God Morgon!

strukturell induktion

$\boxed{\text{bevisa}}$ egenskaper om program

$\left\{\begin{array}{l}\text{enkel induktion} \\ \text{stark induktion}\end{array}\right.$

$\forall n \in \mathbb{N}.$ ....

$\boxed{\text{kolla}}$ om en egenskap gäller:

- testning

t.ex. QuickCheck

$$
\begin{cases}
(+\!\!+) :: [a] \to [a] \to [a] \\
[] \mathbin{+\!\!+} ys = ys \\
(x:xs) \mathbin{+\!\!+} ys = x : (xs \mathbin{+\!\!+} ys)
\end{cases}
$$

all kod

tinyurl.com/dit980

$+\!\!+$ associativ

ex.1 $\quad \forall xs \in [a] . \quad xs \mathbin{+\!\!+} [] = xs$

"trivialt!"

visa : $xs ++ [\,] = xs$ för alla listor xs

bevis : med (strukturell) induktion över xs

Låt $P(xs) =$ " $xs ++ [\,] = xs$ "

basfall : $P([\,])$: $[\,] ++ [\,] = [\,]$ (def. $++$)

stegfall : $P(as) \Rightarrow P(a:as)$

anta : $P(as)$: $as ++ [\,] = as$ (I.H.)

visa : $P(a:as)$: $(a:as) ++ [\,] = a:as$

$$
\begin{aligned}
(a:as) ++ [\,] &= a : (as ++ [\,]) && \text{(def. } ++\text{)} \\
&= a : as && \text{(I.H.)}
\end{aligned}
$$

$\square$

visa : $(+\!\!+)$ är associativ $xs +\!\!+ (ys +\!\!+ zs) = (xs +\!\!+ ys) +\!\!+ zs$

bevis : med strukturell induktion över xs

Låt $P(xs)$ = " $xs +\!\!+ (ys +\!\!+ zs) = (xs +\!\!+ ys) +\!\!+ zs$ för listor ys, zs "

Basfall : $P([\,])$: $[\,] +\!\!+ (ys +\!\!+ zs) = ys +\!\!+ zs$ (def. $+\!\!+$)

$= ([\,] +\!\!+ ys) +\!\!+ zs$ (def. $+\!\!+$)

stegfall : $P(as) \Rightarrow P(a:as)$

anta : $as +\!\!+ (ys +\!\!+ zs) = (as +\!\!+ ys) +\!\!+ zs$ (I.H.)

visa : $(a:as) +\!\!+ (ys +\!\!+ zs) = ((a:as) +\!\!+ ys) +\!\!+ zs$

$(a:as) +\!\!+ (ys +\!\!+ zs) = a : (as +\!\!+ (ys +\!\!+ zs))$ (def. $+\!\!+$)

$= a : ((as +\!\!+ ys) +\!\!+ zs)$ (I.H.)

$= (\underline{a : (as +\!\!+ ys)}) +\!\!+ zs$ (def. $+\!\!+$)

$= (\underline{(a:as) +\!\!+ ys}) +\!\!+ zs$ (def. $+\!\!+$)

$\square$

som (reverse)

$$\begin{cases} rev :: [a] \to [a] \\ rev\ [] = [] \\ rev\ (x:xs) = rev\ xs \mathbin{+\!\!+} [x] \end{cases}$$

$$\forall xs \in [a].\ rev\ (rev\ xs) = xs$$

$$\text{rev } (xs ++ ys) = \text{rev } ys ++ \text{rev } xs$$

$$\text{rev } ([1,2,3] ++ [10,11,12]) =$$

$$[12, 11, 10, 3, 2, 1] =$$

$$\text{rev } ([10,11,12]) ++ \text{rev } [1,2,3]$$

egenskap om

rev / ++

rev (xs ++ ys) =?

visa : $\text{rev}\ (xs \mathbin{+\!\!+} ys) = \text{rev}\ ys \mathbin{+\!\!+} \text{rev}\ xs$

bevis : med induktion över xs

Låt $P(xs) =$ "$\text{rev}\ (xs \mathbin{+\!\!+} ys) = \text{rev}\ ys \mathbin{+\!\!+} \text{rev}\ xs$"

bastfall : $P([\,])$:

$$
\begin{aligned}
\text{rev}\ ([\,] \mathbin{+\!\!+} ys) &= \text{rev}\ ys && (\text{def. } \mathbin{+\!\!+}) \\
&= \text{rev}\ ys \mathbin{+\!\!+} [\,] && (\text{lemma } \mathbin{+\!\!+}/[\,]) \\
&= \text{rev}\ ys \mathbin{+\!\!+} \text{rev}\ [\,] && (\text{def. rev})
\end{aligned}
$$

stegfall : $P(as) \Rightarrow P(a:as)$

anta : $P(as)$: $\text{rev}\ (as \mathbin{+\!\!+} ys) = \text{rev}\ ys \mathbin{+\!\!+} \text{rev}\ as$ (I.H.)

visa : $P(a:as)$: $\text{rev}\ ((a:as) \mathbin{+\!\!+} ys) = \text{rev}\ ys \mathbin{+\!\!+} \text{rev}\ (a:as)$

$$
\begin{aligned}
\text{rev}\ ((a:as) \mathbin{+\!\!+} ys) &= \text{rev}\ (a : (as \mathbin{+\!\!+} ys)) && (\text{def. } \mathbin{+\!\!+}) \\
&= \text{rev}\ (as \mathbin{+\!\!+} ys) \mathbin{+\!\!+} [a] && (\text{def. rev}) \\
&= (\text{rev}\ ys \mathbin{+\!\!+} \text{rev}\ as) \mathbin{+\!\!+} [a] && (\text{I.H.}) \\
&= \text{rev}\ ys \mathbin{+\!\!+} (\text{rev}\ as \mathbin{+\!\!+} [a]) && (\mathbin{+\!\!+} \text{assoc.}) \\
&= \text{rev}\ ys \mathbin{+\!\!+} \text{rev}\ (a:as) && (\text{def. rev})
\end{aligned}
$$

$\square$

visa : rev (rev xs) = xs   för alla listor xs

bevis : med strukt. ind. över xs

Låt $P(xs) =$ "rev (rev xs) = xs"

basfall : $P([])$: rev (rev []) = rev []   (def. rev)

= []   (def. rev)

stegfall : $P(as) \Rightarrow P(a:as)$

anta : $P(as)$: rev (rev as) = as   (I.H.)

visa : $P(a:as)$: rev (rev (a:as)) = a:as

rev (rev (a:as)) = rev (rev as ++ [a])   (def. rev)

= rev [a] ++ rev (rev as)   (lemma rev/++)

= [a] ++ rev (rev as)   (def. rev)

= [a] ++ as   (I.H.)

= a:as   (def. ++)

□

$$
\begin{aligned}
\mathrm{rev}\ [a] &= \mathrm{rev}\ (a : [\,]) && (\text{def. } [\_]) \\
&= \mathrm{rev}\ [\,] +\!\!+ [a] && (\text{def. rev}) \\
&= [\,] +\!\!+ [a] && (\text{def. rev}) \\
&= [a) && (\text{def. } +\!\!+)
\end{aligned}
$$

"varför
är rev [a]=[a]"

rekursiv

```
data [a] = []
         | a : [a]
```

Add (Num 5) (Mul (Num 7)
(Num 8))

5 + 7 · 8

swap

7 · 8 + 5

```
data Expr = Num Integer
          | Add Expr Expr
          | Mul Expr Expr
```

```
data Nat = Zero
         | Plus1 Nat
```

enkel induktion
=
strukturell induktion
över Nat

```
eval :: Expr → Integer
eval (Num n) = n
eval (Add a b) = eval a + eval b
eval (Mul a b) = eval a · eval b
eval

swap :: Expr → Expr
swap (Num n) = Num n
swap (Add a b) = Add (swap b) (swap a)
swap (Mul a b) = Mul (swap a) (swap b)
swap
```

$$\forall e \in Expr.\ eval\ e = eval\ (swap\ e)$$

visa: eval (swap e) = eval e för alla $e \in Expr$

bevis: med strukt. ind. över e

Låt P(e) = "eval (swap e) = eval e"

basfall: P(Num n) : eval (swap (Num n)) = eval (Num n) (def. swap)

stegfall 1: $(P(a) \land P(b)) \Rightarrow P(\text{Add a b})$

anta: P(a) : eval (swap a) = eval a (I.H.1)

P(b) : eval (swap b) = eval b (I.H.2)

visa: P(Add ab) : eval (swap (Add a b)) = eval (Add a b)

eval (swap (Add a b)) = eval (Add (swap b) (swap a)) (def swap)

= eval (swap b) + eval (swap a) (def eval)

= eval b + eval a (I.H1 + I.H2)

= eval a + eval b (+ komm.)

= eval (Add a b)

<u>stegfall 2</u> : $(P(a) \wedge P(b)) \Rightarrow P(\text{Mul } a\ b)$

<u>anta</u> : $P(a)$: eval (swap a) = eval a (I.H.1)

$P(b)$: eval (swap b) = eval b (I.H.2)

<u>visa</u> : $P(\text{Mul } a b) \stackrel{?}{=}$ eval (swap (Mul a b)) = eval (Mul a b)

eval (swap (Mul a b)) = eval (Mul (swap a) (swap b)) (def swap)

= eval (swap a) * eval (swap b) (def eval)

= eval a * eval b (I.H.1 + I.H.2)

= eval (Mul a b) (def eval)

$\square$