17

BASIC

d) $(a^* b^*)^* \stackrel{?}{=} (a^* b)^*$

no!

consider the conter-example "a"

$a \in L((a^* b^*)^*)$

$a \notin L((a^* b)^*)$

e) $(ab+a)^* a \stackrel{?}{=} a(ba+a)^*$

yes!

$(ab+a)^* a$

$= (a(b+\varepsilon))^* a$ (distributivity)

$= a((b+\varepsilon)a)^*$ (shifting)

$= a(ba+a)^*$ (distributivity)

— x —

1>

ADDITIONAL

Show

$$b + ab^* + aa^*b + aa^*ab^* = a^*(b + ab^*)$$

Let's work from R.H.S. I will suppose we have already shown : $e^* = ee^* + \varepsilon$ — ①

perhaps by using a semantic argument.

$$
\begin{aligned}
R.H.S &= \underline{a^*}(b+ab^*) \\
&= \underline{(aa^* + \varepsilon)(b + ab^*)} && \text{by ①} \\
&= \underline{(aa^*+\varepsilon)b} + (aa^* + \varepsilon)ab^* \\
&= aa^*b + b + \underline{(aa^* + \varepsilon)ab^*} \\
&= aa^*b + b + aa^*ab^* + ab^* && \text{distributivity} \\
&= b + ab^* + aa^*b + aa^*ab^* && \text{(by associativity of +)} \\
&= L.H.S.
\end{aligned}
$$

■

—— × ——

2 > BASIC

Given $L = \{ w \mid \#0(w) = 2 \times \#1(w) \}$ ["twice the number of zeroes as ones"]

where $w \in \{0,1\}^*$

⚠ L is not regular.

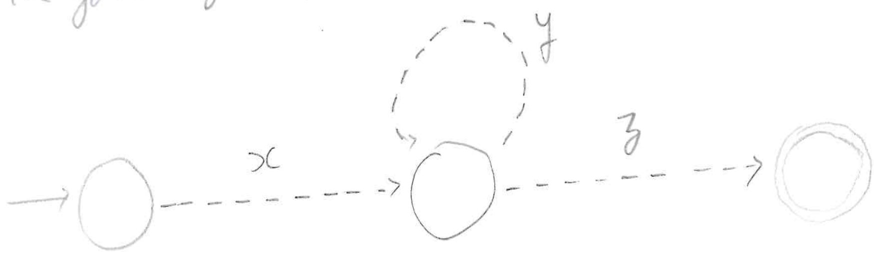
PROOF Assume L is regular.

Then by the pumping lemma, we have that $\exists n$

Such that $\forall w$ with $|w| \geq n$, $w = xyz$ such that:

- $|y| > 0$
- $|xy| \leq n$
- $\forall k \geq 0$, $xy^kz \in L$.

or, in other words, an automaton which accepts L has the following shape (omitting further detail about contents):

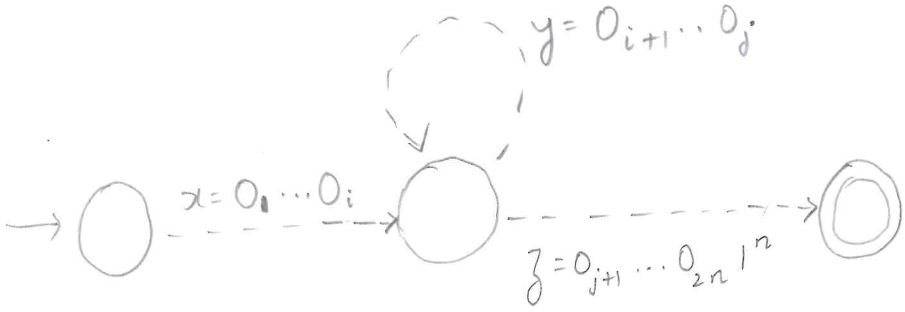


now consider the word $0^{2n}1^n$ (for n given above)

$0^{2n}1^n \in L$, by definition, and also $|0^{2n}1^n| = 3n \geq n$

Since we are given $|y| > 0$ and $|xy| \leq n$, it must be the case that x and y only contain 0s.

That is, graphically, (PTO)



where i and j simply indicate the end position of x and y.

we know $|xy| = j \leq n$

and $|y| = j - i > 0$

now, chose K (in the third "given" of the pumping lemma)

as 2 : $K = 2$

then we get that $xy^2z \in L$

i.e., $0^i (0^{j-i})^2 \, 0^{2n-j} \, 1^n \in L$

i.e., $0^i \, 0^{2j-2i} \, 0^{2n-j} \, 1^n \in L$

i.e., $0^{2n+(j-i)} \, 1^n \in L$ ——— (1)

but since $j - i > 0$, $2n \neq 2n + (j-i)$

and it cannot be the case that $0^{2n+(j-i)} 1^n \in L$

which contradicts our inference (1)

Hence, L must be regular. ∎

(CONTD.)

2) BASIC

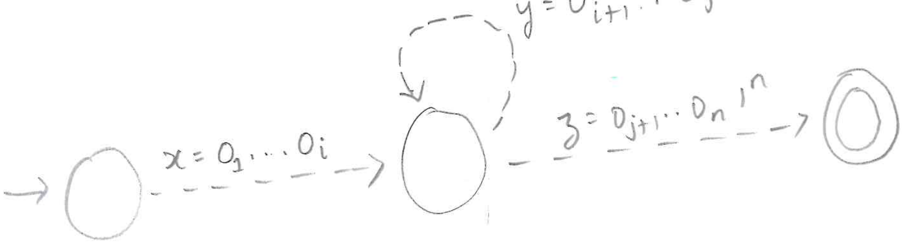
Given $M = \{ w \mid \#0(w) \leq \#1(w) \leq \#0(w)+1 \}$

[That is, "same number of zeros and ones OR atmost an extra one"]

As before, assume that M is regular, and apply the pumping lemma. (not shown here for brevity)

Consider the word $0^n 1^n$, length of which $\geq n$

and hence we (graphically) get the shape:



we know (by the lemma):

$|xy| = j \leq n$ and $|y| = j - i > 0$

chose $K = 2$, then we get that $xy^2z \in L$,

i.e., $0^i \, 0^{2(j-i)} \, 0^{n-j} \, 1^n \in L$

i.e., $0^{n+(j-i)} 1^n \in L$ ——— ①

but since $j - i > 0$, $n + (j-i) > n$,

which violates that $\#0(w) \leq \#1(w)$,

which contradicts inference ①.

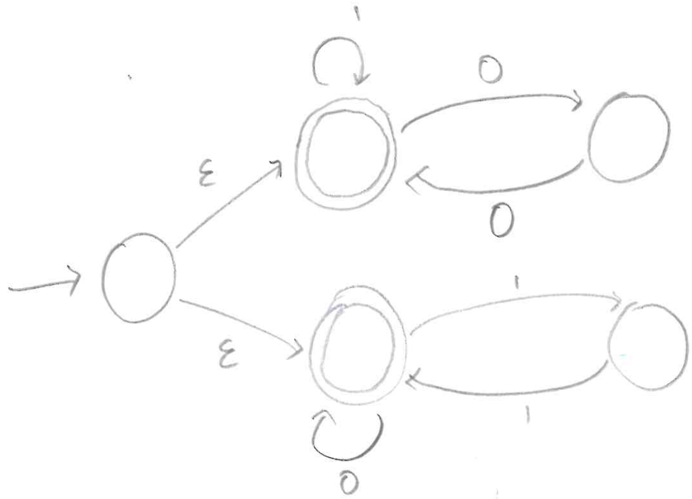
Hence, M must be regular!

(CONTD.)

2) BASIC Given $N = \{ w \mid \#0(w) \times \#1(w) \text{ is even} \}$

⚠ N is regular.

PROOF N is regular since the following automaton, call it A, has $L(A) = N$:



[ε-N.F.A A accepts all strings with even number of 0s or 1s.]

If $\#0(w) \times \#1(w)$ is even,

then either $\#0(w)$ is even or $\#1(w)$ is even

Hence all w in N is accepted by A.

——— × ———

2>
ADDITIONAL

4.1.4,c) Given the language of $(00+11)^*$, apply the pumping lemma to show that we cannot use it to prove that the language is not regular.

(An informal argument follows)

By applying the pumping lemma to language of $(00+11)^*$ (call it L) we get there exists an n s.t. $\forall w. |w| \geq n$, $w = xyz$ with .....

We don't know what this n might be, but it cannot be 0 (since $|y| > 0$ and $|xy| \leq n$).

hence, $n > 0$.

hence, $|w| \geq 1$ (ie, we must pick a non-empty w)

but any non-empty w will have pairs of 00 or 11. If y happens to be one of these pairs, then there's no way to derive a contradiction as any number of these pairs are accepted in the words of L.

——— x ———