Solutions to the selected exercises from E3

11 - Feb - 2020

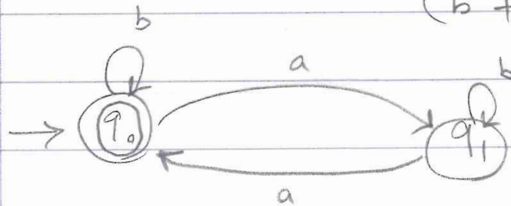
1) Let $\Sigma = \{a, b\}$,

(a) Give one regular expression for the set of words containing an even number of a's and one for the set of words containing an odd number of a's.

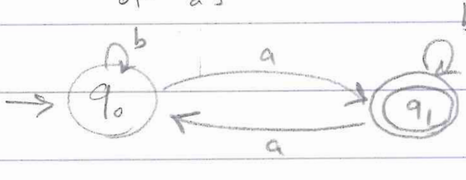
- even number of a's : $b^*(ab^*ab^*)^*$

or

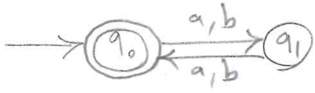
$(b + ab^*ab^*)^*$



- odd number of a's : $b^*(ab^*ab^*)^*ab^*$



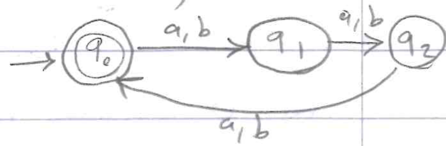
(b) Give one regular expression for strings with even length and one for strings whose length is a multiple of 3.



- even length: $\left((a+b)(a+b)\right)^*$

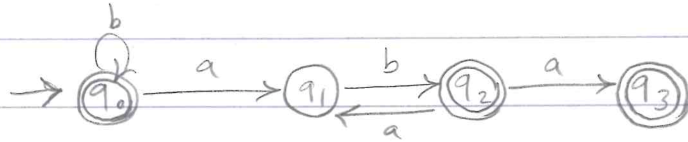
$= (aa+ab+ba+bb)^*$

- multiple of 3: $\left((a+b)(a+b)(a+b)\right)^*$



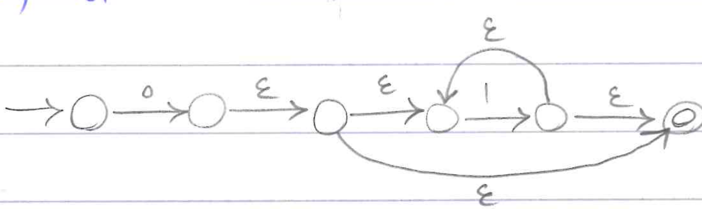
(c) Give a regular expression for the ~~automata~~ strings that do not contain the substring 'aa'.

$(b+ab)^*(a+\varepsilon)$

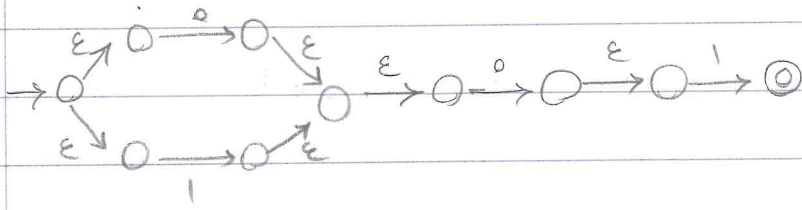


2) Exercise 3.2.4 (Convert RE to ε-NFA)

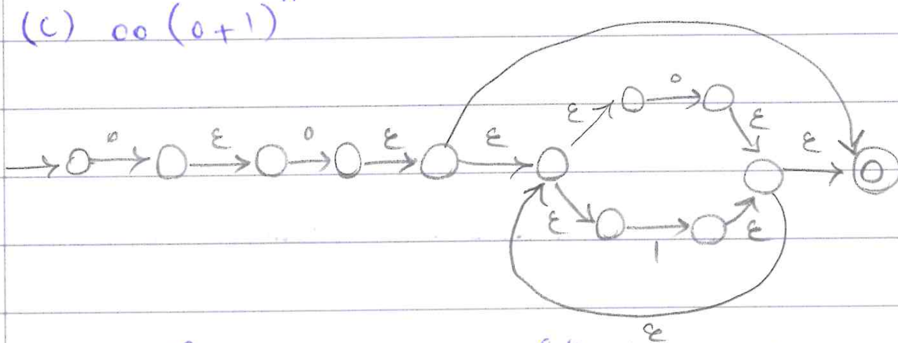
(a) 01^*



(b) $(0+1)01$



(c) $00(0+1)^*$



3) Simplify each of the following regular expressions:

(a) $\varepsilon + ab + abab(ab)^*$

$= \varepsilon + ab(\varepsilon + ab(ab)^*)$

[using $\varepsilon + dd^* = d^*$] $= \varepsilon + ab(ab)^*$

[again] $= (ab)^*$

(b) $aa(b^*+a) + a(ab^*+aa)$

$= aa(b^*+a) + aa(b^*+a)$

[using $d+d=d$] $= aa(b^*+a)$

c) $a(a+b)^* + aa(a+b)^* + aaa(a+b)^*$

$= (a + aa + aaa)(a+b)^*$

$= a(a+b)^*$

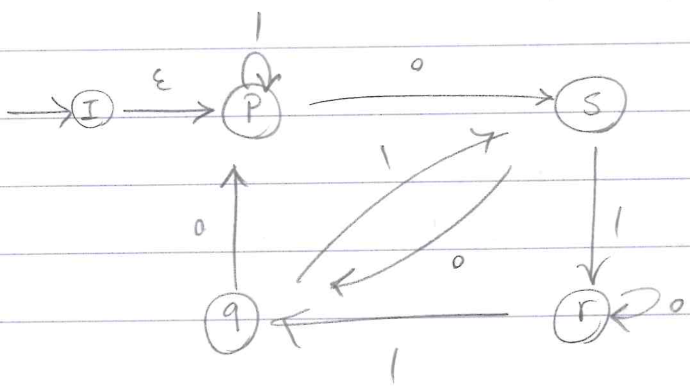
$\hookrightarrow$ [using $\forall n \geq 1.\ (a + aa + \cdots + a^n)(a+b)^* = a(a+b)^*$]
can be proven by induction

4) Exercise 3.2.3 (Convert DFA to RE by state-elimination)

[and by system of linear equations]

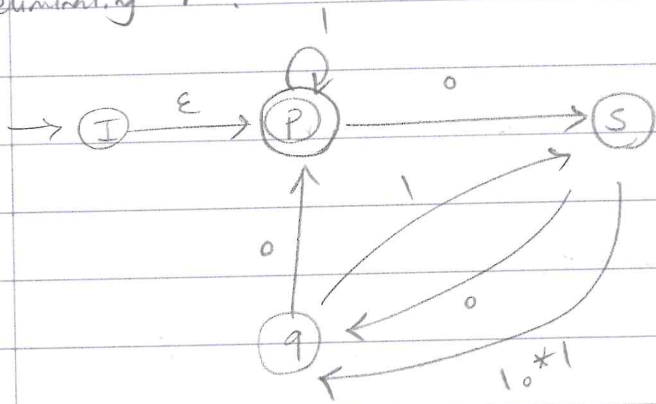
		0	1
$\rightarrow *$	p	s	p
	q	p	s
	r	r	q
	s	q	r

By state-elimination:

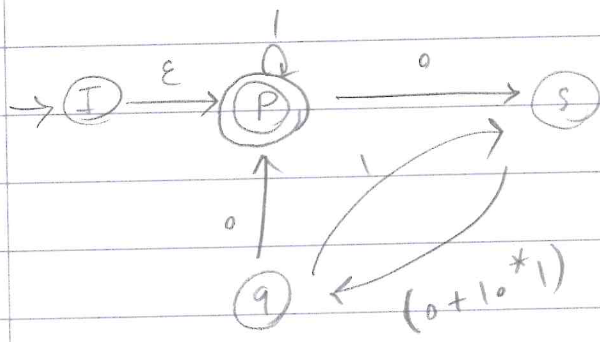


eliminating r :

⇒

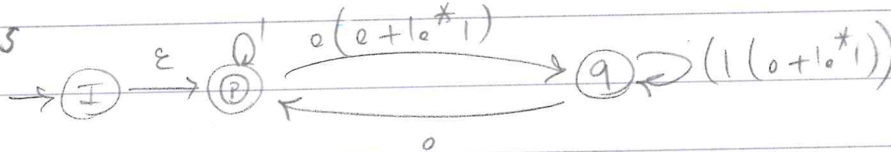


⇒



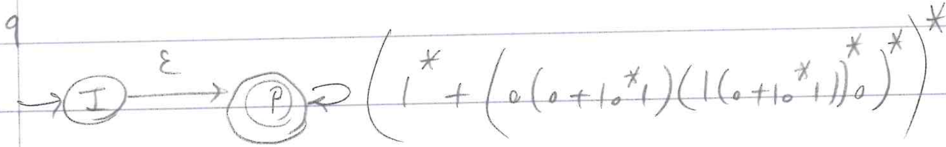
eliminating s

⇒



eliminating q

⇒



⇒ RegEx = $\left(1^* + 0(0+10^*1)(1(0+10^*1))^*0)^*\right)^*$

By system of linear equations (Arden's lemma)

$$\begin{cases} X_1 = 0X_4 + 1X_1 + \varepsilon \\ X_2 = 0X_1 + 1X_4 \\ X_3 = 0X_3 + 1X_2 \\ X_4 = 0X_2 + 1X_3 \end{cases} \xRightarrow{X_2} \begin{cases} X_1 = 0X_4 + 1X_1 + \varepsilon \\ X_3 = 0X_3 + 1(0X_1 + 1X_4) \\ X_4 = 0(0X_1 + 1X_4) + 1X_3 \end{cases}$$

$$\Rightarrow \begin{cases} X_1 = 0X_4 + 1X_1 + \varepsilon \\ X_3 = 0^*(10X_1 + 11X_4) \\ X_4 = (01)^*(00X_1 + 1X_3) \end{cases}$$

$$\Rightarrow \begin{cases} X_1 = 0X_4 + 1X_1 + \varepsilon \\ X_4 = (01)^*(00X_1 + 1(0^*10X_1 + 0^*11X_4)) \end{cases}$$

$$\Rightarrow \begin{cases} X_1 = 0X_4 + 1X_1 + \varepsilon \\ X_4 = (01)^*(10^*11)X_4 + (01)^*(00 + 10^*10)X_1 \end{cases}$$

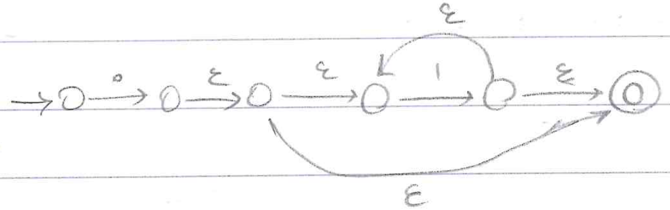
$$\Rightarrow \begin{cases} X_1 = 0X_4 + 1X_1 + \varepsilon \\ X_4 = ((01)^*(10^*11))^*(01)^*(00 + 10^*10)X_1 \end{cases}$$

$$\Rightarrow X_1 = (0((01)^*(10^*11))^*(01)^*(00 + 10^*10)) + 1)^*$$

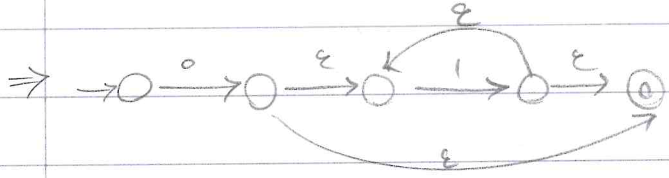
✓ These two RegExes are equivalent.

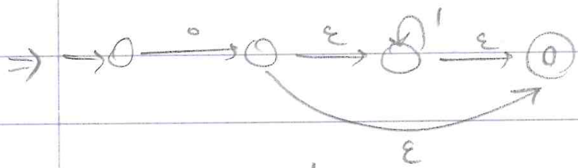
5) 3.2.5) Eliminate ε-transitions from Solutions to 3.2.4.

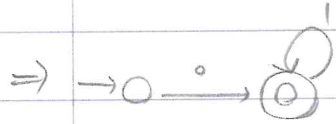
(a)



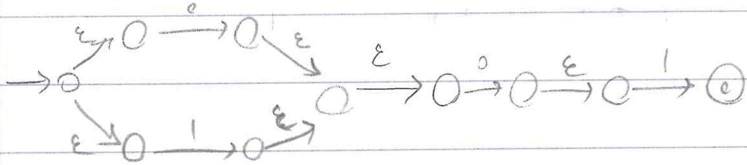
01^*

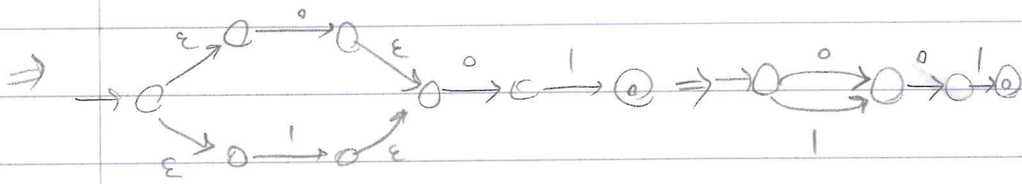




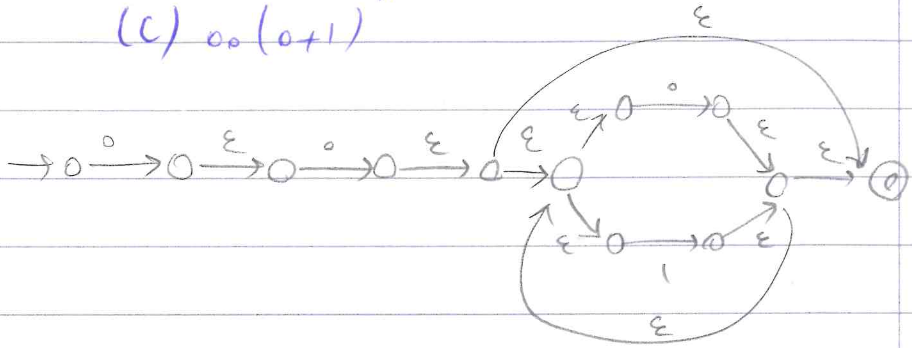


(b) (0+1)01

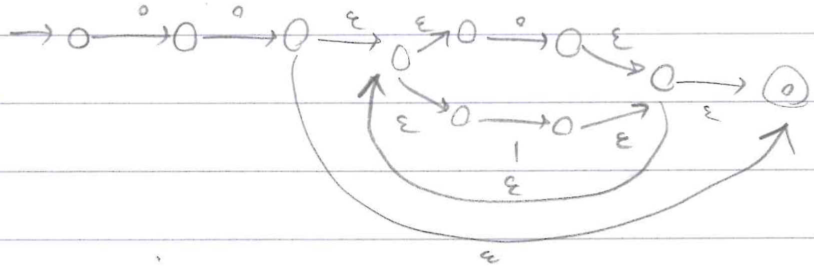




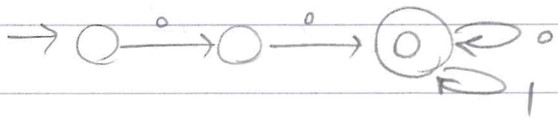
(c) $00(0+1)^*$



$\Rightarrow$



$\Rightarrow$



6) Some parts of 3.4.1 and 3.4.2

(I) $R+S = S+R$

Replace $\begin{cases} R \rightsquigarrow \{a\} \\ S \rightsquigarrow \{b\} \end{cases} \Rightarrow \{a\} \cup \{b\} = \{b\} \cup \{a\} = \{a, b\}$

both languages are the same.

(II) $(R^*)^* = R^*$

Replace R by $\{a\}$. $\Rightarrow (\{a\}^*)^* = \{a\}^*$

$\{a\}^*$: all strings of a's including the empty string

$(\{a\}^*)^*$: all strings consisting of concatenation of strings of a's, which are the same as above.

(III) $(R+S)^* = R^* + S^*$

Replace $\begin{cases} R \rightsquigarrow \{a\} \\ S \rightsquigarrow \{b\} \end{cases}$

$(R+S)^*$: all strings of a's and b's (mixed)

$R^* + S^*$: strings of a's (alone) and strings of b's (alone)

Counter example: "ab" is not $R^* + S^*$

(IV) $R + (R+S)^* = (R+S)^*$

$$R + (R+S)^* = R + \left(\varepsilon + (R+S) + (R+S)^2 (R+S)^*\right)$$

[+ assoc.] $$= \varepsilon + (R+R) + S + (R+S)^2 (R+S)^*$$

[R+R=R] $$= \varepsilon + (R+S) + (R+S)^2 (R+S)^* = (R+S)^*$$

$[R^* = \varepsilon + R + R^2 + \cdots + R^{k-1} + R^k R^*]$