EXERCISE - 2

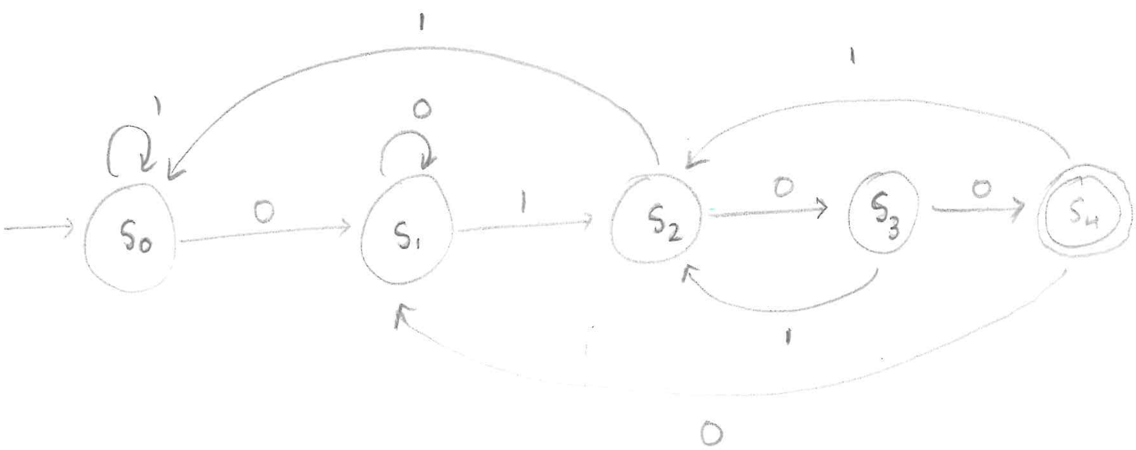
[DFA]

2) BASIC

$\Sigma = \{0, 1\}$ $\quad$ $R = \{ w0100 \mid w \in \Sigma^* \}$

⚠ R is regular

For this purpose, we construct the following DFA "A":



[Claim: $L(A) = R$

should be provable by induction on w in $w0100$.]

Examples:

0100 ✓

00100 ✓

1 01 00 ✓

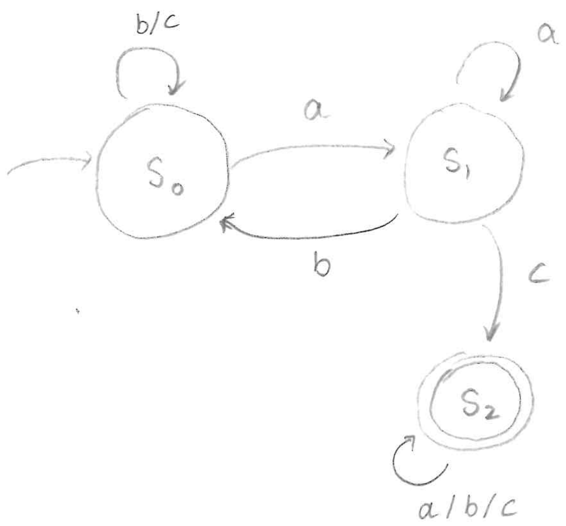
11...11 01 00 ✓

010 01 00 ✓

4)
BASIC

$\Sigma = \{a, b, c\}$

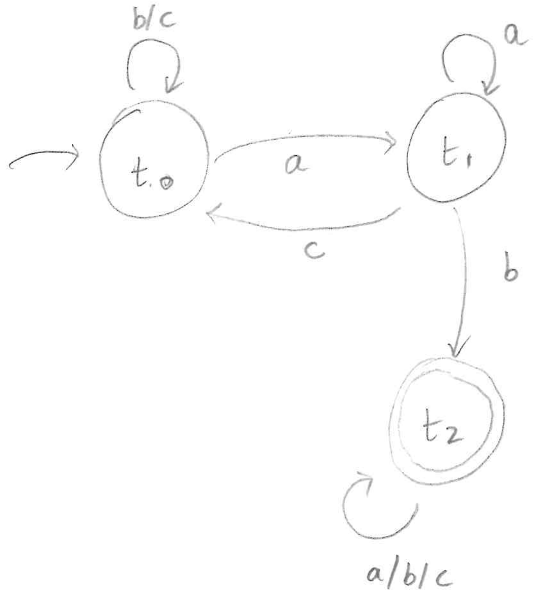
⚠ DFA D_1 s.t. $\mathcal{L}(D_1) = \{ w \in \Sigma^* \mid ac \text{ is a subword of } w \}$



[Claim: $\mathcal{L}(D_1) = \{ w \in \Sigma^* \mid ac \text{ is subword of } w \}$

Should follow from $\forall q\, w, w_2 .\ \hat{\delta}(q, w\, ac\, w_2) = s_2$]

⚠ DFA D_2 s.t. $\mathcal{L}(D_2) = \{ w \in \Sigma^* \mid ab \text{ is a subword of } w \}$



[Proof should follow from $\forall q\, w, w_2 .$ $\hat{\delta}(q, w_1 ab\, w_2) = t_2$]

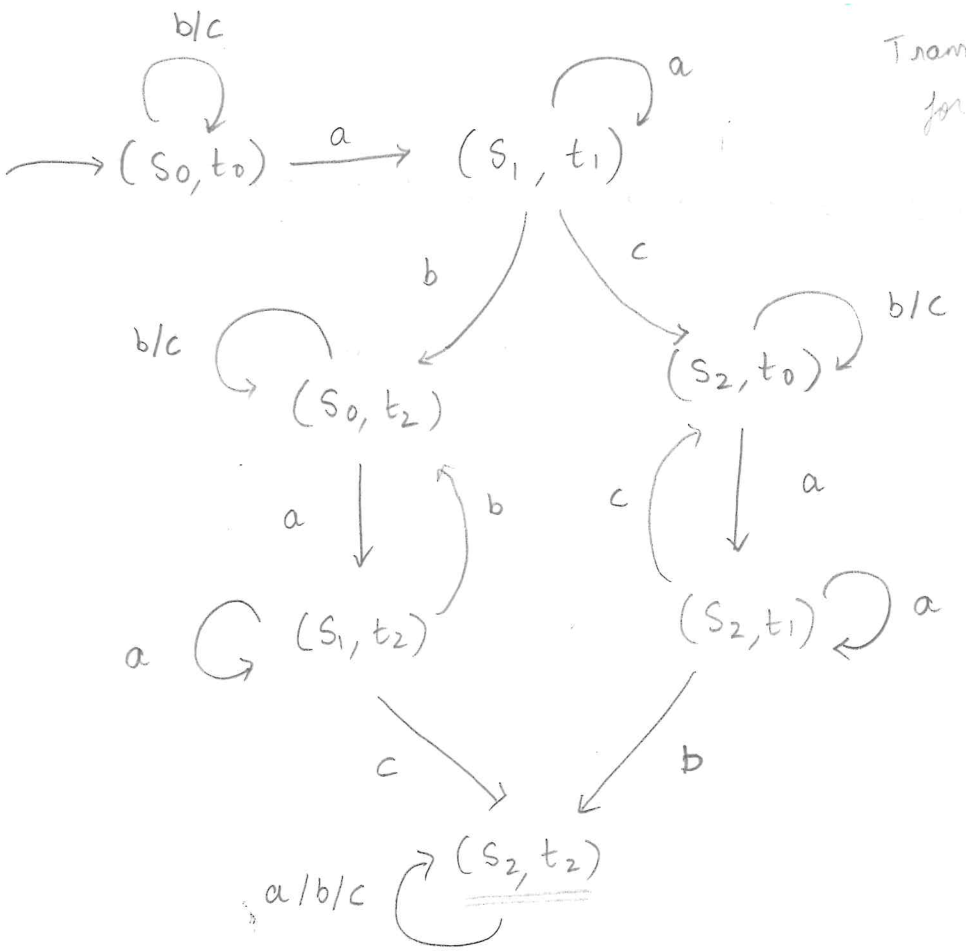
(Continuation of 4)

⚠ $D_1 \otimes D_2$

$$D_1 \otimes D_2 = (Q_1 \times Q_2 \quad = \{s_0, s_1, s_2\} \times \{t_0, t_1, t_2\}$$
$$, \Sigma \quad = \{a, b, c\}$$
$$, \delta_1 \times \delta_2$$
$$, (s_0, t_0)$$
$$, F_1 \times F_2) \quad = \{(s_2, t_2)\}$$

recollect that

$$\delta_1 \times \delta_2 (s, t) = (\delta_1(s), \delta_2(t))$$



Transition diagram for $D_1 \otimes D_2$

$$\mathcal{L}(D_1 \otimes D_2) = \{ w \in \Sigma^* \mid w \in \mathcal{L}(D_1) \cap \mathcal{L}(D_2) \}$$

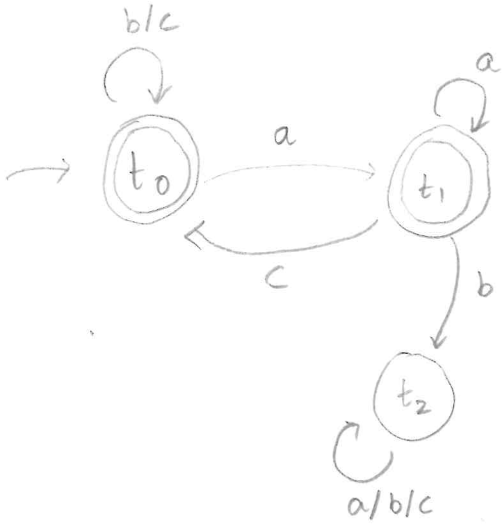
(must be shown separately, taken for granted here)

(Continuation of 4)

⚠ $D_1 \otimes \overline{D_2}$

recollect that $\overline{D_2} = \{ Q_2, \Sigma_2, \delta_2, t_0, \overline{F_2} \}$

i.e.,



$L(\overline{D_2}) = \{ w \in \Sigma^* \mid$ w does not contain ab $\}$

the product $D_1 \otimes \overline{D_2}$ is contructed the same as before.

CLAIM: $L(D_1 \otimes \overline{D_2}) = \{ w \in \Sigma^* \mid$ w contains ac, but not ab $\}$

Follows from $L(D_1 \otimes \overline{D_2}) = L(D_1) \cap \overline{L(D_2)}$

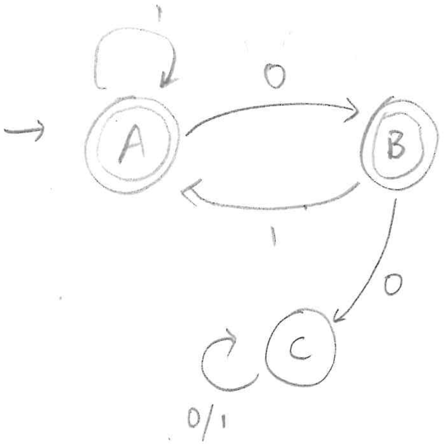
(which must be shown separately, but we take for granted here.)

— × —

2)
ADDITIONAL

		0	1
$\rightarrow$ *	A	B	A
*	B	C	A
	C	C	C

i.e.,



Let's try some sample strings:

0 ✓
1 ✓
00 ✗
01 ✓
10 ✓
11 ✓
00 – ✗
01 – ✓
100 ✗
101 ✓
11 – ✓

Informally, it appears that the DFA accepts all words that don't contain 00.

Let's prove this by induction:

$\forall w \in \Sigma^*.\ \hat{\delta}(A, w) \in F$ if w doesn't contain 00.

Base case: $w = \epsilon$

$\hat{\delta}(A, \epsilon) = A$

since $A \in F$, we get $\hat{\delta}(A, \epsilon) \in F$

Induction step: $w = xv$

by I.H., $\hat{\delta}(A, v) \in F$ if v doesn't contain 00.

by assumption, xv doesn't contain 00

⚠ to show that $\hat{\delta}(A, xv) \in F$

(P.T.O.)

Case analysis on x :

Case $x = 1$

then $\hat{\delta}(A, 1v) = \hat{\delta}(\delta(A,1), v)$

but by defn. $\delta(A,1) = A$

and hence $\hat{\delta}(A, 1v) = \hat{\delta}(A, v)$

and by I.H. $\hat{\delta}(A, 1v) \in F$

since $\hat{\delta}(A, v) \in F$.

Case $x = 0$

then $\hat{\delta}(A, 0v) = \hat{\delta}(\delta(A, 0), v)$

by defn $\delta(A, 0) = B$

hence $\hat{\delta}(A, 0v) = \hat{\delta}(B, v)$

to show $\hat{\delta}(B, v) \in F$

we do case analysis on v :

Case $v = \epsilon$

$\hat{\delta}(B, \epsilon) = B$
$\in F$

Case $v = 1v'$

$\hat{\delta}(B, 1v') = \hat{\delta}(\delta(B, 1), v')$
$= \hat{\delta}(A, v')$
$\in F$ (by I.H. on v')

Case $v = 0v'$

impossible!

since we know xv doesn't contain 00.

Hence, $\hat{\delta}(B, v) \in F$

and $\hat{\delta}(A, 0v) \in F$

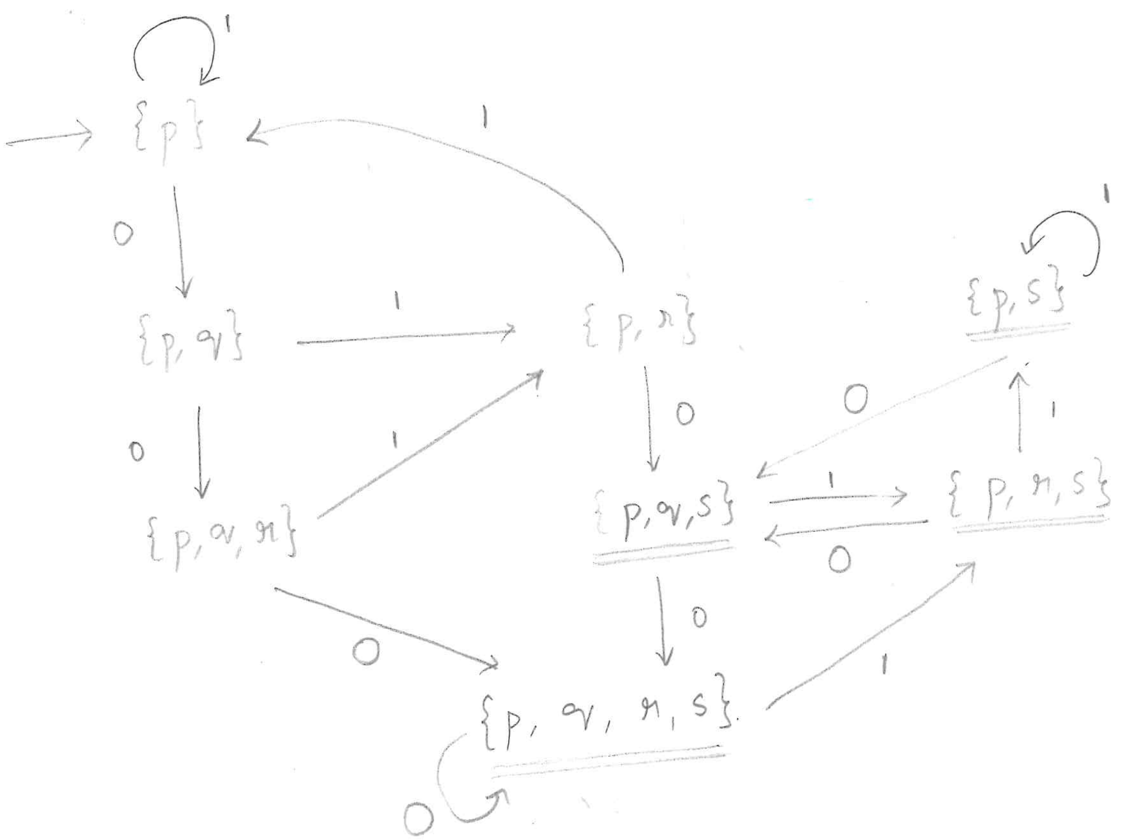
and $\hat{\delta}(A, xv) \in F$ (for all x)

which completes our induction step and hence the proof!

[NFA]

1 - 2.3.1)
BASIC

		0	1
→	p	{p, q}	{p}
	q	{r}	{r}
	r	{s}	∅
*	s	{s}	{s}



3- 2.5.1)

BASIC

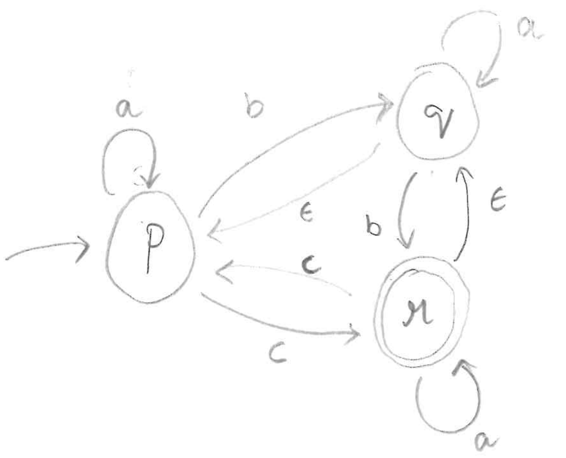
		ϵ	a	b	c
$\rightarrow$	p	$\emptyset$	$\{p\}$	$\{q\}$	$\{r\}$
	q	$\{p\}$	$\{q\}$	$\{r\}$	$\emptyset$
*	r	$\{q\}$	$\{r\}$	$\emptyset$	$\{p\}$

a) ϵ-closure $(p) = \{p\}$

ϵ-closure $(q) = \{q, p\}$

ϵ-closure $(r) = \{r, q, p\}$

b)



c ✓

a c ✓

b c, b b ✓

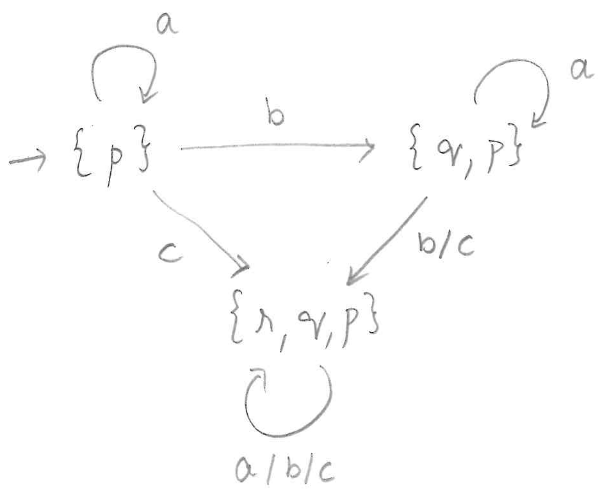
c a, c b ✓

a a c ✓

a b b ✓

a b c ✓

c)



3)
ADDITIONAL

Given $L \subseteq \Sigma^*$ is regular,

show that $L^R = \{ rev(w) \mid w \in L \}$

is regular as well.

Since L is regular,

there must exist a DFA $D = (Q, \Sigma, \delta, q_0, F)$

Construct a new ϵ-NFA $D^R : (Q \cup \{q_0^R\}$

$, \Sigma \cup \{\epsilon\}$

$, \delta^R$

$, q_0^R$

$, \{q_0\})$

where

$$\delta^R(q_0^R, \epsilon) = F$$

$$\delta^R(q_0^R, -) = \phi$$

$$\forall q_t \in Q.\ \delta^R(q_t, a) = \{ q_s \in Q \mid \delta(q_s, a) = q_t \}$$

(i.e., δ^R simply reverses all transitions in $\mathcal{D}$)

CLAIM: $L(\mathcal{D}^R) = \{ rev(w) \mid w \in L \}$

(needs to be proved !)

Hence L^R is regular (by the construction of D^R, which can be converted ~~to~~ a DFA)