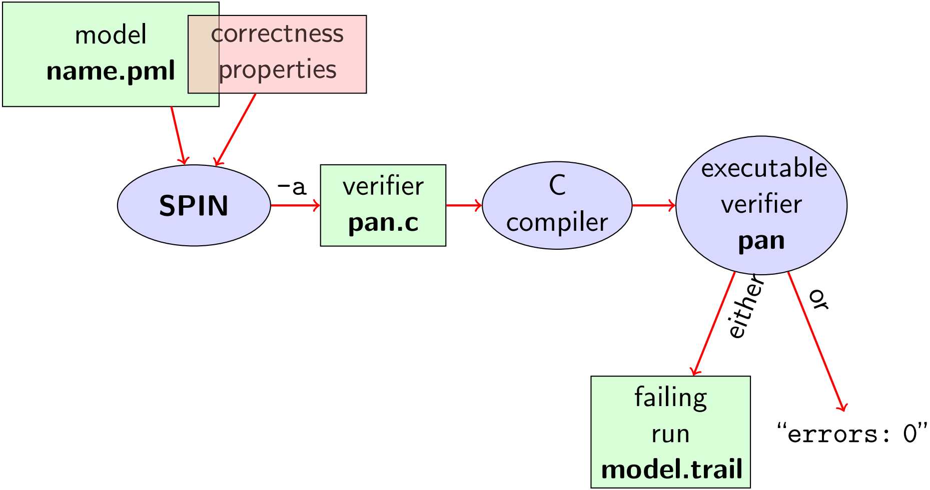
Formal Methods for Software Development

Model Checking with Temporal Logic

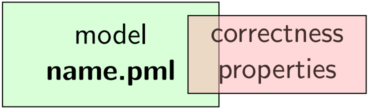
Wolfgang Ahrendt

18th September 2020

Model Checking with SPIN



Stating Correctness Properties



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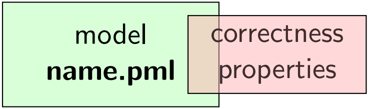

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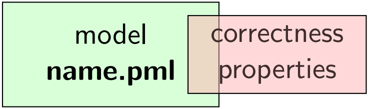

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- meta labels
  - `end` labels ✔
  - `accept` labels
  - `progress` labels

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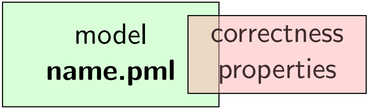

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  - `accept` labels (briefly)
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Preliminaries

1. Accept labels in PROMELA $\leftrightarrow$ Büchi automata
2. Fairness

Preliminaries 1: Acceptance Cycles

Definition (Accept Location)

A location marked with an accept label of the form "`accept`*xxx*`:`" is called an accept location.

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Accept locations can be used to specify cyclic behavior

Definition (Acceptance Cycle)

A run which infinitely often passes through an accept location is called an acceptance cycle.

Acceptance cycles are mainly used in never claims (see below), to define (undesired) infinite behavior

Preliminaries 2: Fairness

Does this model terminate in each run?

Simulate: `start/fair.pml`

```
byte n = 0;
bool flag = false;

active proctype P() {
  do ::  flag -> break
     ::  else -> n = 5 - n
  od
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active proctype Q() {
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```

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Termination guaranteed only if scheduling is (weakly) fair!

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Definition (Weak Fairness)

A run is called weakly fair iff the following holds:
each continuously executable statement is executed eventually.

Model Checking of Temporal Properties

Many correctness properties not expressible by assertions

- All properties that involve state changes
- Temporal logic expressive enough to characterize many (but not all) Linear Time properties

In this course: "temporal logic" synonymous with "linear temporal logic"

Today: model checking of properties formulated in temporal logic

Beyond Assertions

Locality of Assertions

Assertions talk only about the state at their location in the code

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Mutual exclusion enforced by adding assertion to each critical section

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assert( critical <= 1 );
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- No separation of concerns (model vs. correctness property)
- Changing assertions is error prone (easily out of sync)
- Easy to forget assertions:
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Drawbacks

- No separation of concerns (model vs. correctness property)
- Changing assertions is error prone (easily out of sync)
- Easy to forget assertions:
  safety property might be violated at unexpected locations
- Many interesting properties not expressible via assertions

Temporal Correctness Properties

Examples of properties where assertions are suboptimal (too local):

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Something will hold eventually, infinitely often, ...

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These are temporal properties $\Rightarrow$ use temporal logic

Boolean Temporal Logic

Numerical variables in expressions

- Expressions such as `i <= len-1` contain numerical variables
- Propositional LTL as introduced so far only knows propositions
- Slight generalisation of LTL required

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- Slight generalisation of LTL required

In Boolean Temporal Logic, atomic building blocks are *Boolean expressions* over PROMELA variables

Boolean Temporal Logic over PROMELA

Set For_{BTL} of Boolean Temporal Formulas (simplified)

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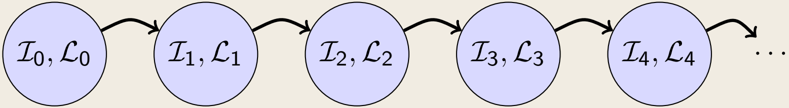
- if ϕ and ψ are formulas $\in For_{BTL}$, then all of

$$!\,\phi, \quad \phi \;\&\&\; \psi, \quad \phi \;||\; \psi, \quad \phi \;\text{->}\; \psi, \quad \phi \;\text{<->}\; \psi$$
$$[\,]\phi, \quad <>\phi, \quad \phi\,\mathtt{U}\,\psi$$

  are $\in For_{BTL}$

Semantics of Boolean Temporal Logic

A trace τ through a PROMELA model M



- $\mathcal{I}_j$ maps each variable in M to its current value
- $\mathcal{L}_j$ maps each running process to its current location counter
- From $\mathcal{L}_j$ to $\mathcal{L}_{j+1}$, *only one* of the location counters has advanced (exception: channel rendezvous)

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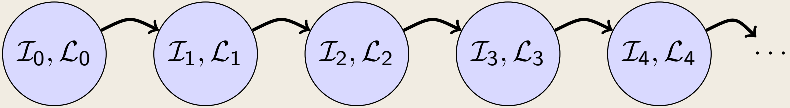

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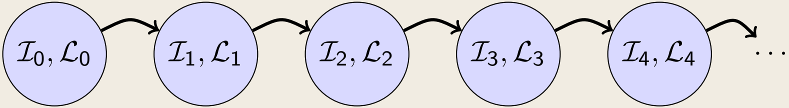

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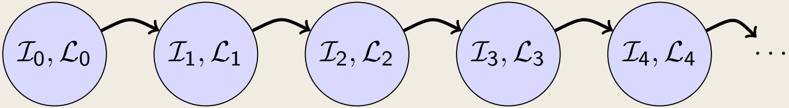

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Evaluating other formulas $\in For_{BTL}$ in traces τ: see previous lecture

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or, equivalently:
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Any violating run would have `(critical > 1)` after *finite* time

Applying Temporal Logic to Critical Section Problem

We want to verify `[](critical<=1)` as a correctness property of:

```
active proctype P() {
  do :: /* non-critical activity */
        atomic {
          !inCriticalQ;
          inCriticalP = true
        }
        critical++;
        /* critical activity */
        critical--;
        inCriticalP = false
  od
}

/* similarly for process Q */
```

Model Checking a Safety Property with Spin

Command Line Execution

Add definition of TL formula to Promela file

Example `ltl atMostOne { [](critical <= 1) }`

General `ltl` *name* `{` *TL-formula* `}`

can define more than one formula

```
> spin -a file.pml
> gcc -DSAFETY -o pan pan.c
> ./pan -N name
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Demo: `target/safety1.pml`

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`ltl` definitions not part of Ben Ari's book (SPIN $\leq$ 6): ignore 5.3.2, etc.

Model Checking a Safety Property using Web Interface

1. add definition of TL formula to Promela file

   Example `ltl atMostOne { [](critical <= 1) }`

   General `ltl` *name* { *TL-formula* }

   can define more than one formula

2. load Promela file into web interface
3. ensure `Safety` is selected
4. enter name of LTL formula in according field
5. select `Verify`

Demo: `safety1.pml`

Model Checking a Safety Property using JSPIN

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2. load PROMELA file into JSPIN
3. write *name* in 'LTL formula' field
4. ensure `Safety` is selected
5. select `Verify`
   - (corresponds to command line `./pan -N` *name* `...`)
6. (if necessary) select `Stop` to terminate too long verification

Demo: `safety1.pml`

Temporal Model Checking without Ghost Variables

We want to verify mutual exclusion without using ghost variables.

```
bool inCriticalP = false , inCriticalQ = false;

active proctype P() {
  do :: atomic {
          !inCriticalQ;
          inCriticalP = true
        }
     cs: /* critical activity */
        inCriticalP = false
  od
}

/* similar for process Q with same label cs: */

ltl mutualExcl { []!(P@cs && Q@cs) }
```

Demo: `start/noGhost.pml`

Never Claims: Processes trying to show user wrong

Büchi automaton, as PROMELA process, for negated property

1. Negated TL formula translated to '**never**' process
2. Accepting locations in Büchi automaton represented with help of accept labels ("`accept`*xxx*`:`")
3. If one of these reached infinitely often, the orig. property is violated

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Example (Never claim for `<>p`, simplified for readability)

```
never {  /* !<>p */
  accept_xyz: /* passed ∞ often iff !<>p holds */
  do
  :: !p
  od
}
```

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Example

`<>csp`

(with `csp` a variable only true in the critical section of `P`)

"in each run, process `P` visits its critical section eventually"

Applying Temporal Logic to Starvation Problem

We want to verify `<>csp` as a correctness property of:

```
active proctype P() {
  do :: /* non-critical activity */
        atomic {
          !inCriticalQ;
          inCriticalP = true
        }
        csp = true;
        /* critical activity */
        csp = false;
        inCriticalP = false
  od
}

/* similarly for process Q */
/* there, using csq         */
```

Model Checking a Liveness Property using jSpin

1. open Promela file `liveness1.pml`
2. write `ltl PwillEnterCS { <>csp }` in Promela file
   (as first `ltl` formula)
3. ensure that `Acceptance` is selected (for liveness properties)
   (Spin will search for *accepting* cycles through the never claim)
4. *for the moment* uncheck `Weak Fairness` (see discussion below)
5. select `Verify`

Verification Fails

Demo: `start/liveness1.pml`

Verification fails!

Why?

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Verification fails!

Why?

The liveness property on one process "had no chance".
Not even weak fairness was switched on!

Model Checking Liveness with Weak Fairness using JSPIN

Always check `Weak fairness` when verifying liveness

1. open PROMELA file
2. write `ltl PwillEnterCS { <>csp }` in PROMELA file
   (as first `ltl` formula)
3. ensure that `Acceptance` is selected (for liveness properties)
   (SPIN will search for *accepting* cycles through the never claim)
4. ensure `Weak fairness` is checked
5. select `Verify`

Model Checking Liveness using Web Interface

1. add definition of TL formula to PROMELA file

   Example `ltl pWillEnterC { <>csp }`

   General `ltl` *name* { *TL-formula* }

   can define more than one formula
2. load PROMELA file into web interface
3. ensure `Acceptance` is selected (for liveness properties)
4. enter name of LTL formula in according field
5. ensure `Weak fairness` is checked
6. select `Verify`

Demo: `liveness1.pml`

Model Checking Liveness using SPIN directly

Command Line Execution

Make sure `ltl` *`name`* `{` *`TL-formula`* `}` is in *`file`*`.pml`

```
> spin -a file.pml
> gcc -o pan pan.c
> ./pan -a -f [-N name]
```

`-a` check acceptance cycles, `-f` weak fairness

Demo: `start/liveness1.pml`

Limitation of Weak Fairness

Verification fails again!

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Weak fairness is too weak ...

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Note that `!inCriticalQ` is not continuously executable!

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A run is called weakly fair iff the following holds:
each continuously executable statement is executed eventually.

Note that `!inCriticalQ` is not continuously executable!

Restriction to weak fairness is principal limitation of SPIN

Here, liveness needs strong fairness, which is not supported by SPIN.

Revisit `fair.pml`

- Specify liveness of `fair.pml` using labels

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Demo: `target/fair.pml`

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- Here, weak fairness is needed, *and sufficient*

Demo: `target/fair.pml`

LTL Model Checking—Theory

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Given:

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Verification task: is the LTL formula ϕ satisfied in all traces of $\mathcal{T}$, i.e.,

$$\mathcal{T} \models \phi \quad ?$$

LTL Model Checking—Preview

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   If no such π is found, then
$$\mathcal{T} \models \phi$$

When What?

this lecture

3.–5. product of transition system and Büchi automaton (construction and analysis)

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this lecture

3.–5. product of transition system and Büchi automaton (construction and analysis)

next lecture

1. translating LTL into generalised Büchi automata
2. generalised Büchi automata and their normalisation

Product of Transition System and Büchi Automaton

A model checking graph is a directed graph with initial and accepting nodes.

Definition (Model Checking Graph)

A model checking graph $(N, \to, N_0, N_a)$ is composed of:

- finite, non-empty set of nodes N
- an 'arrow' relation $\to \subseteq N \times N$
- a non-empty set of initial nodes $N_0 \subseteq N$
- a set of accepting nodes $N_a \subseteq N$

Product of Transition System and Büchi Automaton

In the following, we assume without further mention:

1. transition systems without terminal states:
   $\{s' \in S | s \to s'\} \neq \emptyset$ for all states $s \in S$

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Can always be achieved by adding 'trap states' or 'trap locations', resp.

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We assume a set of atomic propostions AP.

Definition (Product of Transition System and Büchi Automaton)

Let $\mathcal{T} = (S, \rightarrow, S_o, L)$ be a transition system over AP and $\mathcal{B} = (Q, \delta, Q_0, F)$ be a Büchi automaton over the alphabet 2^{AP}.
Then, $\mathcal{T} \otimes \mathcal{B}$ is the following model checking graph:

$$\mathcal{T} \otimes \mathcal{B} = (S \times Q, \rightarrow', N_0, N_a)$$

where:

- $\langle s, q \rangle \rightarrow' \langle s', q' \rangle$ iff $s \rightarrow s'$ and $(q, L(s'), q') \in \delta$

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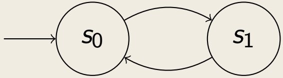
Model Checking Example

Assume $AP = \{red, green\}$

$\mathcal{T}$:



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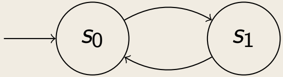

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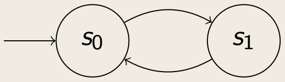

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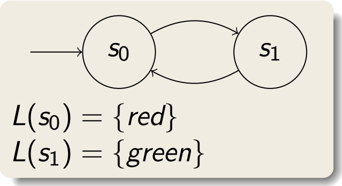

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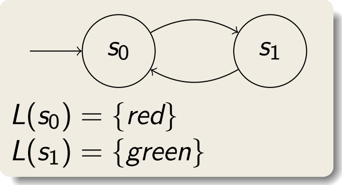

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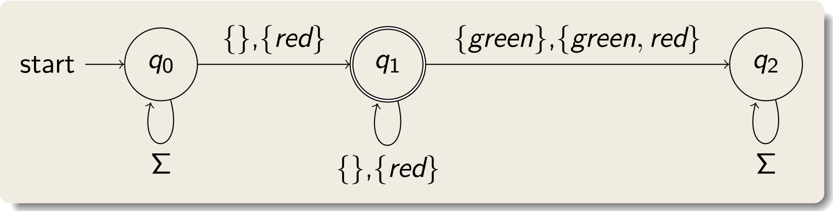

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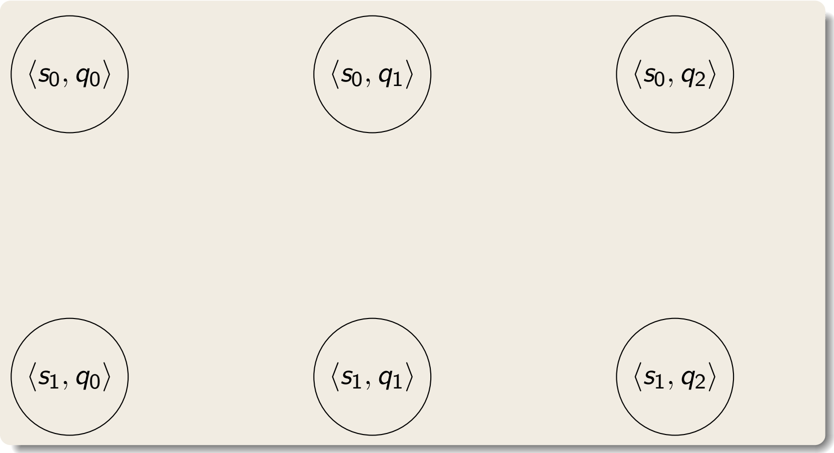
Model checking graph $\mathcal{T} \otimes \mathcal{B}_{\neg\phi}$:



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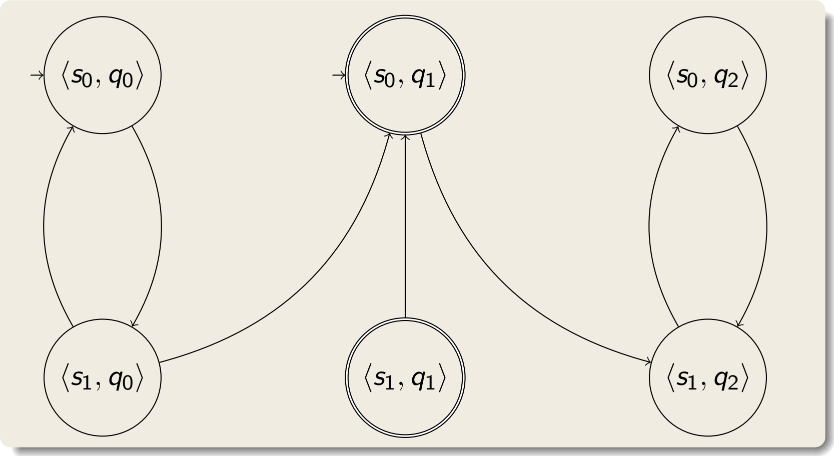

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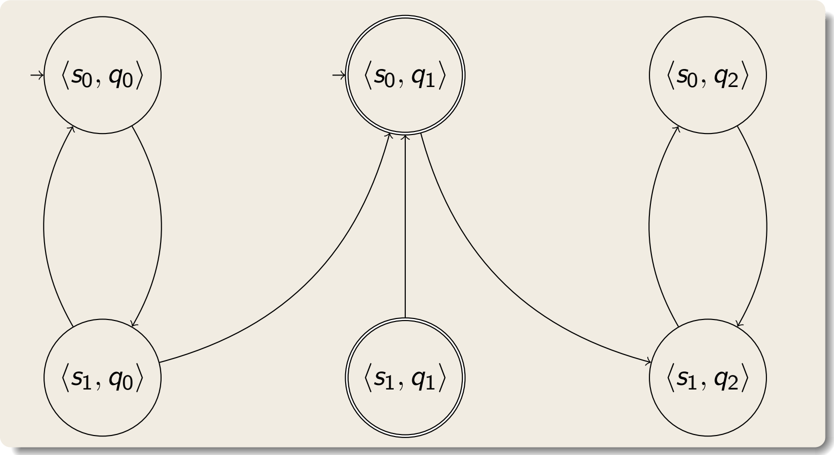

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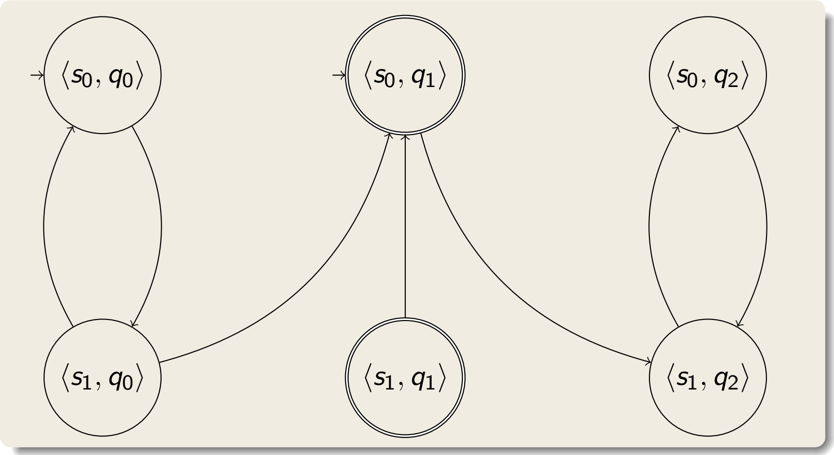

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$\mathcal{T} \models \phi$

Literature for this Lecture

Ben-Ari Chapter 5
except Sections 5.3.2, 5.3.3, 5.4.2
(`ltl` construct replaces **#define** and `-f` option of SPIN)