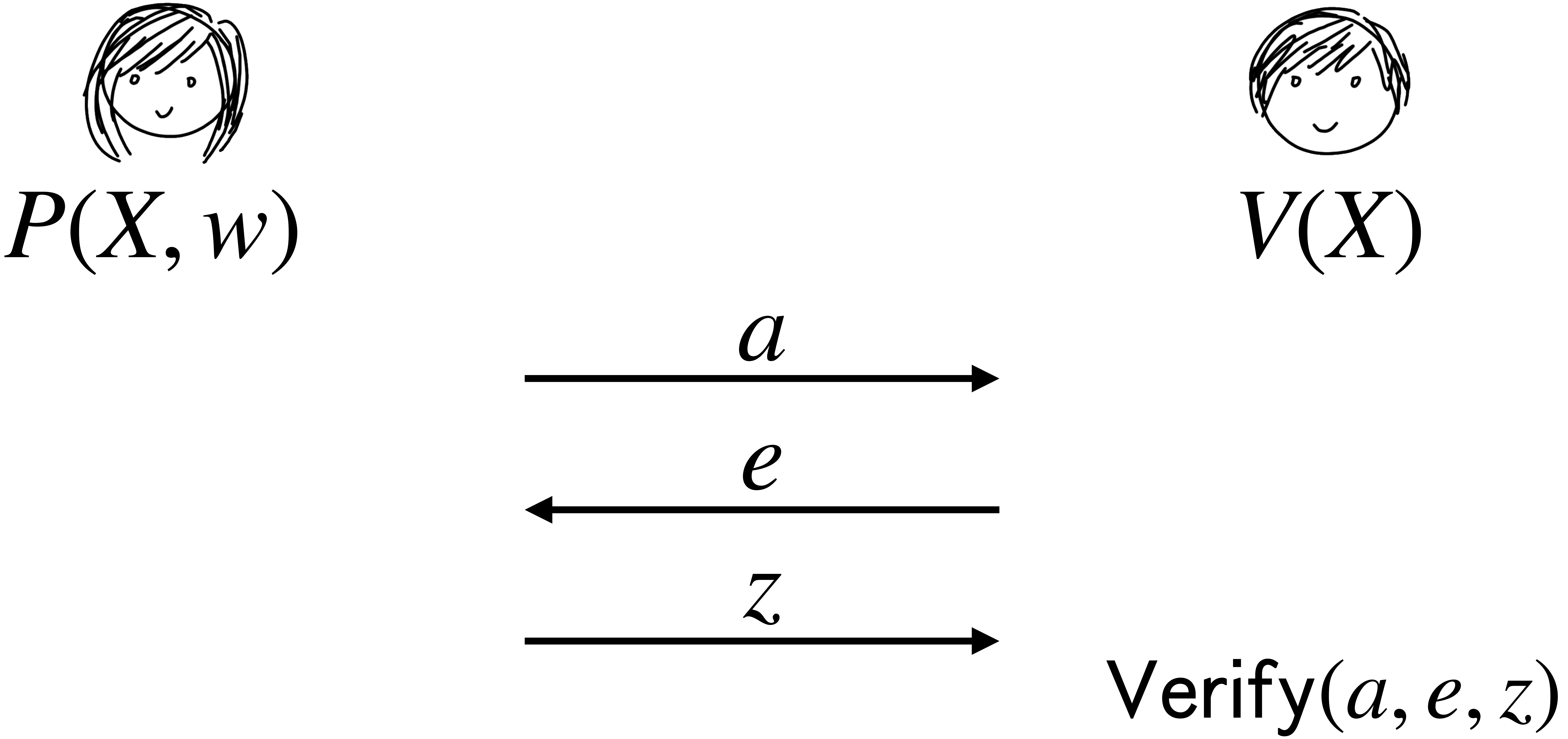

$P(X, w)$

$V(X)$

a

e

z

$\mathsf{Verify}(a, e, z)$

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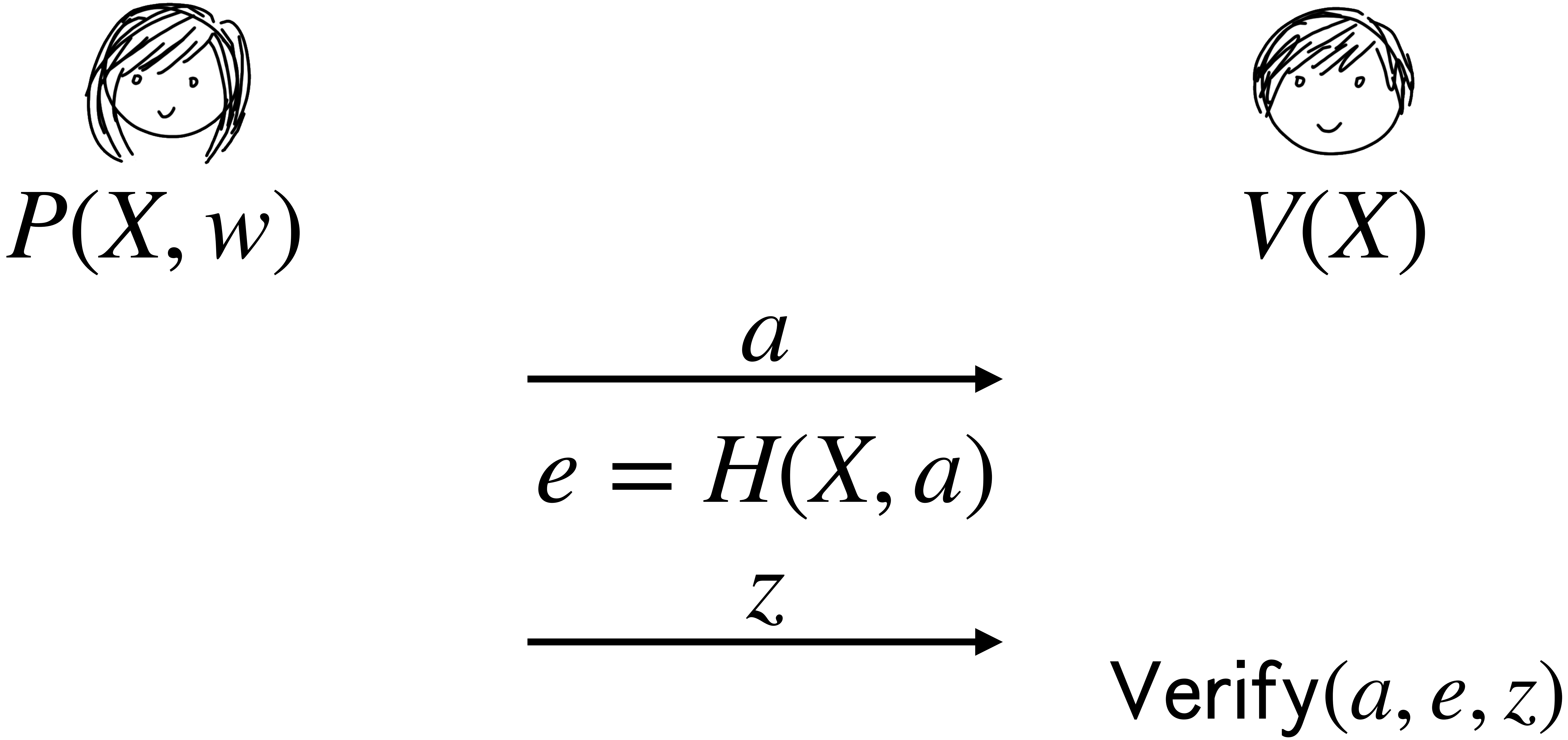

$P(X, w)$

$V(X)$

$$P(X,w) \xrightarrow{a} V(X)$$

$$e = H(X, a)$$

$$P(X,w) \xrightarrow{z} V(X)$$

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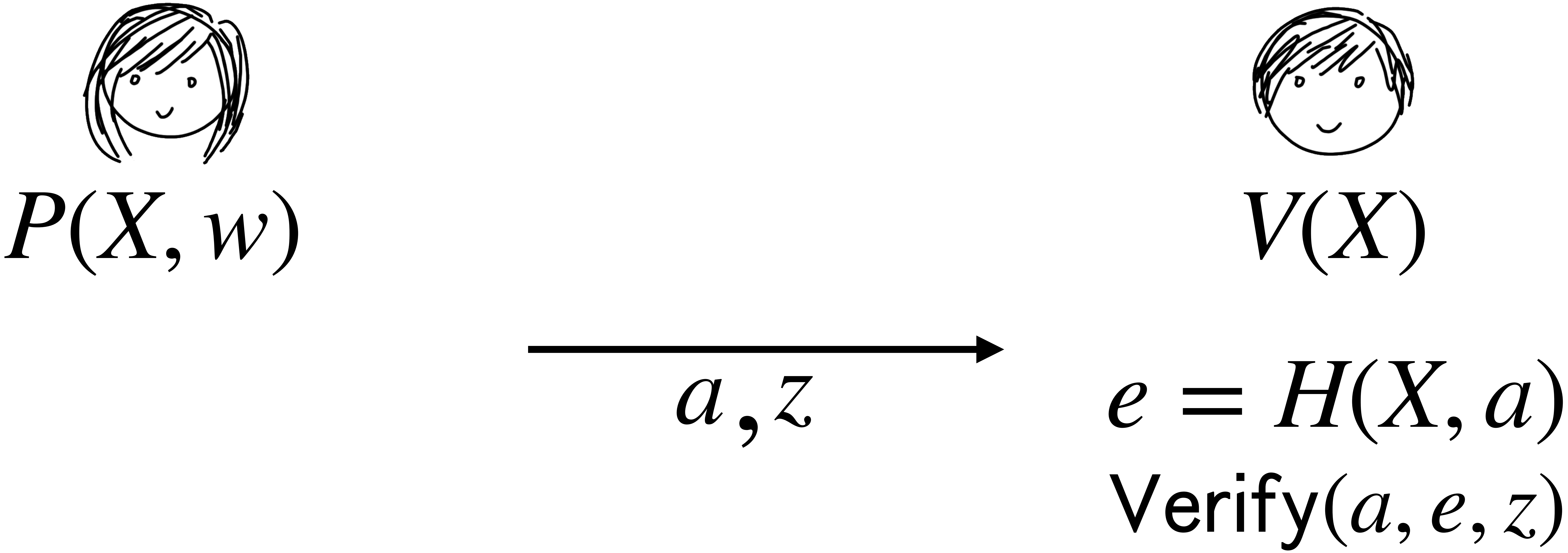


$P(X, w)$

$V(X)$

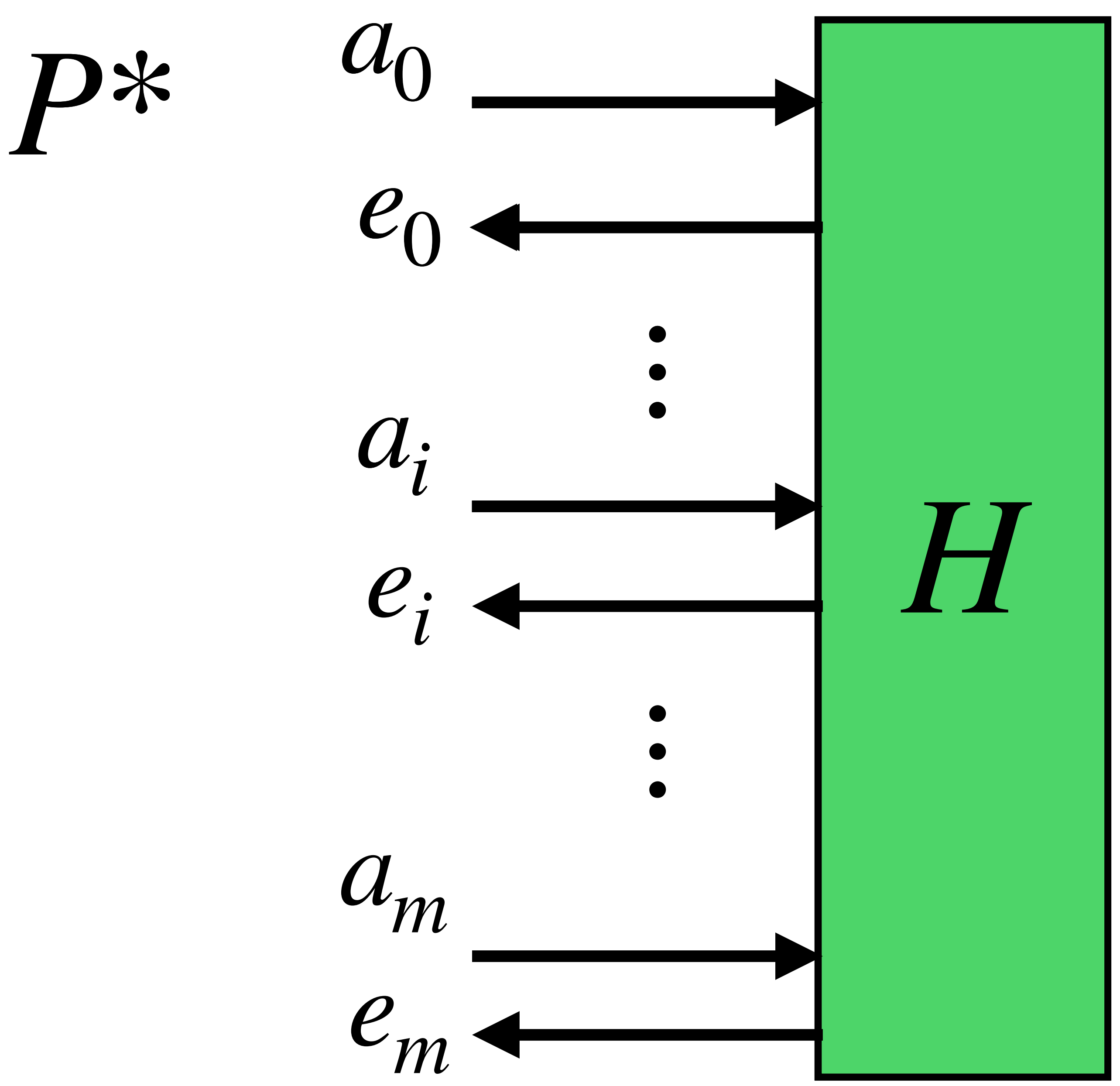
$$\xrightarrow[\quad a,z \quad]{}$$

$e = H(X, a)$

$\mathsf{Verify}(a, e, z)$

Fiat-Shamir: Security

- "Forking" extraction strategy in Random Oracle Model [Pointcheval Stern 96]:



Output (a_i, e_i, z_i)

Fiat-Shamir: Security

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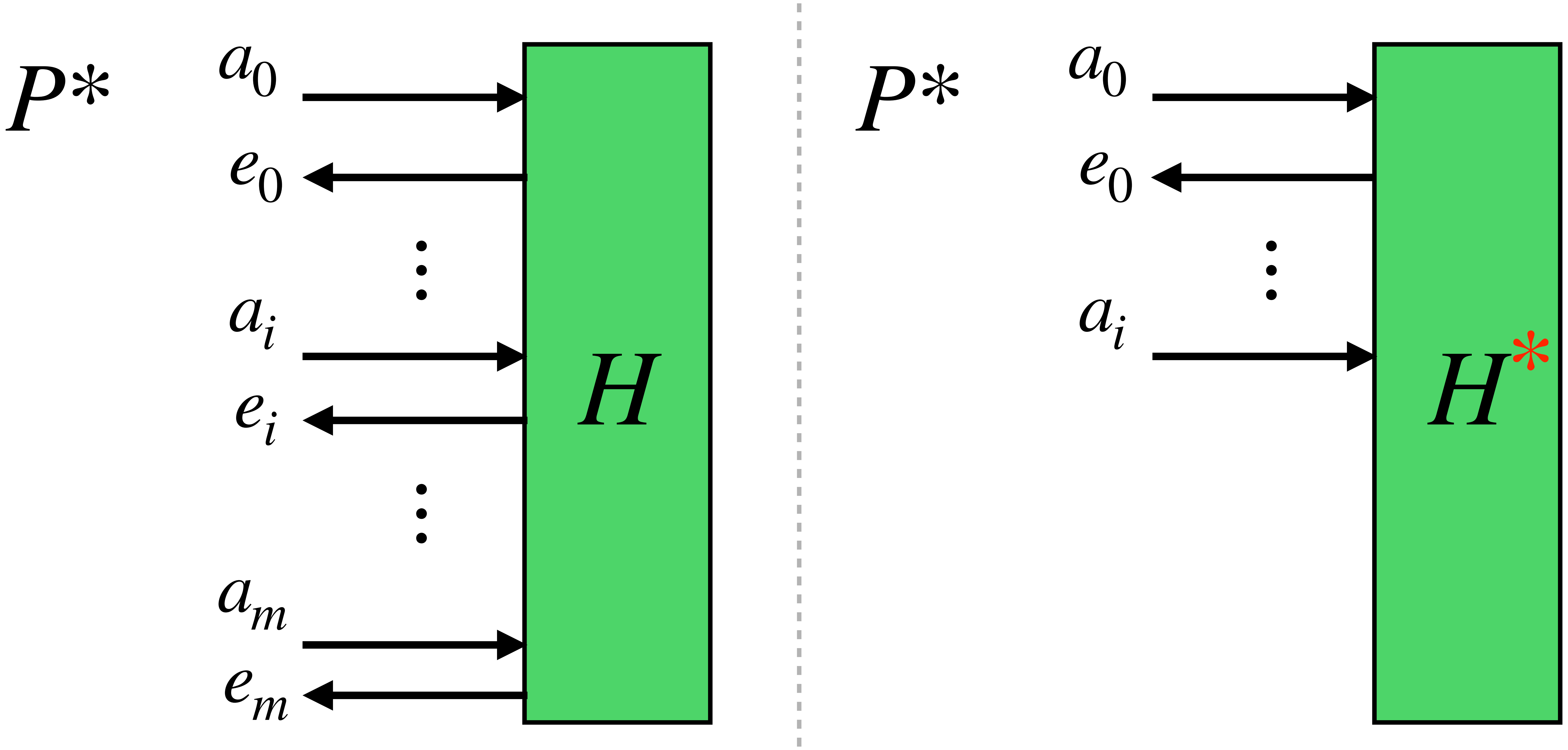

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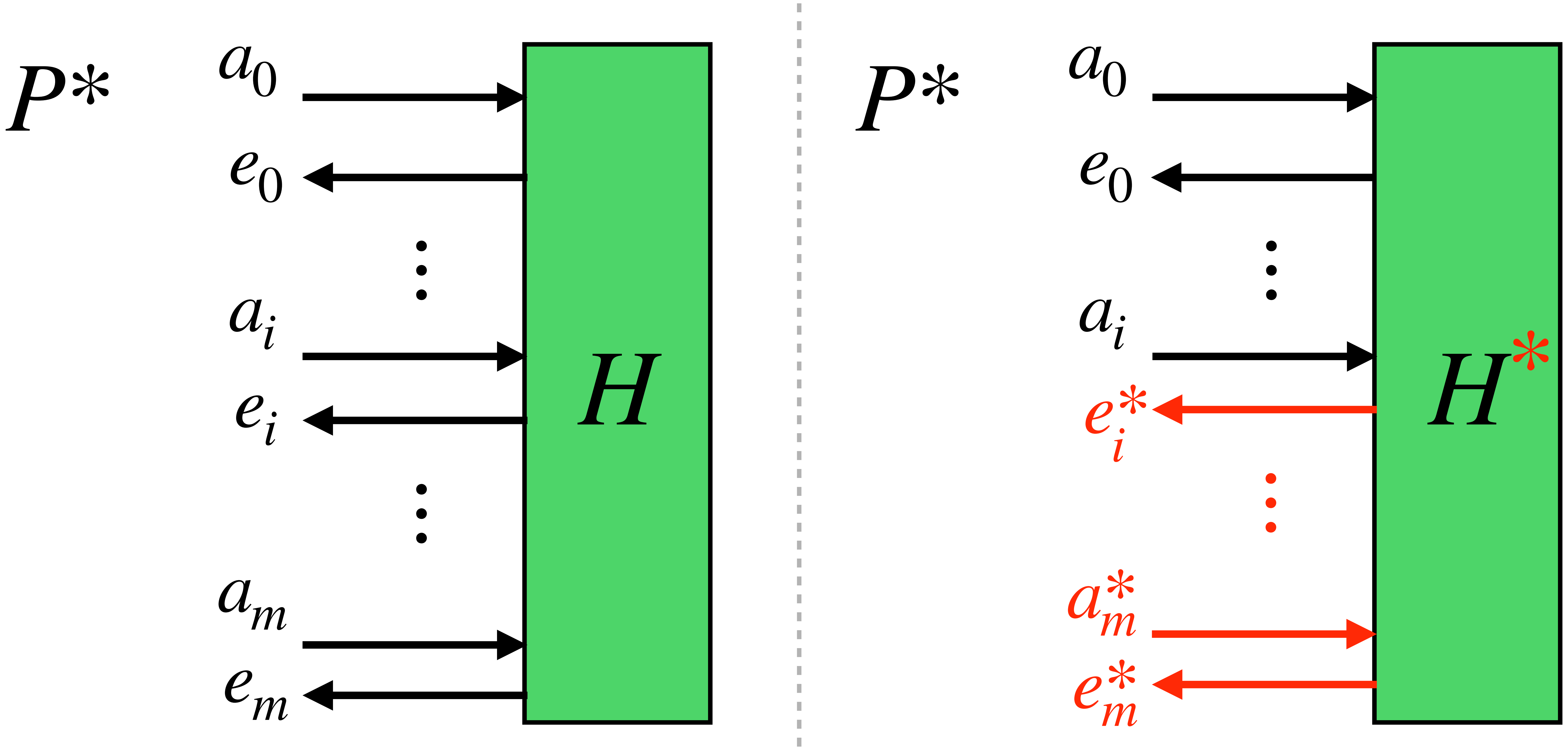

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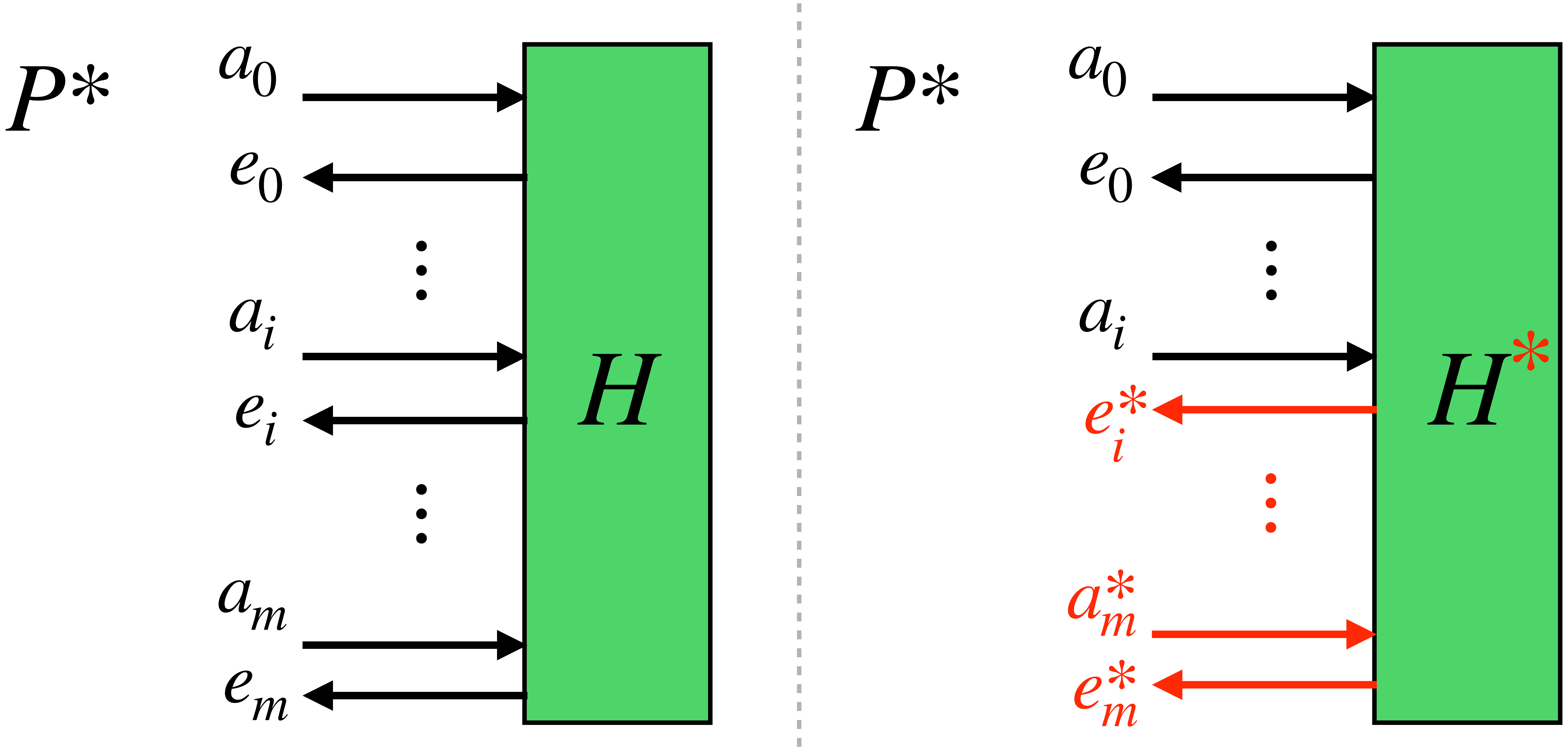

Output (a_i, e_i, z_i)

Output (a_i, e_i^*, z_i^*)

Fiat-Shamir: Security

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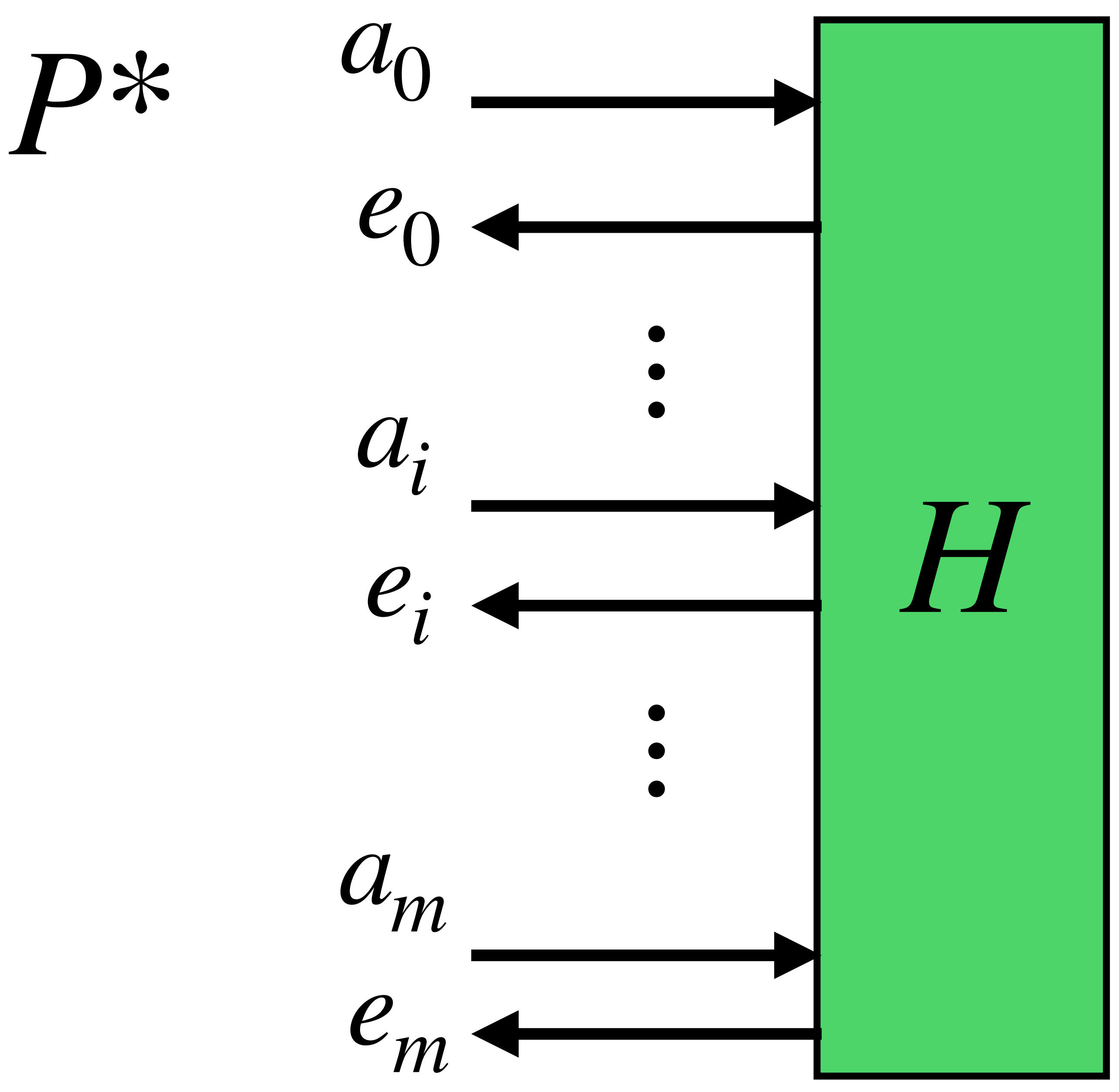

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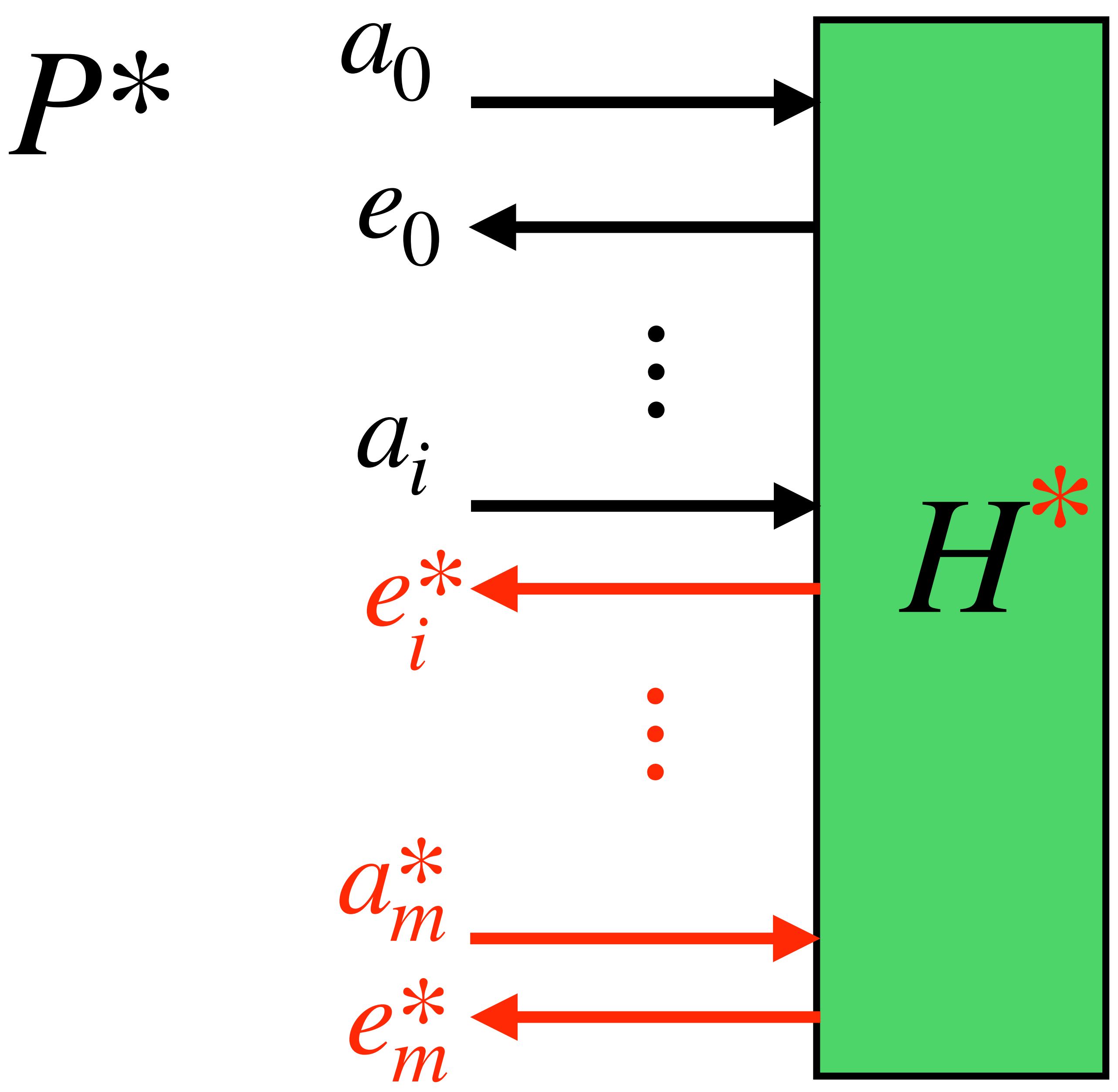


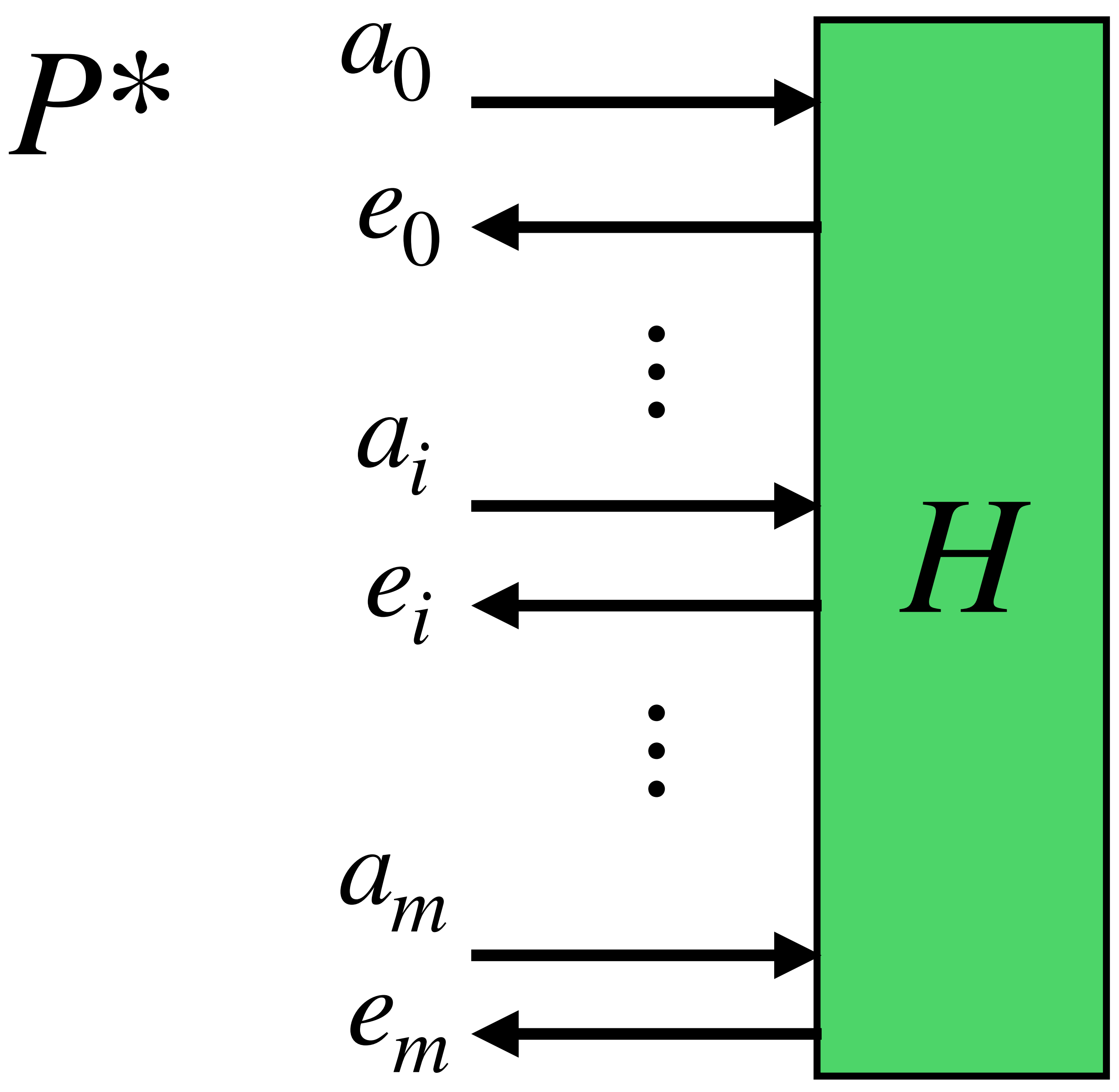
Output (a_i, e_i^*, z_i^*)

$$\mathsf{Ext}\begin{pmatrix}(a_i, e_i) & (a_i, e_i) \\ \multicolumn{2}{c}{z_i, z_i^*}\end{pmatrix}$$

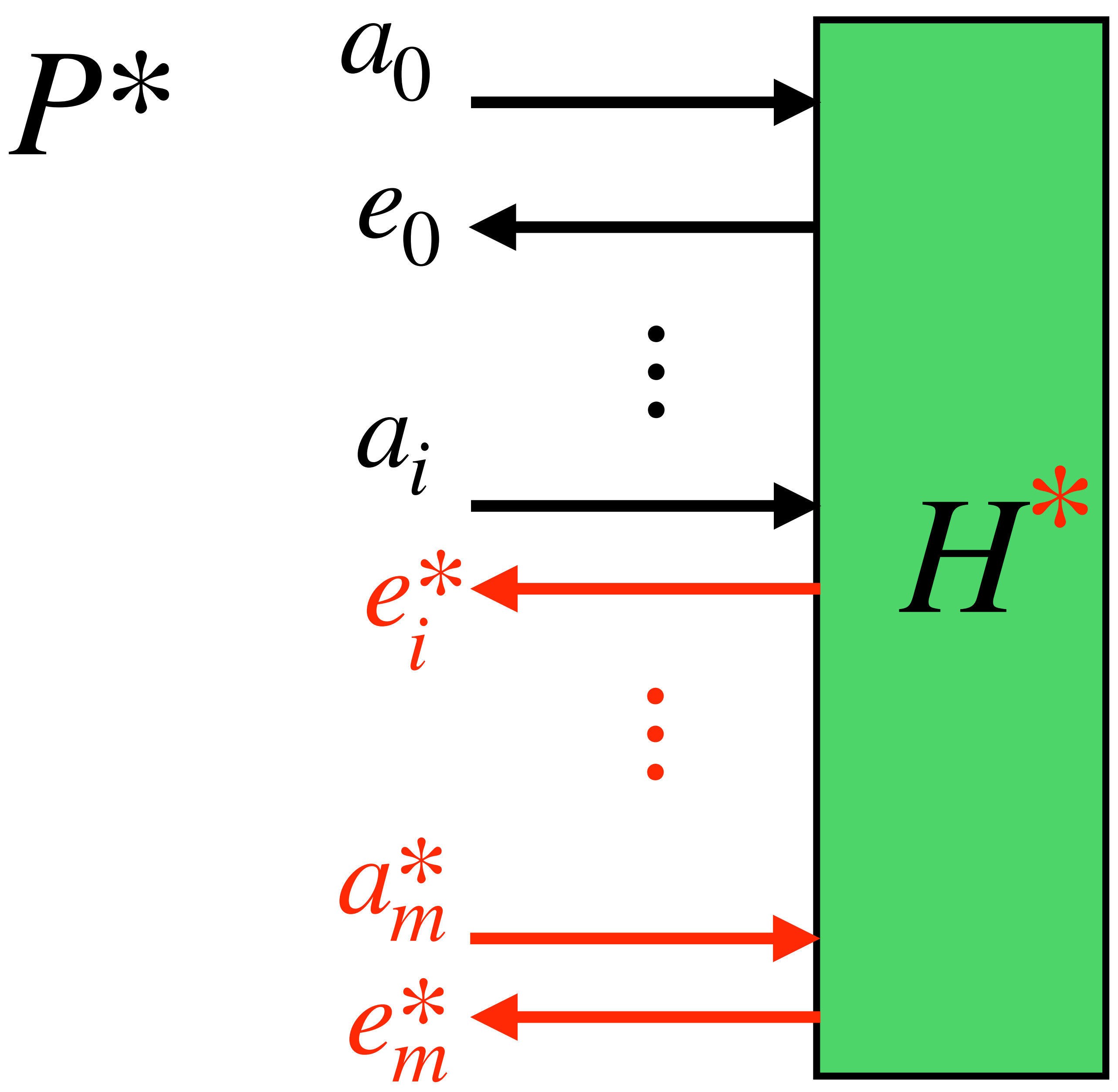
Outputs witness w

Fiat-Shamir: Security

- “Forking” extraction strategy in Random Oracle Model [Pointcheval Stern 96]:



Output (a_i, e_i, z_i)



Output (a_i, e_i^*, z_i^*)

$$\mathsf{Ext}\begin{pmatrix} (a_i, e_i) \;\; (a_i, e_i) \\ z_i, z_i^* \end{pmatrix}$$

Outputs witness w

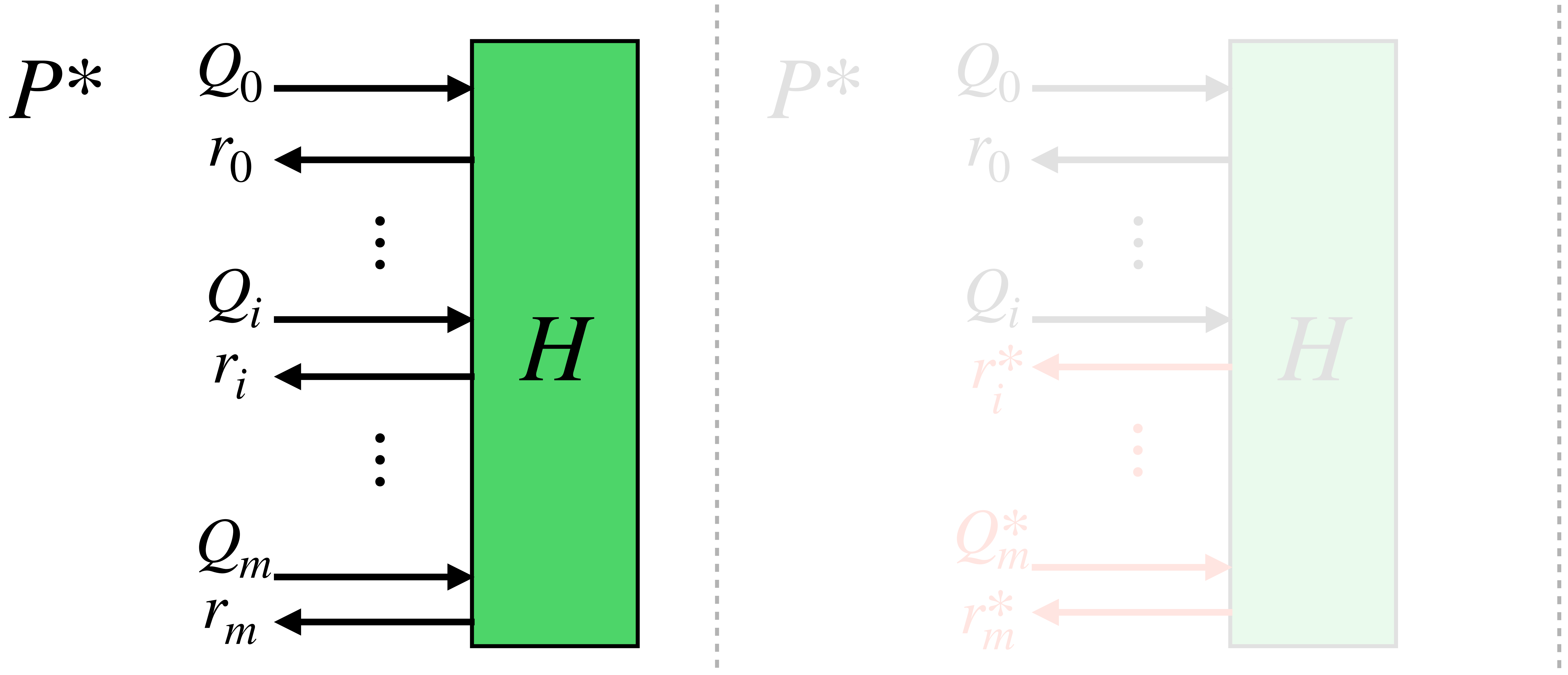
Probability of success: p $\qquad$ p $\qquad$ $\approx p^2$

Fiat-Shamir Compilation

- Advantages:
  - Simple to describe/implement
  - Very efficient; proving, verification cost exactly the same as input Σ-protocol
- Downsides:
  - Forking strategy does not compose; unclear how to prove concurrent security
  - Quadratic security loss

Straight-line Extraction

- Formalized by [Pass 03] in the Random Oracle Model:



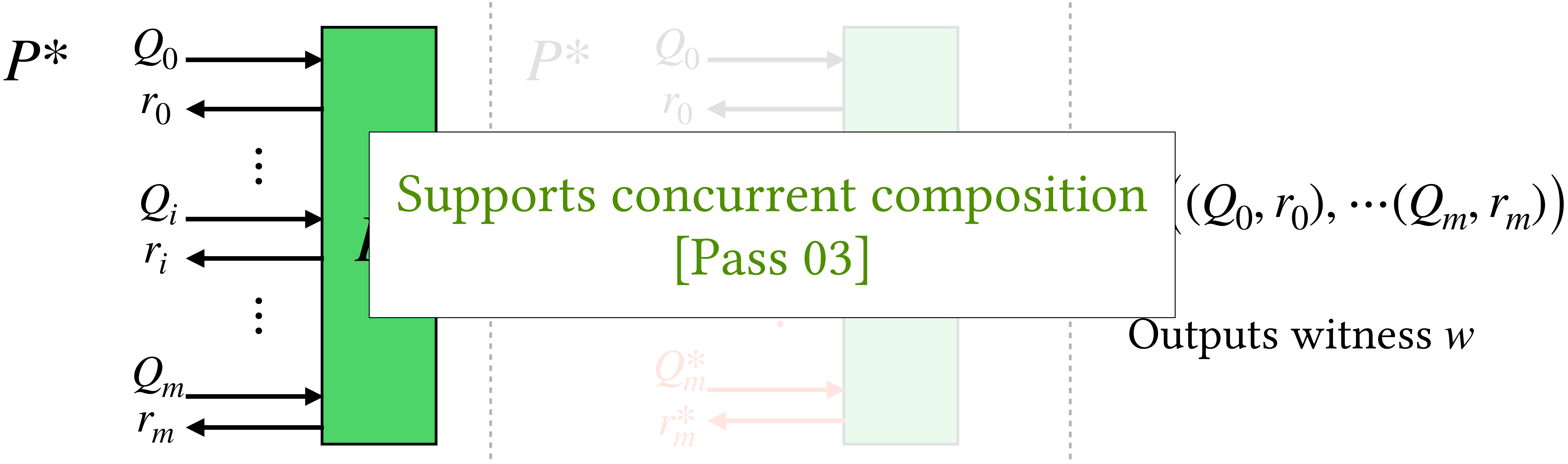
$\mathsf{Ext}\left((Q_0, r_0), \cdots (Q_m, r_m)\right)$

Outputs witness w

Probability of success: p p $\approx p$

Straight-line Extraction

- Formalized by [Pass 03] in the Random Oracle Model:



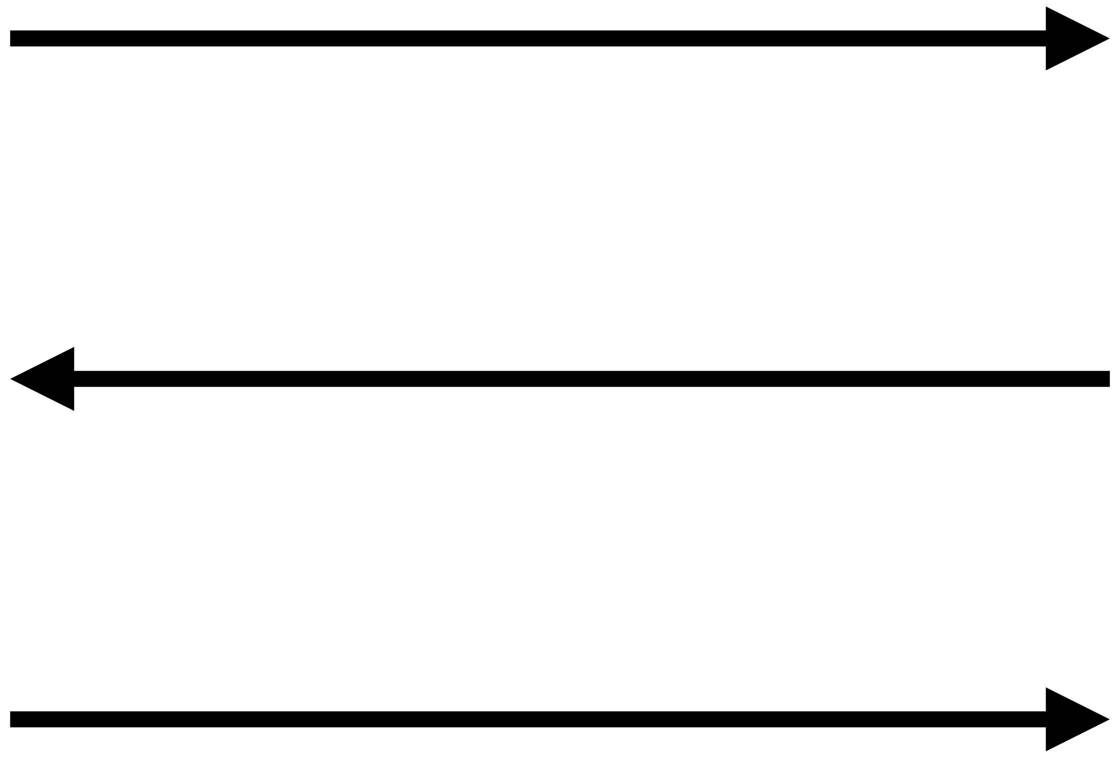
Supports concurrent composition [Pass 03]

Probability of success: p p $\approx p$

Logical OR-Composition of Σ Protocols

[Cramer Damgård Schoenmakers 94]

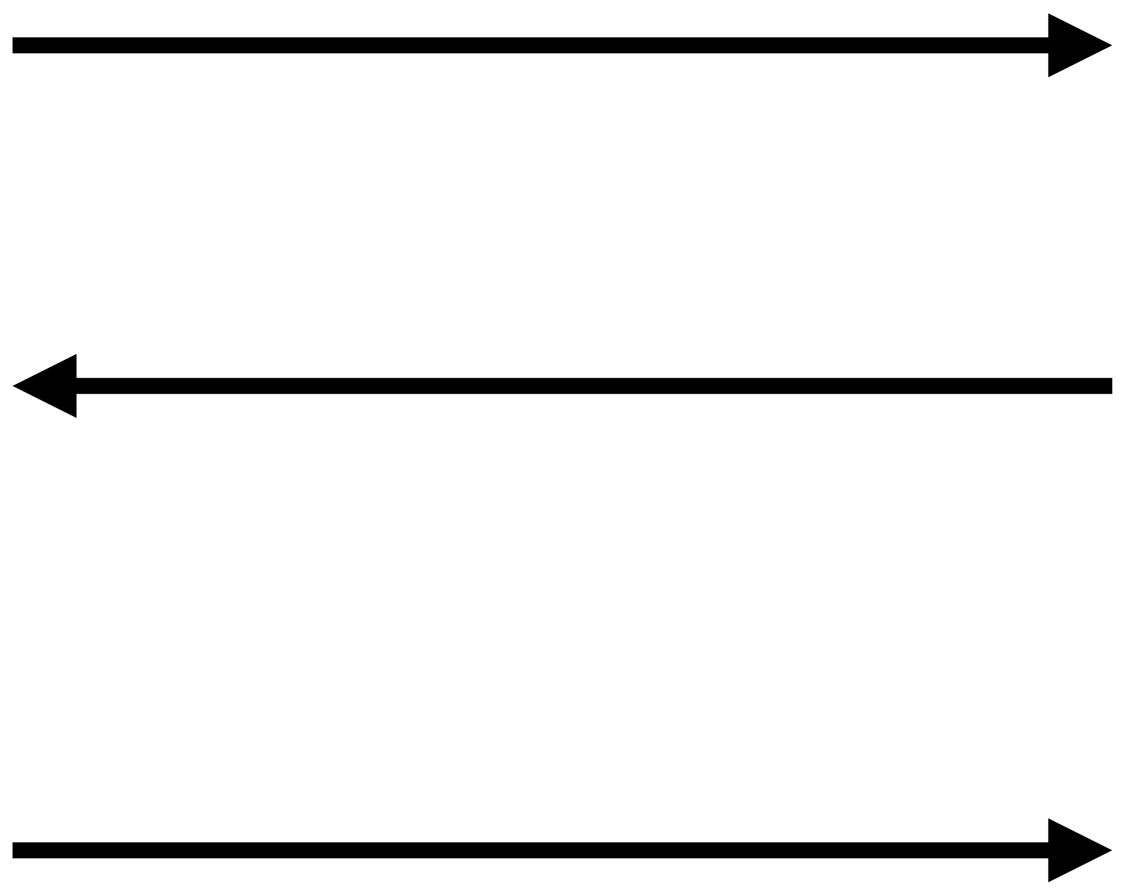
$P_{\mathsf{OR}}(w_b)$ x_0, x_1 V



Logical OR-Composition of Σ Protocols

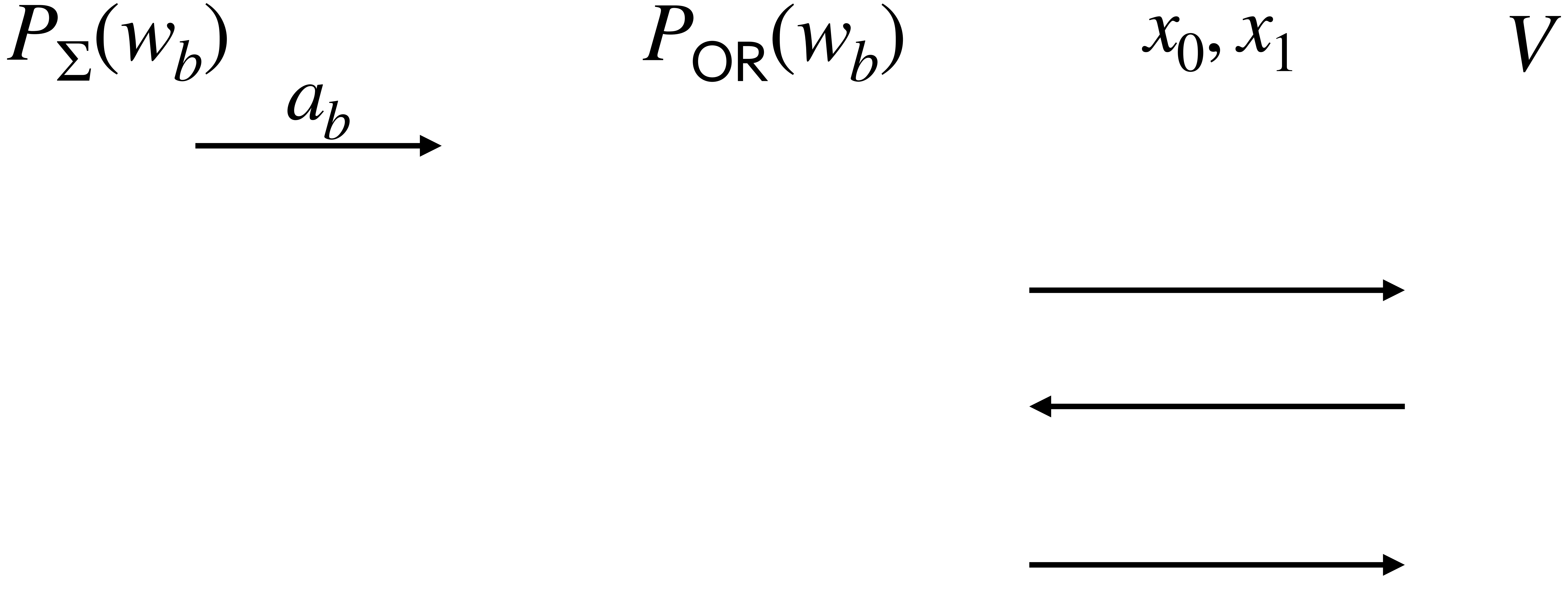
[Cramer Damgård Schoenmakers 94]

$P_{\Sigma}(w_b)$ $P_{\mathsf{OR}}(w_b)$ x_0, x_1 V



Logical OR-Composition of Σ Protocols

[Cramer Damgård Schoenmakers 94]



$P_{\Sigma}(w_b)$

a_b

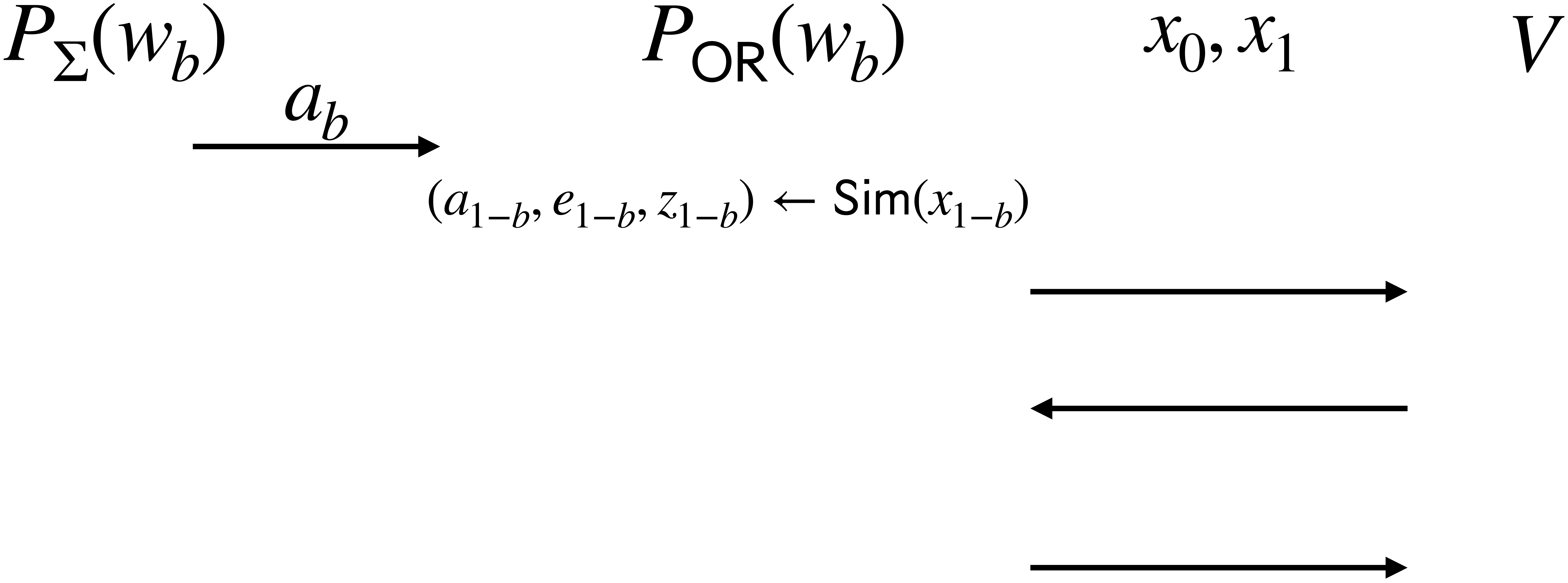
$P_{\mathsf{OR}}(w_b)$

x_0, x_1

V

Logical OR-Composition of Σ Protocols

[Cramer Damgård Schoenmakers 94]



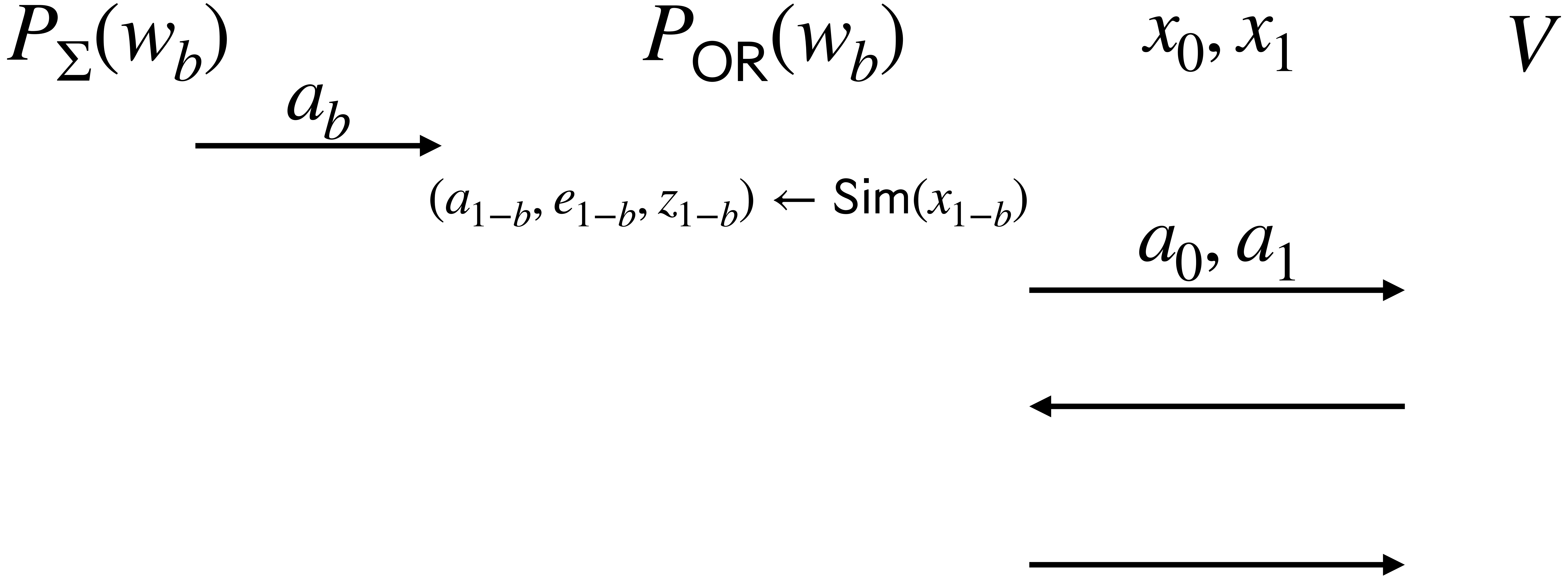
$P_\Sigma(w_b)$ $\quad$ $P_{\mathsf{OR}}(w_b)$ $\quad$ x_0, x_1 $\quad$ V

$\xrightarrow{a_b}$

$(a_{1-b}, e_{1-b}, z_{1-b}) \leftarrow \mathsf{Sim}(x_{1-b})$

Logical OR-Composition of Σ Protocols

[Cramer Damgård Schoenmakers 94]



$P_\Sigma(w_b)$

$P_{\mathsf{OR}}(w_b)$

x_0, x_1

V

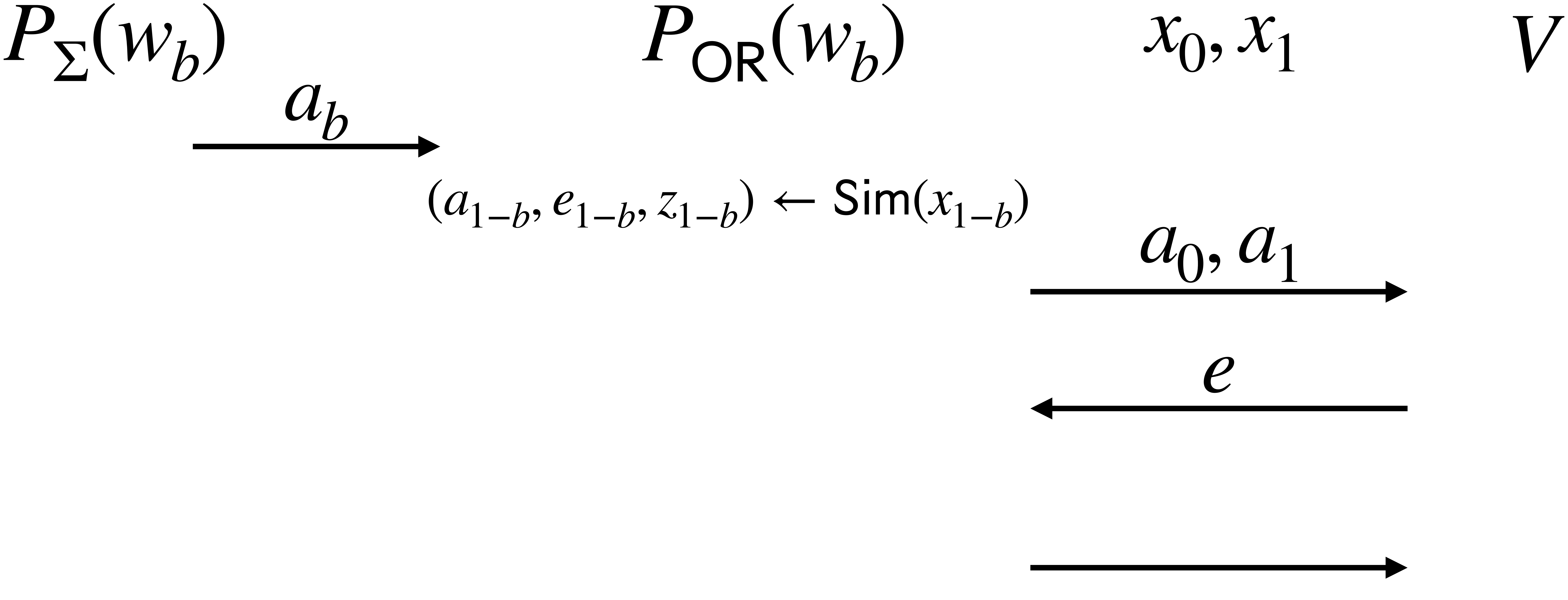
a_b

$(a_{1-b}, e_{1-b}, z_{1-b}) \leftarrow \mathsf{Sim}(x_{1-b})$

a_0, a_1

Logical OR-Composition of Σ Protocols

[Cramer Damgård Schoenmakers 94]



$P_\Sigma(w_b)$ $\quad$ $P_{\mathsf{OR}}(w_b)$ $\quad$ x_0, x_1 $\quad$ V

$\xrightarrow{a_b}$

$(a_{1-b}, e_{1-b}, z_{1-b}) \leftarrow \mathsf{Sim}(x_{1-b})$

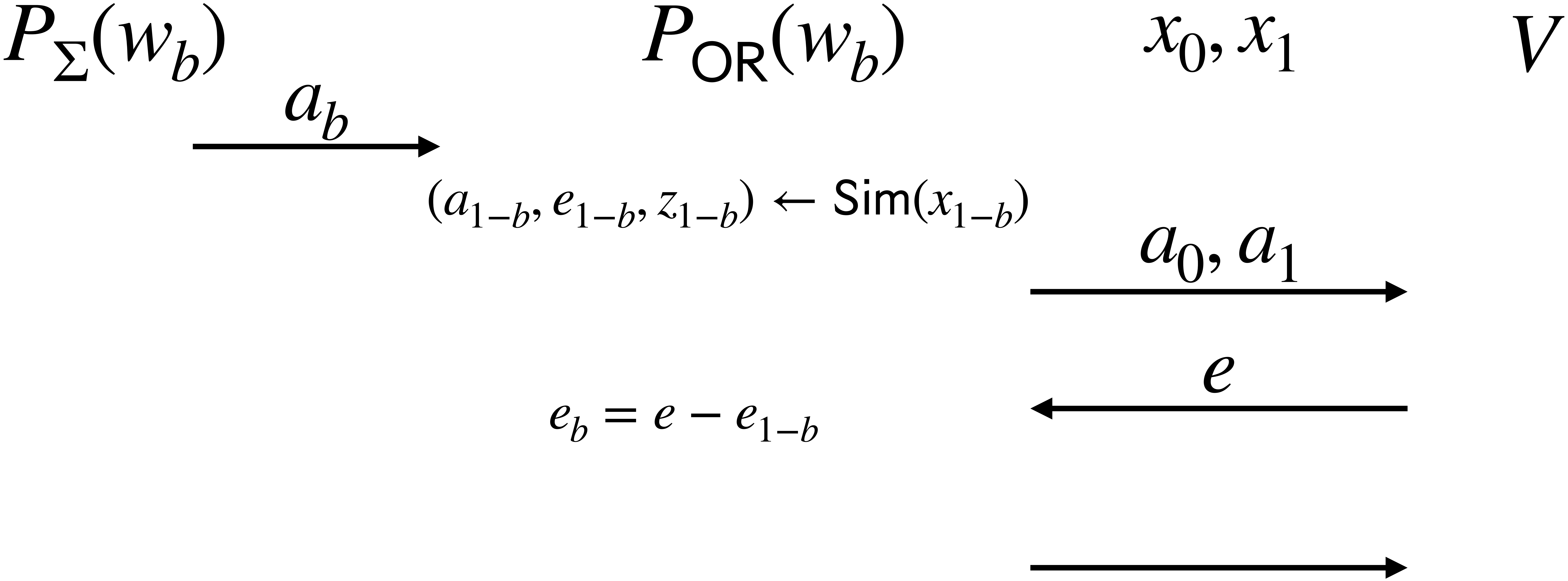
$\xrightarrow{a_0, a_1}$

$\xleftarrow{e}$

$\xrightarrow{\quad}$

Logical OR-Composition of Σ Protocols

[Cramer Damgård Schoenmakers 94]



$P_{\Sigma}(w_b)$ $P_{\mathsf{OR}}(w_b)$ x_0, x_1 V

$\xrightarrow{a_b}$

$(a_{1-b}, e_{1-b}, z_{1-b}) \leftarrow \mathsf{Sim}(x_{1-b})$

$\xrightarrow{a_0, a_1}$

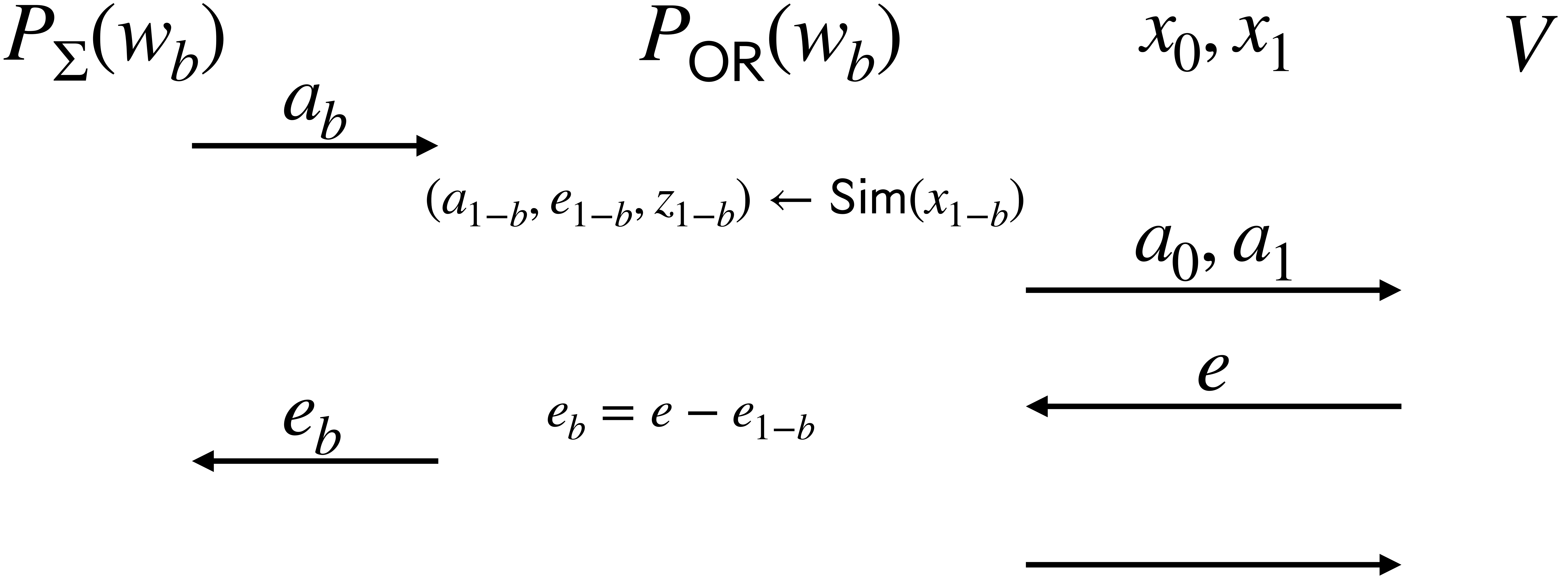
$\xleftarrow{e}$

$e_b = e - e_{1-b}$

$\longrightarrow$

Logical OR-Composition of Σ Protocols

[Cramer Damgård Schoenmakers 94]



$P_\Sigma(w_b)$

$P_{\mathsf{OR}}(w_b)$

x_0, x_1

V

a_b

$(a_{1-b}, e_{1-b}, z_{1-b}) \leftarrow \mathsf{Sim}(x_{1-b})$

a_0, a_1

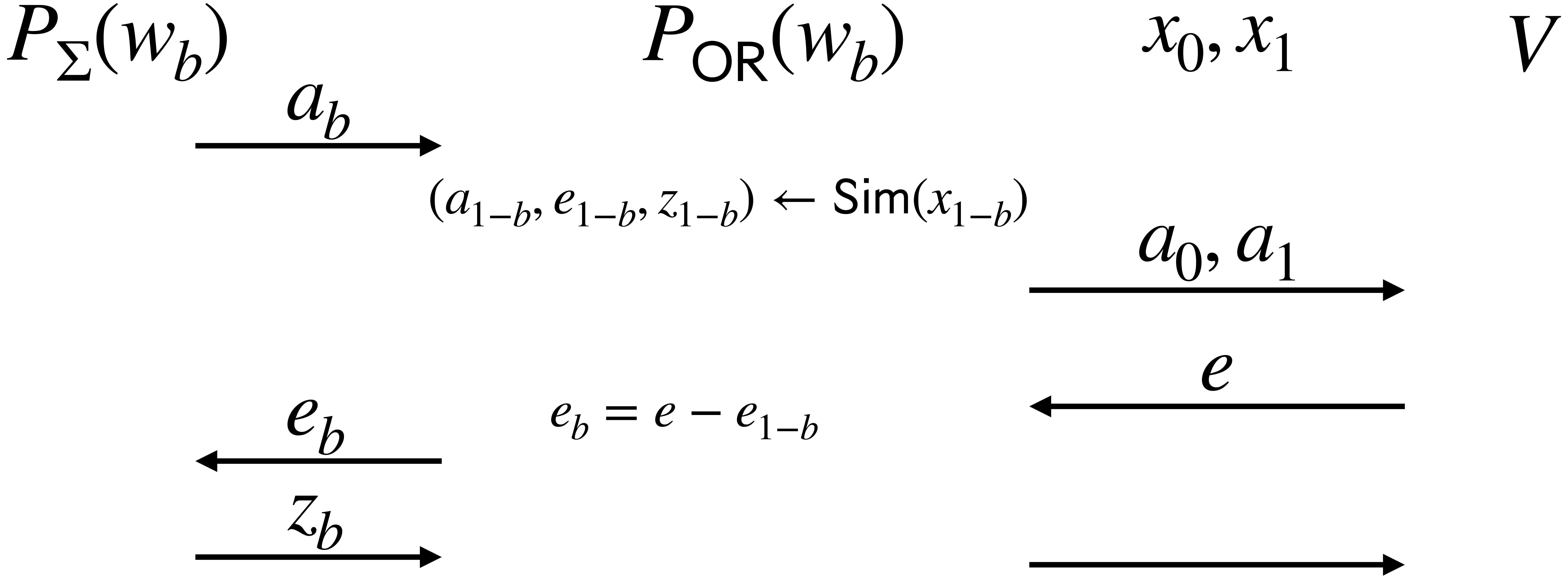
e

e_b

$e_b = e - e_{1-b}$

Logical OR-Composition of Σ Protocols

[Cramer Damgård Schoenmakers 94]



$P_{\Sigma}(w_b)$ $P_{\mathsf{OR}}(w_b)$ x_0, x_1 V

$\xrightarrow{a_b}$

$(a_{1-b}, e_{1-b}, z_{1-b}) \leftarrow \mathsf{Sim}(x_{1-b})$

$\xrightarrow{a_0, a_1}$

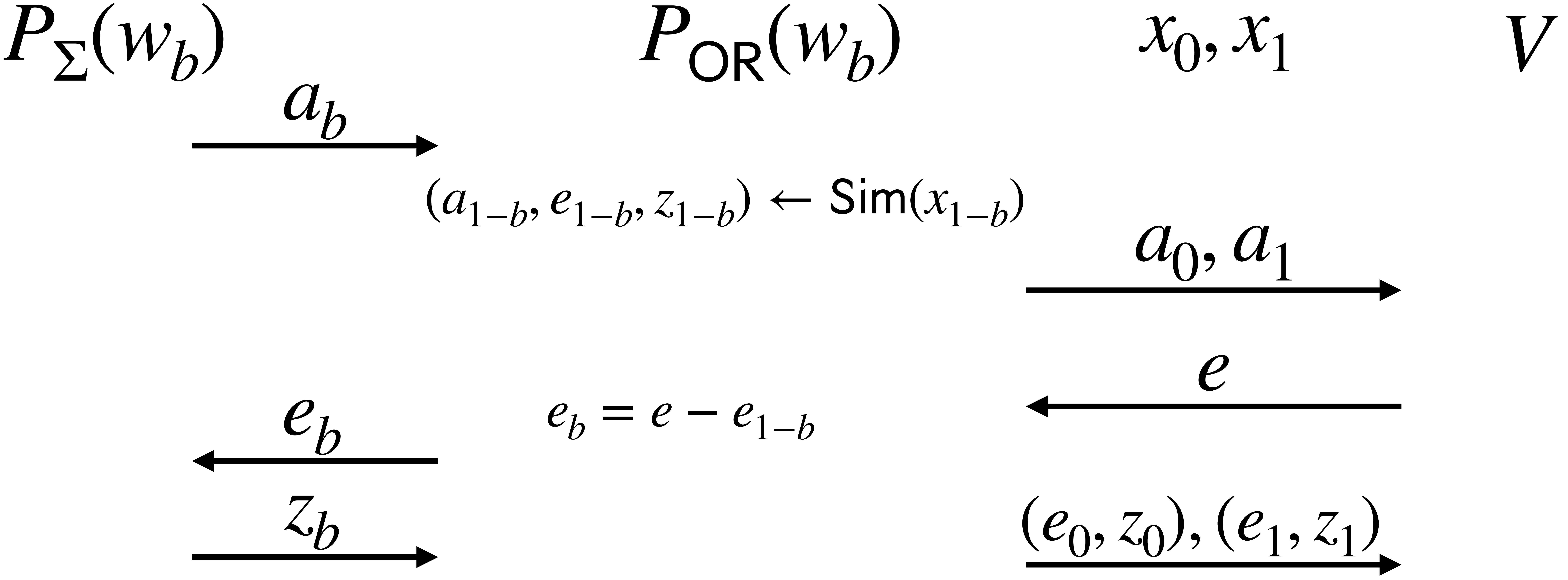
$\xleftarrow{e}$

$\xleftarrow{e_b}$ $e_b = e - e_{1-b}$

$\xrightarrow{z_b}$ $\xrightarrow{}$

Logical OR-Composition of Σ Protocols

[Cramer Damgård Schoenmakers 94]



$P_\Sigma(w_b)$ $\quad P_{\mathsf{OR}}(w_b)$ $\quad x_0, x_1$ $\quad V$

$$P_\Sigma(w_b) \xrightarrow{a_b} P_{\mathsf{OR}}(w_b)$$

$$(a_{1-b}, e_{1-b}, z_{1-b}) \leftarrow \mathsf{Sim}(x_{1-b})$$

$$P_{\mathsf{OR}}(w_b) \xrightarrow{a_0, a_1} V$$

$$P_{\mathsf{OR}}(w_b) \xleftarrow{e} V$$

$$e_b = e - e_{1-b}$$

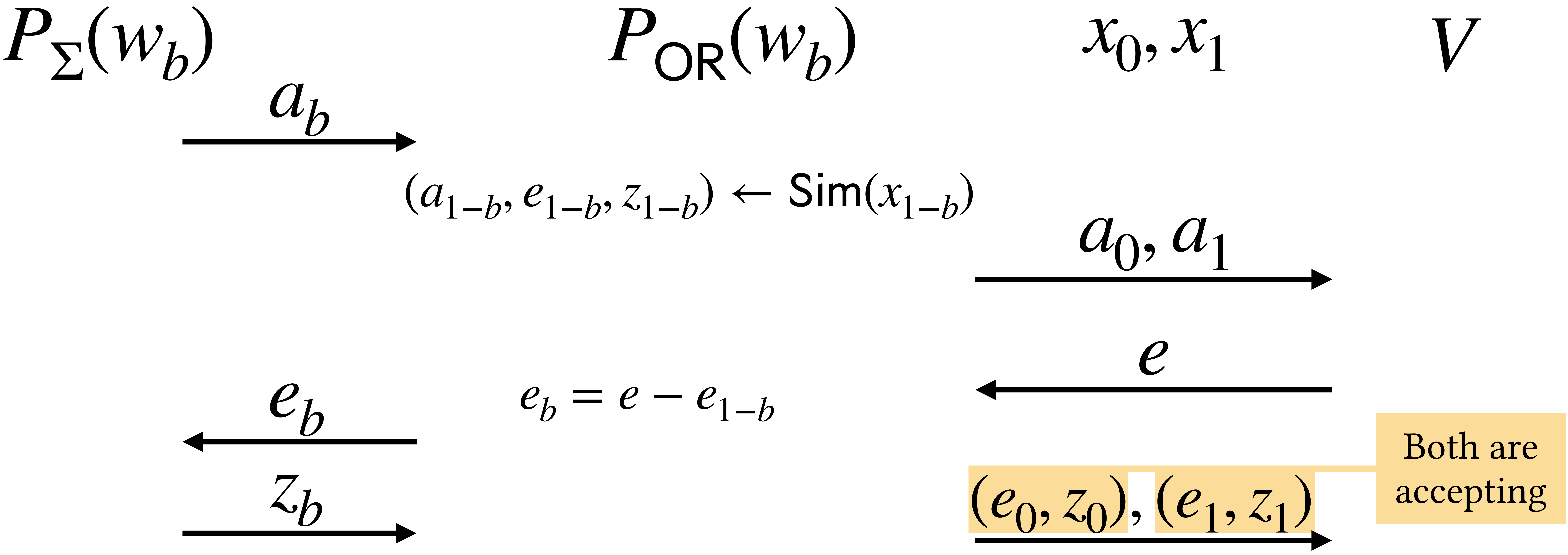
$$P_\Sigma(w_b) \xleftarrow{e_b} P_{\mathsf{OR}}(w_b)$$

$$P_\Sigma(w_b) \xrightarrow{z_b} P_{\mathsf{OR}}(w_b)$$

$$P_{\mathsf{OR}}(w_b) \xrightarrow{(e_0, z_0), (e_1, z_1)} V$$

Logical OR-Composition of Σ Protocols

[Cramer Damgård Schoenmakers 94]



$P_{\Sigma}(w_b)$

$P_{\mathsf{OR}}(w_b)$

x_0, x_1

V

a_b

$(a_{1-b}, e_{1-b}, z_{1-b}) \leftarrow \mathsf{Sim}(x_{1-b})$

a_0, a_1

e

e_b

$e_b = e - e_{1-b}$

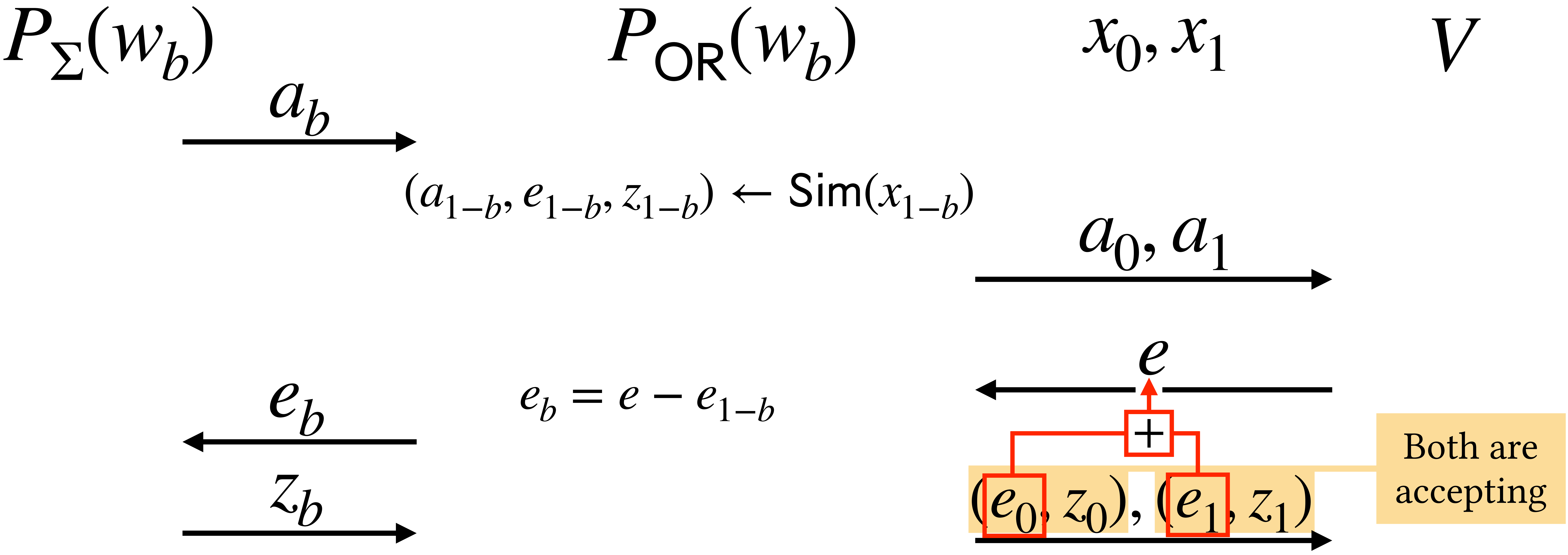
z_b

$(e_0, z_0), (e_1, z_1)$

Both are accepting

Logical OR-Composition of Σ Protocols

[Cramer Damgård Schoenmakers 94]



$P_\Sigma(w_b)$ $\qquad$ $P_{\mathsf{OR}}(w_b)$ $\qquad$ x_0, x_1 $\qquad$ V

$$P_\Sigma(w_b) \xrightarrow{a_b} P_{\mathsf{OR}}(w_b)$$

$$(a_{1-b}, e_{1-b}, z_{1-b}) \leftarrow \mathsf{Sim}(x_{1-b})$$

$$P_{\mathsf{OR}}(w_b) \xrightarrow{a_0, a_1} V$$

$$P_{\mathsf{OR}}(w_b) \xleftarrow{e} V$$

$$P_\Sigma(w_b) \xleftarrow{e_b} P_{\mathsf{OR}}(w_b)$$

$$e_b = e - e_{1-b}$$

$$P_\Sigma(w_b) \xrightarrow{z_b} P_{\mathsf{OR}}(w_b)$$

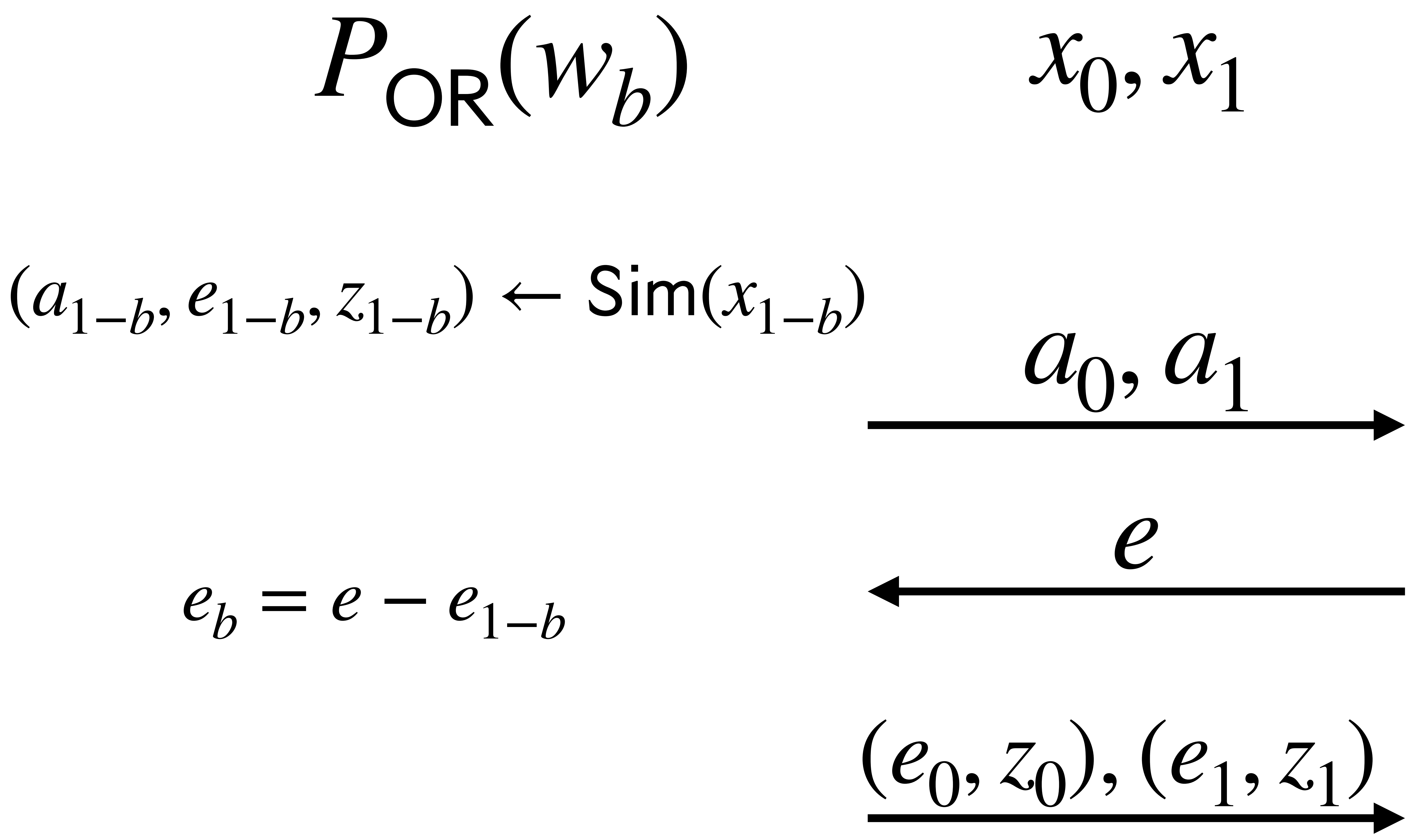
$$P_{\mathsf{OR}}(w_b) \xrightarrow{(\boxed{e_0}, z_0), (\boxed{e_1}, z_1)} V$$

$$e = e_0 + e_1$$

Both are accepting

Logical OR-Composition of Σ Protocols

[Cramer Damgård Schoenmakers 94]



$P_{\mathsf{OR}}(w_b)$ x_0, x_1 V

$(a_{1-b}, e_{1-b}, z_{1-b}) \leftarrow \mathsf{Sim}(x_{1-b})$

$\xrightarrow{a_0, a_1}$

$\xleftarrow{e}$

$e_b = e - e_{1-b}$

$\xrightarrow{(e_0, z_0), (e_1, z_1)}$

Recall: $(a, e, z, z') \leftarrow \mathscr{A}(\mathsf{pp})$ violates unique responses

Logical OR-Composition of Σ Protocols

[Cramer Damgård Schoenmakers 94]

$P_{\mathsf{OR}}(w_b)$ $\qquad$ x_0, x_1 $\qquad$ V

$(a_{1-b}, e_{1-b}, z_{1-b}) \leftarrow \mathsf{Sim}(x_{1-b})$

$$\xrightarrow{\quad a_0, a_1 \quad}$$

$$\xleftarrow{\quad e \quad}$$

$e_b = e - e_{1-b}$

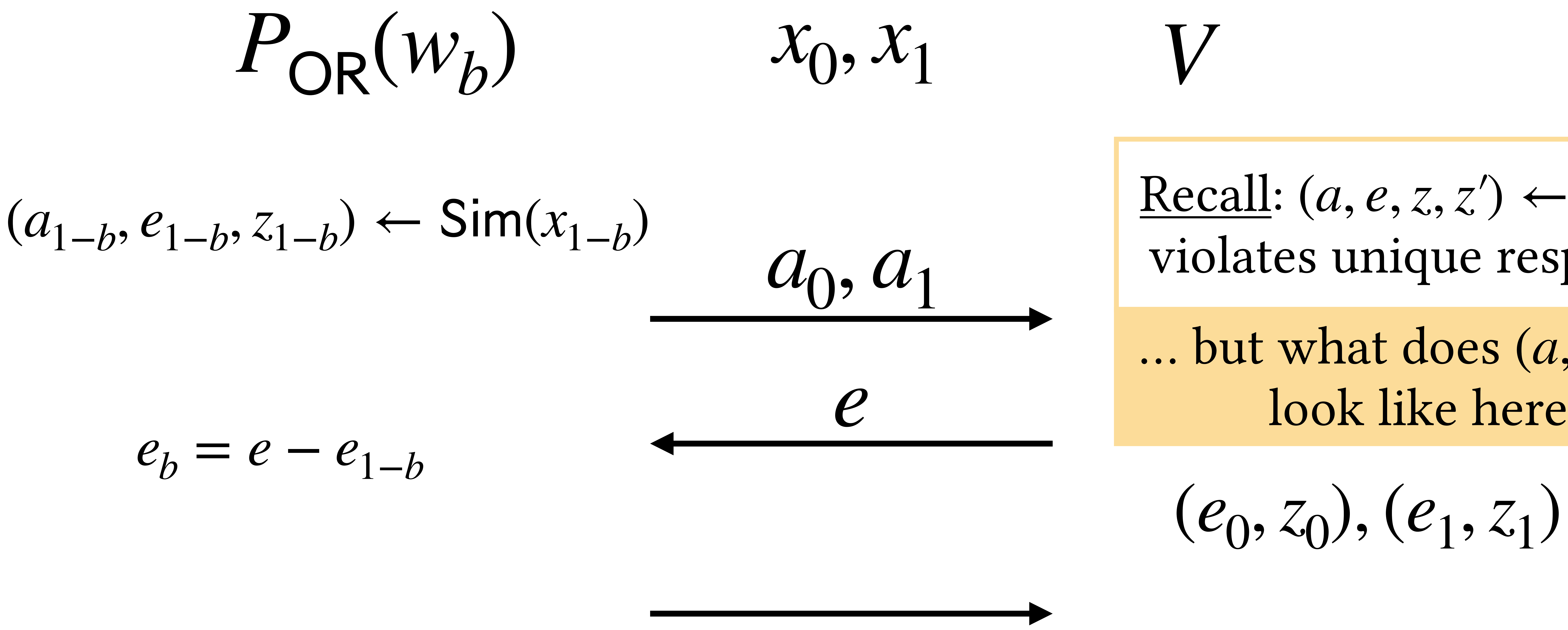
$$\xrightarrow{\quad (e_0, z_0), (e_1, z_1) \quad}$$

Recall: $(a, e, z, z') \leftarrow \mathscr{A}(\mathsf{pp})$ violates unique responses

… but what does (a, e, z, z') look like here?

Logical OR-Composition of Σ Protocols

[Cramer Damgård Schoenmakers 94]



$P_{\mathsf{OR}}(w_b)$ $\quad$ x_0, x_1 $\quad$ V

$(a_{1-b}, e_{1-b}, z_{1-b}) \leftarrow \mathsf{Sim}(x_{1-b})$

a_0, a_1

Recall: $(a, e, z, z') \leftarrow \mathscr{A}(\mathsf{pp})$ violates unique responses

… but what does (a, e, z, z') look like here?

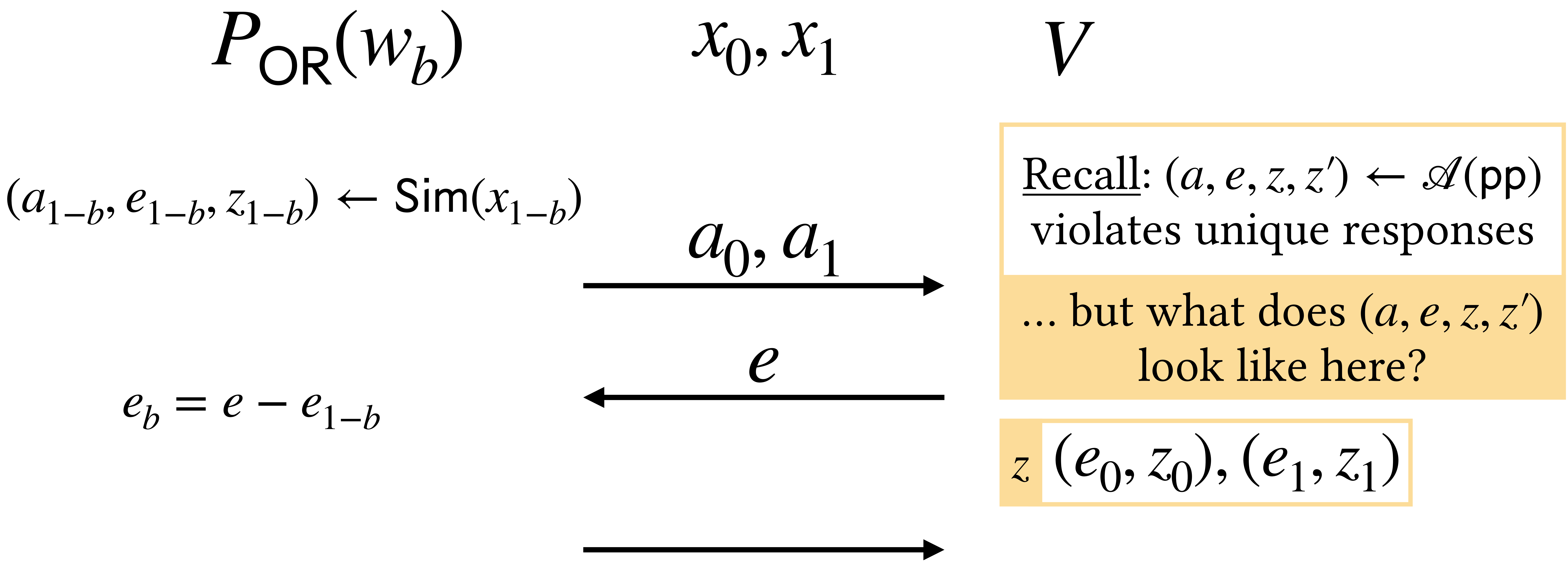
e

$e_b = e - e_{1-b}$

$(e_0, z_0), (e_1, z_1)$

Logical OR-Composition of Σ Protocols

[Cramer Damgård Schoenmakers 94]



$P_{\mathsf{OR}}(w_b)$

x_0, x_1

V

$(a_{1-b}, e_{1-b}, z_{1-b}) \leftarrow \mathsf{Sim}(x_{1-b})$

$\xrightarrow{a_0, a_1}$

<u>Recall</u>: $(a, e, z, z') \leftarrow \mathscr{A}(\mathsf{pp})$ violates unique responses

... but what does (a, e, z, z') look like here?

$\xleftarrow{e}$

$e_b = e - e_{1-b}$

z $(e_0, z_0), (e_1, z_1)$

$\longrightarrow$

Logical OR-Composition of Σ Protocols

[Cramer Damgård Schoenmakers 94]

$P_{\mathsf{OR}}(w_b)$ $\quad x_0, x_1 \quad$ V

$(a_{1-b}, e_{1-b}, z_{1-b}) \leftarrow \mathsf{Sim}(x_{1-b})$

$$\xrightarrow{\quad a_0, a_1 \quad}$$

$$\xleftarrow{\quad e \quad}$$

$e_b = e - e_{1-b}$

$$\xrightarrow{(e'_0, z'_0), (e'_1, z'_1) \quad z'}$$

$z \quad (e_0, z_0), (e_1, z_1)$

Recall: $(a, e, z, z') \leftarrow \mathscr{A}(\mathsf{pp})$ violates unique responses

… but what does (a, e, z, z') look like here?

Logical OR-Composition of Σ Protocols

[Cramer Damgård Schoenmakers 94]

$P_{\mathsf{OR}}(w_b)$ $\qquad$ x_0, x_1 $\qquad$ V

$(a_{1-b}, e_{1-b}, z_{1-b}) \leftarrow \mathsf{Sim}(x_{1-b})$

$$\xrightarrow{\quad a_0, a_1 \quad}$$

$$\xleftarrow{\quad e \quad}$$

$e_b = e - e_{1-b}$

$$\xrightarrow{\quad (e_0', z_0'), (e_1', z_1') \quad} \; z'$$

Recall: $(a, e, z, z') \leftarrow \mathscr{A}(\mathsf{pp})$ violates unique responses

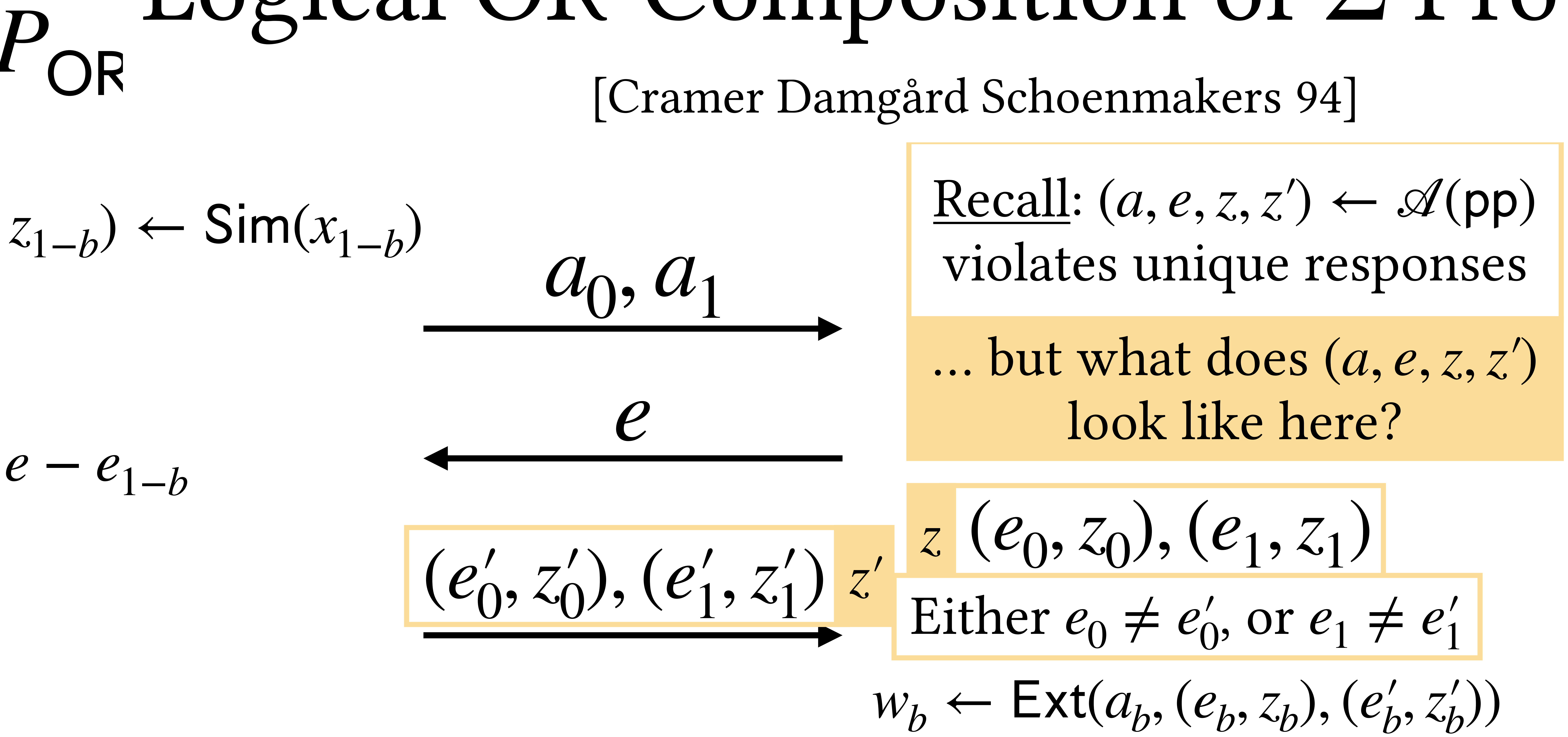
... but what does (a, e, z, z') look like here?

z $\quad (e_0, z_0), (e_1, z_1)$

Either $e_0 \neq e_0'$, or $e_1 \neq e_1'$

Logical OR-Composition of Σ Protocols

[Cramer Damgård Schoenmakers 94]



P_{OR}

$z_{1-b}) \leftarrow \mathsf{Sim}(x_{1-b})$

a_0, a_1

e

$e - e_{1-b}$

$(e_0', z_0'), (e_1', z_1')$ z'

<u>Recall</u>: $(a, e, z, z') \leftarrow \mathscr{A}(\mathsf{pp})$ violates unique responses

... but what does (a, e, z, z') look like here?

z $(e_0, z_0), (e_1, z_1)$

Either $e_0 \neq e_0'$, or $e_1 \neq e_1'$

$w_b \leftarrow \mathsf{Ext}(a_b, (e_b, z_b), (e_b', z_b'))$

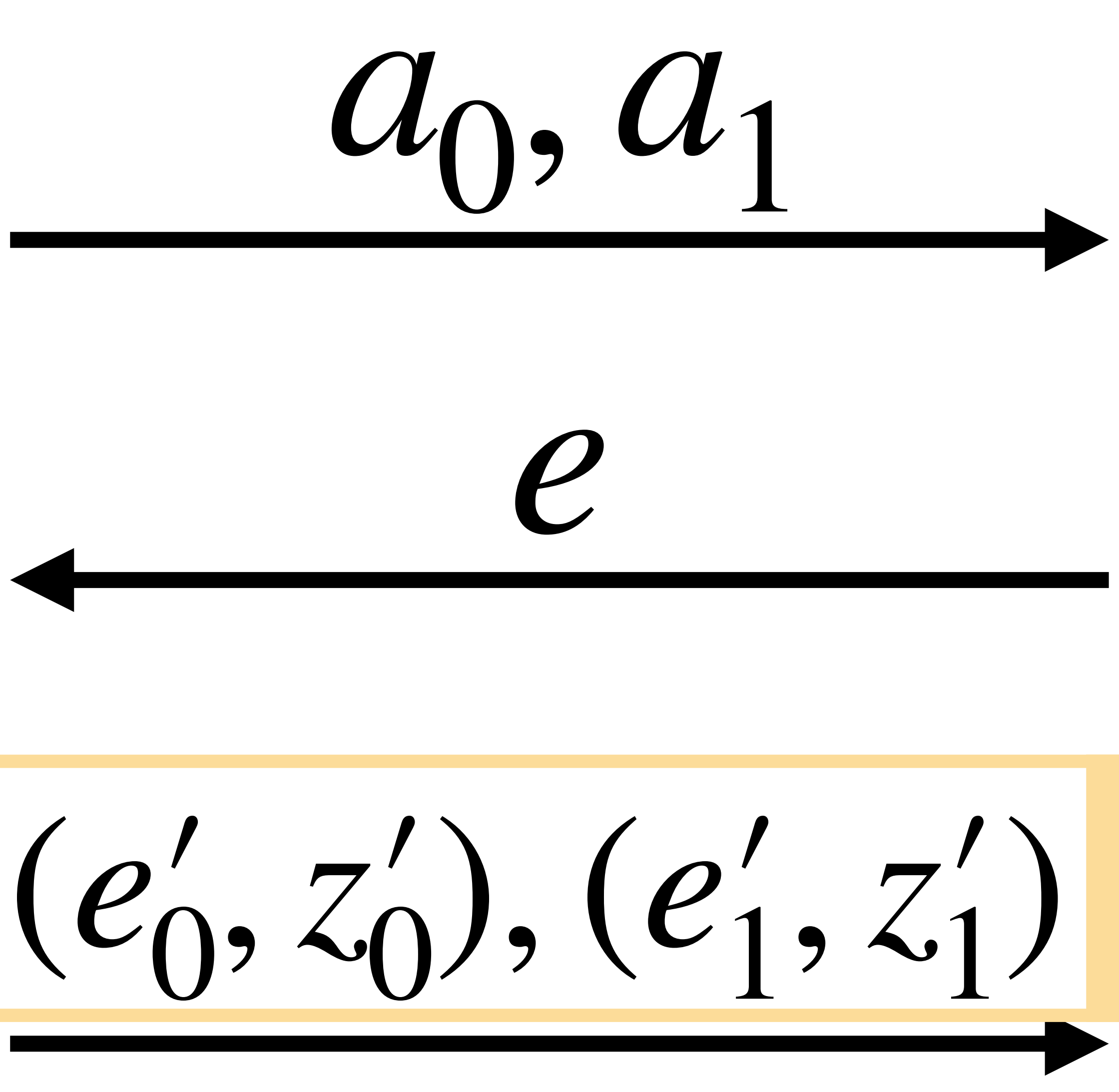
P_{OR}

Logical OR-Composition of Σ Protocols

[Cramer Damgård Schoenmakers 94]

$z_{1-b}) \leftarrow \mathsf{Sim}(x_{1-b})$

$e - e_{1-b}$



a_0, a_1

e

$(e'_0, z'_0), (e'_1, z'_1)$ z'

Recall: $(a, e, z, z') \leftarrow \mathscr{A}(\mathsf{pp})$ violates unique responses

... but what does (a, e, z, z') look like here?

z $(e_0, z_0), (e_1, z_1)$

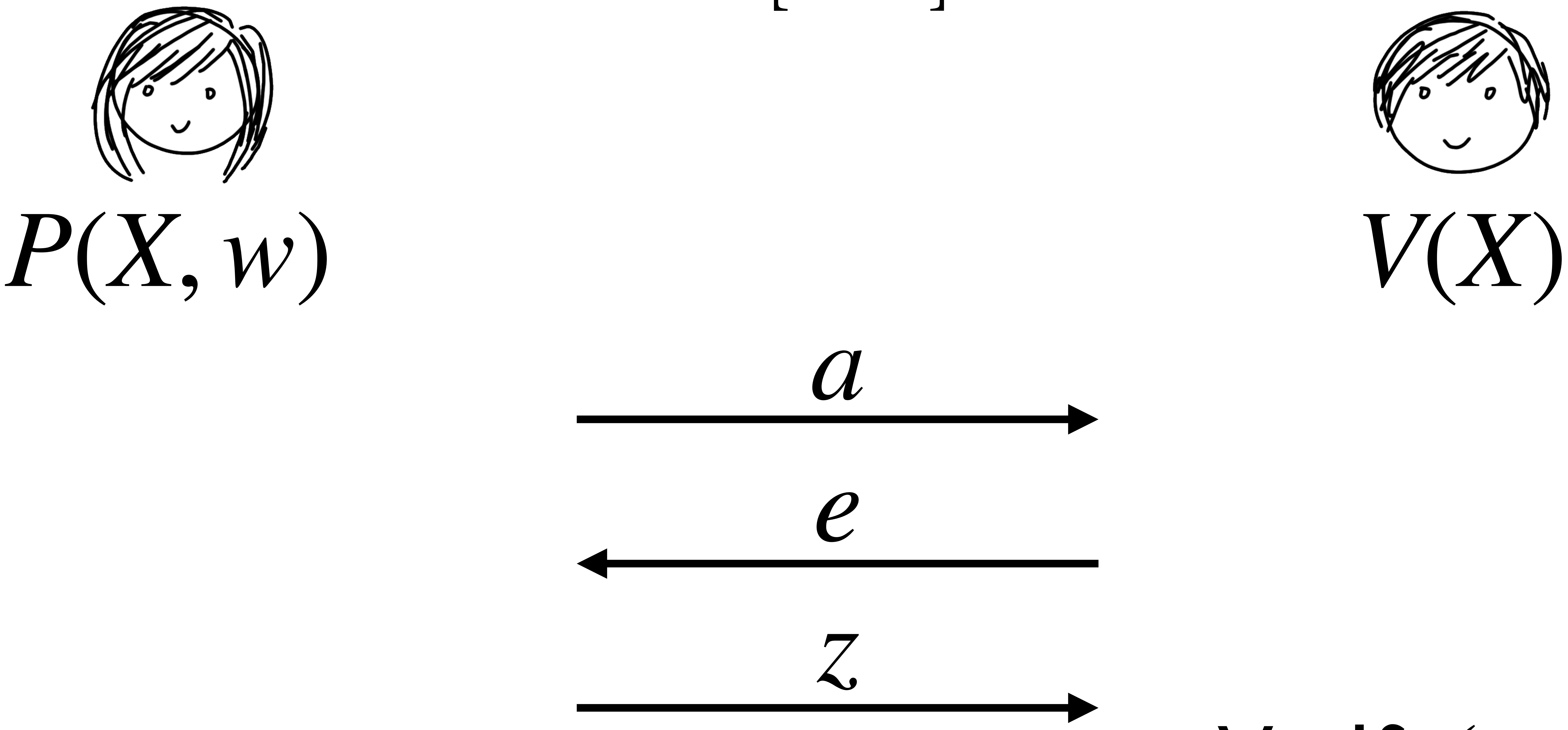
Either $e_0 \neq e'_0$, or $e_1 \neq e'_1$

$w_b \leftarrow \mathsf{Ext}(a_b, (e_b, z_b), (e'_b, z'_b))$

Quasi-unique responses not *strictly* necessary for extraction (folklore)

Tightening Conditions for Extraction

[Ks 22]



$P(X, w)$

$V(X)$

$\xrightarrow{a}$

$\xleftarrow{e}$

$\xrightarrow{z}$

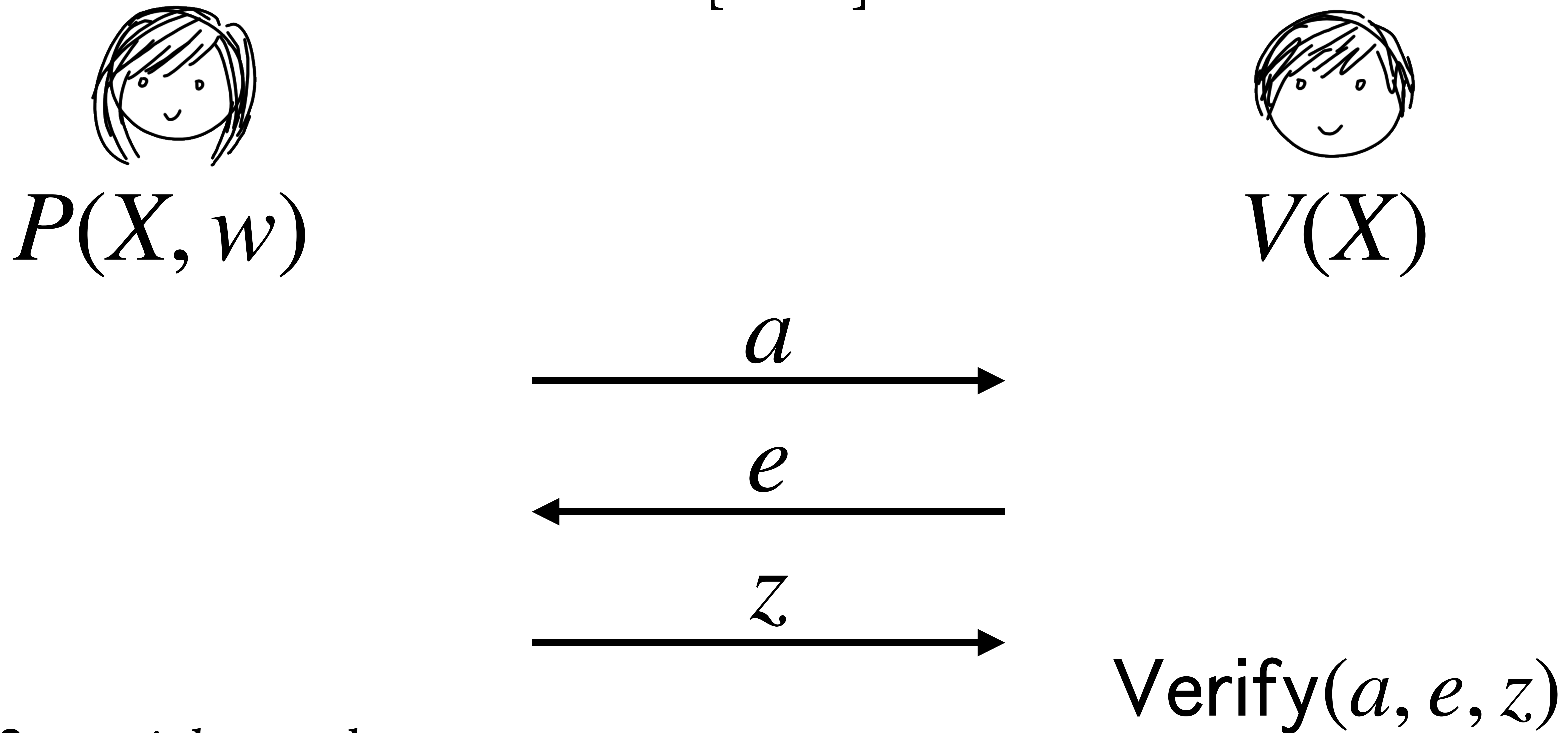
$\mathsf{Verify}(a, e, z)$

2-special soundness:

$$w \leftarrow \mathsf{Ext}(X, a, (e_1, z_1), (e_2, z_2)) \text{ such that } R(X, w) = 1$$

Tightening Conditions for Extraction

[Ks 22]



$P(X, w)$

$V(X)$

$\xrightarrow{a}$

$\xleftarrow{e}$

$\xrightarrow{z}$

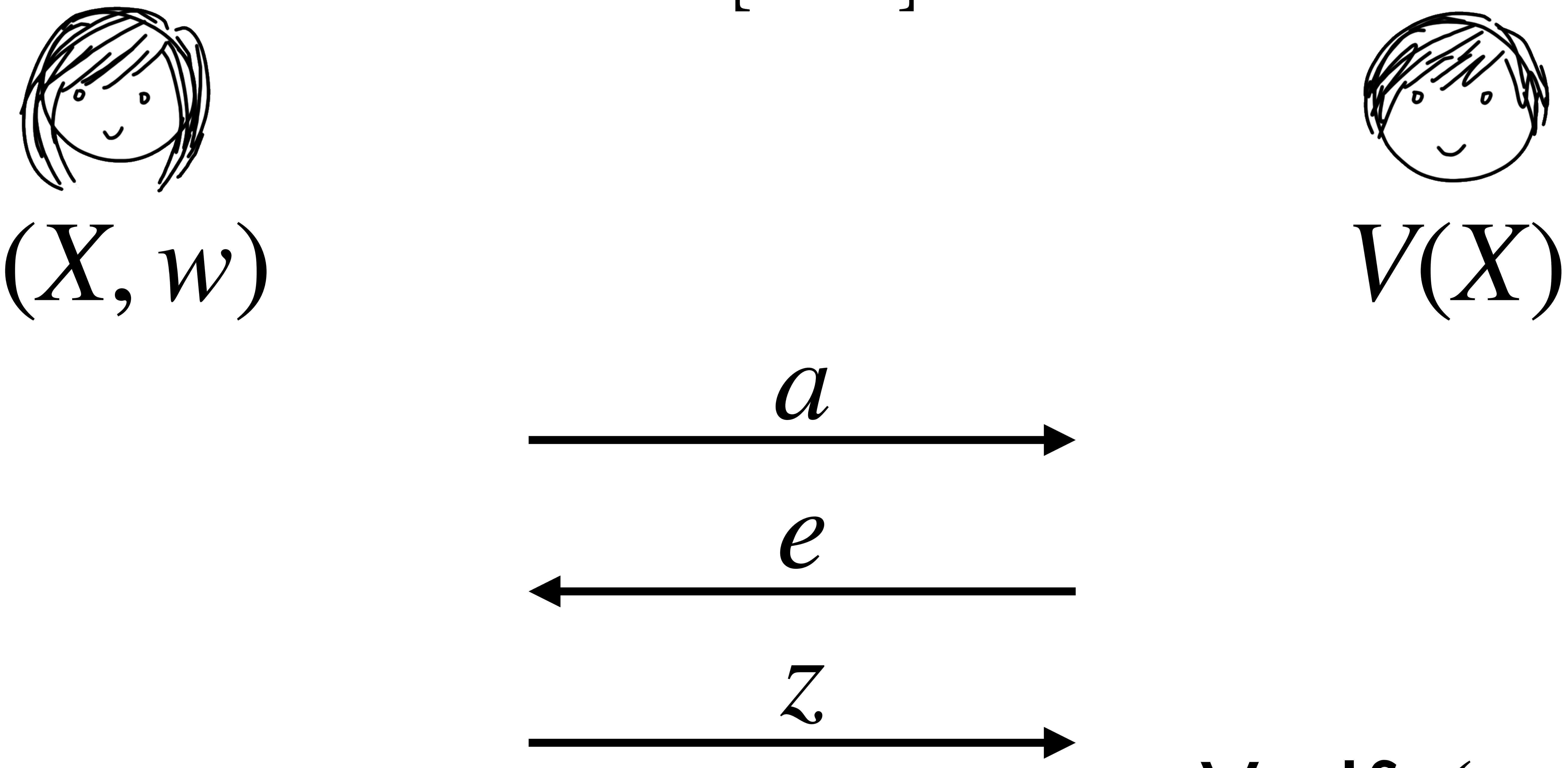
$\mathsf{Verify}(a, e, z)$

Strong 2-special soundness:

$$w \leftarrow \mathsf{Ext}(X, a, (e_1, z_1), (e_2, z_2)) \text{ such that } R(X, w) = 1$$

Tightening Conditions for Extraction

[Ks 22]



$P(X, w)$

$V(X)$

a

e

z

$\mathsf{Verify}(a, e, z)$

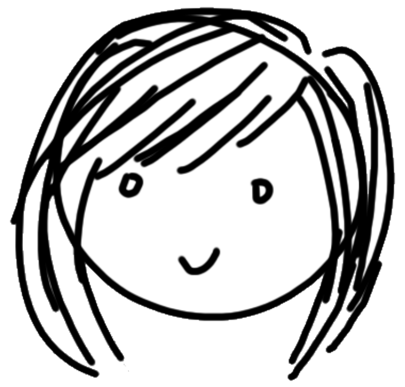
Strong 2-special soundness:

$$w \leftarrow \mathsf{Ext}(X, a, (e_1, z_1), (e_2, z_2)) \text{ such that } R(X, w) = 1$$

$$e_1 \neq e_2 \;\text{ OR }\; z_1 \neq z_2$$

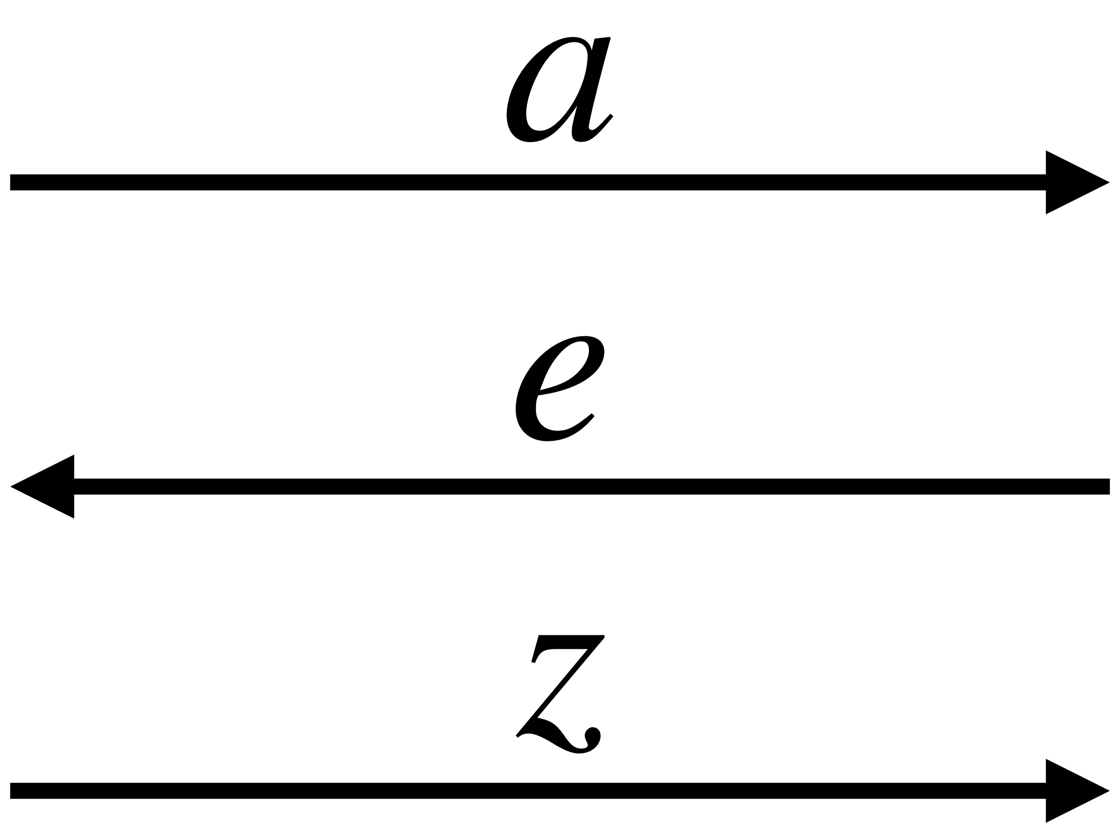
Tightening Conditions for Extraction

[Ks 22]



$P(X, w)$

$V(X)$



a

e

z

$\mathsf{Verify}(a, e, z)$

Strong 2-special soundness:

$$w \leftarrow \mathsf{Ext}(X, a, (e_1, z_1), (e_2, z_2)) \text{ such that } R(X, w) = 1$$

$$e_1 \neq e_2 \ \text{ OR } \ z_1 \neq z_2$$

...are we done?