Formal Methods for Software Development

Proof Obligations

Wolfgang Ahrendt

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This Part

making the connection between

JML

and

Dynamic Logic / KeY

- generating,
- understanding,
- and proving

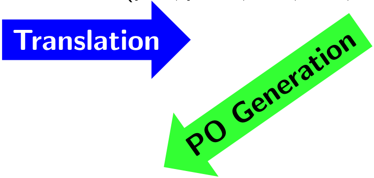
DL proof obligations from JML specifications

From JML Contracts via Intermediate Format to Proof Obligations (PO)

```
public class A {
 /*@ public normal_behavior
   @ requires <Precondition>;
   @ ensures <Postcondition>;
   @ assignable <locations>;
   @*/
 public int m(params) {..}
}
```

Intermediate Format

(pre, $post$, div, var, mod)



Proof obligation as DL formula

$$pre \rightarrow \langle \texttt{this.m(params);} \rangle (post \;\wedge\; frame)$$

JML Translation: Normalizing JML Contracts

Normalization of JML Contracts

1. Flattening of nested specifications
2. Making implicit specifications explicit
3. Processing of modifiers
4. Adding of default clauses if not present
5. Contraction of several clauses

Tho following introduces principles of this process

Normalisation:
Making Implicit Information Explicit

Implicit Information

- Meaning of `normal_` and `exceptional_behavior`
- `non_null` by default
- `\invariant_for(this)` in `requires`, `ensures`, `signals` clauses

Turn into general `behavior` spec. case

1. Add to
   - `normal_behavior` the clause **`signals (Throwable t) false;`**
   - `exceptional_behavior` the clause **`ensures false;`**
2. Replace `normal_behavior`/`exceptional_behavior` by `behavior`

Normalisation: Making Implicit Information Explicit

Implicit Information

- Meaning of `normal_` and `exceptional_behavior`
- `non_null` by default
- `\invariant_for(this)` in `requires`, `ensures`, `signals` clauses

Making `non_null` explicit in method specifications

1. Where `nullable` is absent, add **`o != null`** to preconditions (for parameters[a]) and postconditions (for return values[a]).
   E.g., for method `void m(Object o)` add **`requires o != null;`**
2. Thereafter add **`nullable`**, where absent, to *all* parameter[a] and return type[a] declarations

[a] of reference type

Normalisation: Making Implicit Information Explicit

Implicit Information

- Meaning of `normal_` and `exceptional_behavior`
- `non_null` by default
- `\invariant_for(this)` in `requires`, `ensures`, `signals` clauses

Making `\invariant_for(this)` explicit in method specifications

1. Add explicit `\invariant_for(this)` to non-helper method specs:
   - `requires \invariant_for(this);`
   - `ensures \invariant_for(this);`
   - `signals (Throwable t) \invariant_for(this);`
2. Thereafter add **`helper`**, where absent, to *all* methods

Normalisation: Example

```
/*@ public normal_behavior
  @ requires c.id >= 0;
  @ ensures \result == ( ... );
  @*/
  public boolean addCategory(Category c) {
```

becomes

```
/*@ public behavior
  @ requires c.id >= 0;
  @ ensures \result == ( ... );
  @ signals (Throwable exc) false;
  @*/
  public boolean addCategory(Category c) {
```

Normalisation: Example

```
/*@ public behavior
  @ requires c.id >= 0;
  @ ensures \result == ( ... );
  @ signals (Throwable exc) false;
  @*/
  public boolean addCategory(Category c) {
```

becomes

```
/*@ public behavior
  @ requires c.id >= 0;
  @ requires c != null;
  @ ensures \result == (...);
  @ signals (Throwable exc) false;
  @*/
  public boolean addCategory(/*@ nullable @*/ Category c) {
```

Normalisation: Example

```
/*@ public behavior
  @ requires c.id >= 0;
  @ requires c != null;
  @ ensures \result == (...);
  @ signals (Throwable exc) false;
  @*/
  public boolean addCategory(/*@ nullable @*/ Category c) {
```

becomes

```
/*@ public behavior
  @ requires c.id >= 0;
  @ requires c != null;
  @ requires \invariant_for(this);
  @ ensures \result == (...);
  @ ensures \invariant_for(this);
  @ signals (Throwable exc) false;
  @ signals (Throwable exc) \invariant_for(this);
  @*/
public /*@ helper @*/
 boolean addCategory(/*@ nullable @*/Category c) {
```

Normalisation

Next Normalisation Steps (Not detailed)

- Expanding `pure` modifier:
  - add to each specification case
    - **`assignable \nothing;`**
    - **`diverges false;`**
  - remove `pure`
- Where clauses with defaults (e.g., `diverges`, `assignable`) are absent, add explicit clauses

Normalisation: Clause Contraction

Merge multiple clauses of the same kind into a single one of that kind.

For instance,

```
/*@ public behavior
  @ requires R1;
  @ requires R2;
  @ ensures E1;
  @ ensures E2;
  @ signals (T1 exc) S1;
  @ signals (T2 exc) S2:
  @*/
```

```
/*@ public behavior
  @ requires R1 && R2;
  @ ensures E1 && E2;
  @ signals (Throwable exc)
  @  (exc instanceof T1 ==> S1)
  @  &&
  @  (exc instanceof T2 ==> S2);
  @*/
```

Translating JML into Intermediate Format

Intermediate format for contract of method m

$$(pre, post, div, var, mod)$$

with

- a precondition DL formula pre,
- a postcondition DL formula $post$,
- a divergence indicator $div \in \{TOTAL, PARTIAL\}$,
- a variant term var
- a modifies set mod, either of type `LocSet` or `\strictly_nothing`

Translating JML Expressions to DL-Terms: Arithmetic Expressions

Translation replaces arithmetic JAVA operators by generalized operators

Generic towards various integer semantics (JAVA, Math).

Example:

"+" becomes "`javaAddInt`" or "`javaAddLong`"
"-" becomes "`javaSubInt`" or "`javaSubLong`"
...

Translating JML Expressions to DL-Terms: The `this` Reference

The this reference, explicit or implicit, has only a meaning within a program (refers to currently executing instance).

On logic level (outside the modalities) no such context exists.

`this` reference translated to a program variable (named by convention) `self`

e.g., given class

```
public class MyClass {
  int f;
}
```

JML expressions `f` and **`this`**`.f`
translated to
DL term `select(heap, self, f)`, pretty-printed as `self.f`

Translating Boolean JML Expressions

First-order logic treated fundamentally different in JML and KeY logic

JML

- Formulas no separate syntactic category
- Instead: JAVA's `boolean` expressions extended with first-order concepts (i.p. quantifiers)

Dynamic Logic

- Formulas and expressions completely separate
- `true`, `false` are formulas,
  `boolean` constants `TRUE`, `FALSE` are terms
- Atomic formulas take terms as arguments; e.g.:
  - `x - y < 5`
  - `b = TRUE`

Translating Boolean JML Expressions

$$
\begin{array}{lcl}
\mathcal{F}(\texttt{v}) & = & \texttt{v = TRUE} \\
\mathcal{F}(\texttt{o.f}) & = & \mathcal{E}(\texttt{o.f})\texttt{ = TRUE} \\
\mathcal{F}(\texttt{m()}) & = & \mathcal{E}(\texttt{m})\texttt{() = TRUE} \\
\mathcal{F}(\texttt{!b\_0}) & = & \texttt{!}\mathcal{F}(\texttt{b\_0}) \\
\mathcal{F}(\texttt{b\_0 \&\& b\_1}) & = & \mathcal{F}(\texttt{b\_0})\texttt{ \& }\mathcal{F}(\texttt{b\_1}) \\
\mathcal{F}(\texttt{b\_0 || b\_1}) & = & \mathcal{F}(\texttt{b\_0})\texttt{ | }\mathcal{F}(\texttt{b\_1}) \\
\mathcal{F}(\texttt{b\_0 ==> b\_1}) & = & \mathcal{F}(\texttt{b\_0})\texttt{ -> }\mathcal{F}(\texttt{b\_1}) \\
\mathcal{F}(\texttt{b\_0 <==> b\_1}) & = & \mathcal{F}(\texttt{b\_0})\texttt{ <-> }\mathcal{F}(\texttt{b\_1}) \\
\mathcal{F}(\texttt{e\_0 == e\_1}) & = & \mathcal{E}(\texttt{e\_0})\texttt{ = }\mathcal{E}(\texttt{e\_1}) \\
\mathcal{F}(\texttt{e\_0 != e\_1}) & = & \texttt{!(}\mathcal{E}(\texttt{e\_0})\texttt{ = }\mathcal{E}(\texttt{e\_1})\texttt{)} \\
\mathcal{F}(\texttt{e\_0 >= e\_1}) & = & \mathcal{E}(\texttt{e\_0})\texttt{ >= }\mathcal{E}(\texttt{e\_1})
\end{array}
$$

`v`/`f`/`m()` **`boolean`** variables/fields/pure methods

b_0, b_1 **`boolean`** JML expressions, e_0, e_1 JML expressions

$\mathcal{E}$ translates JML expressions to DL terms

$\mathcal{F}$ Translates `boolean` JML Expressions to Formulas

Quantified formulas over reference types:

```
F((\forall T x; e_0; e_1)) =
 \forall T x; (
      (!x=null  &  x.<created> = TRUE  &  F(e_0))
   -> F(e_1))

F((\exists T x; e_0; e_1)) =
 \exists T x; (
      (!x=null  &  x.<created> = TRUE  &  F(e_0))
   &  F(e_1))
```

$\mathcal{F}$ Translates `boolean` JML Expressions to Formulas

Quantified formulas over primitive types, e.g., **int**

$\mathcal{F}$(`(\forall int x; e_0; e_1)`) =
  `\forall int x;` ((`inInt(x) & `$\mathcal{F}$(e_0)) `->` $\mathcal{F}$(e_1))

$\mathcal{F}$(`(\exists int x; e_0; e_1)`) =
  `\exists int x;` (`inInt(x) & `$\mathcal{F}$(e_0) `&` $\mathcal{F}$(e_1))

`inInt` (similar `inLong`, `inByte`):

Predefined predicate symbol with fixed interpretation

Meaning: Argument is within the range of the Java **int** datatype.

Translating Class Invariants

$$\mathcal{F}(\texttt{\textbackslash invariant\_for(e)}) \quad = \quad \texttt{Object}::\texttt{<inv>}(\texttt{heap}, \mathcal{E}(\texttt{e}))$$

- \invariant_for(e) translated to built-in predicate $\texttt{Object}::\texttt{<inv>}$, applied to heap and the translation of e
- $\texttt{Object}::\texttt{<inv>}$ is considered a specification-only field $\texttt{<inv>}$ of class Object (inherited by all sub-types of Object)
- Given that o is of type T, KeY can expand (during proof construction) '$\texttt{Object}::\texttt{<inv>}(\texttt{heap}, o)$' to the invariant of T
- $\texttt{Object}::\texttt{<inv>}(\texttt{heap}, o)$ pretty printed as $o.\texttt{<inv>}$
- Read 'invariant of o'

Translating JML into Intermediate Format

Intermediate format for contract of method m

$$(pre, post, div, var, mod)$$

with

- a precondition DL formula pre ✔,
- a postcondition DL formula $post$ ✔?almost,
- a divergence indicator $div \in \{TOTAL, PARTIAL\}$,
- a variant term var
- a modifies set mod, either of type `LocSet` or `\strictly_nothing`

Translation of Ensures Clauses

What is missing for `ensures` clauses?

- Translation of `\result`
- Translation of `\old(.)` expressions

Translating `\result`

For `\result` used in ensures clause of method $T\ m(\ldots)$:

$$\mathcal{E}(\texttt{\textbackslash result}) = \texttt{result}$$

where $\texttt{result} \in \mathit{PVar}$ of type T does not occur in the program.

Translating \old Expressions

$\backslash\texttt{old}(e)$ evaluates e in the prestate of the method

Accesses to `heap` must be evaluated w.r.t. to the 'old' heap

1. Introduce a global program variables `heapAtPre` of type Heap
   (Intention: `heapAtPre` refers to heap in method's pre-state)
2. Define:
   $\mathcal{E}(\backslash\texttt{old}(e)) = \mathcal{E}_{\texttt{heap}}^{\texttt{heapAtPre}}(e)$
   ($\mathcal{E}_x^y(e)$ replaces all occurrences of x in $\mathcal{E}(e)$ by y)

Example

$\mathcal{F}(\texttt{o.f == \textbackslash old(o.f)+ 1}) =$
$\mathcal{E}(\texttt{o.f}) \texttt{ = } \mathcal{E}(\texttt{\textbackslash old(o.f)+ 1}) =$
$\mathcal{E}(\texttt{o.f}) \texttt{ = } \mathcal{E}(\texttt{\textbackslash old(o.f)}) \texttt{ + } \mathcal{E}(1) =$
$\mathcal{E}(\texttt{o.f}) \texttt{ = } \mathcal{E}_{\texttt{heap}}^{\texttt{heapAtPre}}(\texttt{o.f}) \texttt{ + 1} =$
`select(heap, o, f) = select(heapAtPre, o, f) + 1` $=$
`o.f = o.f@heapAtPre + 1` *(by pretty printing)*

Translation of Ensures and Signals Clauses

Given the normalised JML contract

```
/*@ public behavior
  @ ...
  @ ensures E;
  @ signals (Throwable exc) S;
  @ ...
  @*/
```

Define

$\mathcal{F}_{\text{ensures}} = \mathcal{F}(\mathtt{E})$

$\mathcal{F}_{\text{signals}} = \mathcal{F}(\mathtt{S})$

Recall (p.16) that `S` is either `false`, or it has the form

```
(exc instanceof ExcType1 ==> ExcPost1) && ...;
```

In the following, assume `exc` is fresh program variable of type `Throwable`

Combining Ensures and Signals to *post*

The DL formula *post* is then defined as

$$(\texttt{exc} = \textbf{null} \rightarrow \mathcal{F}_{\textsf{ensures}}) \;\wedge\; (\texttt{exc} \neq \textbf{null} \rightarrow \mathcal{F}_{\textsf{signals}})$$

Important special case:

Normalisation of `normal_behavior` contract gives

`signals (Throwable exc) false;`

In that case, *post* is:

$$\begin{array}{ll} & (\texttt{exc} = \textbf{null} \rightarrow \mathcal{F}_{\textsf{ensures}}) \;\wedge\; (\texttt{exc} \neq \textbf{null} \rightarrow \mathcal{F}_{\textsf{signals}}) \\ \Leftrightarrow & (\texttt{exc} = \textbf{null} \rightarrow \mathcal{F}_{\textsf{ensures}}) \;\wedge\; (\texttt{exc} \neq \textbf{null} \rightarrow \mathcal{F}(\texttt{false})) \\ \Leftrightarrow & (\texttt{exc} = \textbf{null} \rightarrow \mathcal{F}_{\textsf{ensures}}) \;\wedge\; (\texttt{exc} \neq \textbf{null} \rightarrow \texttt{false}) \\ \Leftrightarrow & (\texttt{exc} = \textbf{null} \rightarrow \mathcal{F}_{\textsf{ensures}}) \;\wedge\; \texttt{exc} = \textbf{null} \\ \Leftrightarrow & \texttt{exc} = \textbf{null} \;\wedge\; \mathcal{F}_{\textsf{ensures}} \end{array}$$

Translating JML into Intermediate Format

Intermediate format for contract of method m

$$(pre, post, div, var, mod)$$

with

- a precondition DL formula *pre* ✔,
- a postcondition DL formula *post* ✔,
- a divergence indicator $div \in \{TOTAL, PARTIAL\}$,✔
- a variant term *var* (postponed to later lecture),
- a modifies set *mod*, either of type `LocSet` or `\strictly_nothing`

The Divergence Indicator

$$div = \begin{cases} TOTAL & \text{if normalised JML contract contains clause } \texttt{diverges}\ \mathbf{false}; \\ PARTIAL & \text{if normalised JML contract contains clause } \texttt{diverges}\ \mathbf{true}; \end{cases}$$

Translating Assignable Clauses: The DL Type `LocSet`

Assignable clauses are translated to

a term of type `LocSet` or the special value `\strictly_nothing`

Intention: A term of type `LocSet` represents a set of locations

Definition (Locations)

A location is a tuple (o, f) with $o \in D^{\texttt{Object}}$, $f \in D^{\texttt{Field}}$

The DL Type `LocSet`

Predefined type with $D(\texttt{LocSet}) = 2^{Location}$
and the functions (all with result type `LocSet`):

`empty`	empty set of locations: $\mathcal{I}(\texttt{empty}) = \emptyset$
`allLocs`	set of all locations, i.e., $\mathcal{I}(\texttt{allLocs}) = \{(d,f) \mid f.a.\ d \in D^{\texttt{Object}}, f \in D^{\texttt{Field}}\}$
`singleton(Object, Field)`	singleton set
`union(LocSet, LocSet)`	
`intersect(LocSet, LocSet)`	
`allFields(Object)`	set of all locations for the given object
`allObjects(Field)`	set of all locations for the given field; e.g., $\{(d,f) \mid \text{f.a. } d \in D^{\texttt{Object}}\}$
`arrayRange(Object, int, int)`	set representing all array locations in the specified range (both inclusive)

Translating Assignable Clauses—Example

Example

```
assignable \everything;
```

is translated into the DL term

$$\mathtt{allLocs}$$

Example

```
assignable this.next, this.content[5..9];
```

is translated into the DL term

$$\mathtt{union}(\mathtt{singleton}(\mathtt{self}, \mathtt{next}),\ \mathtt{arrayRange}(\mathtt{self.content}, 5, 9))$$

Translating JML into Intermediate Format

Intermediate format for contract of method *m*

$$(pre, post, div, var, mod)$$

with

- a precondition DL formula *pre* ✔,
- a postcondition DL formula *post* ✔,
- a divergence indicator $div \in \{TOTAL, PARTIAL\}$ ✔,
- a variant *var* a term of type `any` (postponed),
- a modifies set *mod*, either of type `LocSet` or `\strictly_nothing` ✔

From JML Contracts via Intermediate Format to Proof Obligations (PO)

```
public class A {
 /*@ public normal_behavior
   @ requires <Precondition>;
   @ ensures <Postcondition>;
   @ assignable <locations>;
   @*/
 public int m(params) {..}
}
```

Translation

Intermediate Format
$(pre, post, div, var, mod)$

PO Generation

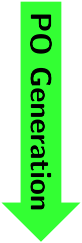
Proof obligation as DL formula

$$pre \rightarrow \langle \texttt{this.m(params);} \rangle (post \ \wedge \ frame)$$

Generating a PO from the Intermediate Format: Idea

Given intermediate format of contract of `m` implemented in class `C`:

$$(\mathit{pre}, \mathit{post}, \texttt{TOTAL}, \mathit{var}, \mathit{mod})$$



$$\mathit{pre} \rightarrow \langle \texttt{self.m(args)} \rangle (\mathit{post} \ \land \ \underbrace{\mathit{frame}}_{\text{correctness of assignable}} \ )$$

In case of $\mathit{div} = \texttt{PARTIAL}$, box modality is used

Generating a PO from Intermediate Format: Method Identification

$$pre \rightarrow \langle \texttt{self.m(args)} \rangle (post \ \wedge \ frame)$$

- Dynamic dispatch: `self.m(...)` causes split into all possible implementations
- Special statement Method Body Statement:

  `m(`*args*`)@C`

  Meaning: implementation of `m` in class `C`

Generating a PO from Intermediate Format: Exceptions

$$pre \rightarrow \langle \texttt{self.m(args)@C} \rangle (post \;\wedge\; frame)$$

Postcondition *post* states either

- that no exception is thrown or
- that in case of an exception the exceptional postcondition holds

but: $\langle \textbf{throw}\ \texttt{exc;} \rangle \varphi$ is trivially false

How to refer to an exception in post-state?

$$pre \rightarrow \left\langle \begin{array}{l} \texttt{exc = } \textbf{null}\texttt{;} \\ \textbf{try}\ \texttt{\{} \\ self.m(args)@C \\ \texttt{\}}\ \textbf{catch}\ \texttt{(Throwable e)\{exc = e;\}} \end{array} \right\rangle (post \;\wedge\; frame)$$

Recall: generation of *post* (p.28) uses program variable `exc`

The Generic Precondition *genPre*

$$pre \rightarrow \langle \texttt{exc=}\textbf{null}\texttt{;}\ \textbf{try}\ \texttt{\{self.m(args)@C\}}\ \textbf{catch}\ \ldots\ \rangle(post\ \wedge\ frame)$$

is still not complete.

Additional properties (known to hold in Java, but not in DL), e.g.,

- **this** is not **null**
- created objects can only point to created objects (no dangling references)
- integer parameters have correct range
- ...

Need to make these assumption on initial state explicit in DL.

Idea: Formalise assumption as additional precondition *genPre*

$$(genPre \wedge pre) \rightarrow \langle \texttt{exc=}\textbf{null}\texttt{;}\ \textbf{try}\ \texttt{\{self.m(args)@C\}}\ \textbf{catch}\ \ldots\ \rangle(post\ \wedge\ frame)$$

The Generic Precondition *genPre* (background info)

$$
\begin{aligned}
\color{blue}{\mathit{genPre}} := \quad & \mathtt{wellFormed(heap)} \\
\land\, & \mathtt{self} \neq \mathbf{null} \\
\land\, & \mathtt{self.<created>} = \mathtt{TRUE} \\
\land\, & \mathtt{C} :: \mathtt{exactInstance(self)} \\
\land\, & \mathit{paramsInRange}
\end{aligned}
$$

- `wellFormed(h)`: predefined predicate;
  true iff. `h` is regular Java heap
- `C :: exactInstance(o)`: predefined predicate;
  true iff. `o` has exact type `C` (not just subtype of `C`)
- *paramsInRange* formula stating that method arguments are in range

The Generic Precondition *genPre*

$$(genPre \wedge pre) \rightarrow \langle \texttt{exc=}\textbf{null}\texttt{;}\ \textbf{try}\ \texttt{\{self.m(args)@C\}}\ \textbf{catch}\ \ldots\ \rangle(post\ \wedge\ frame)$$

is still not complete.

- Need to refer to prestate in post, e.g. for old-expressions

$$(genPre \wedge pre) \rightarrow \{\texttt{heapAtPre} := \texttt{heap}\} \langle \texttt{exc=}\textbf{null}\texttt{;}\ \textbf{try}\ \texttt{\{self.m(args)@C\}}\ \textbf{catch}\ \ldots\ \rangle(post\ \wedge\ frame)$$

Recall: `heapAtPre` was used in translation of `\old`, p.26

Generating a PO from Intermediate Format: The *frame* DL Formula

$$(genPre \land pre) \rightarrow \{\mathtt{heapAtPre} := \mathtt{heap}\}$$
$$\langle \mathtt{exc=}\mathbf{null}\mathtt{;}\ \mathbf{try}\ \mathtt{\{self.m(args)\}}\ \mathbf{catch}\ \ldots\ \rangle$$
$$(post \ \land \ frame)$$

If mod = `\strictly_nothing` then *frame* is defined as:

$$\forall o; \forall f; (o.f = o.f@\mathtt{heapAtPre})$$

Generating a PO from Intermediate Format: The *frame* DL Formula

$$(genPre \land pre) \rightarrow \{\texttt{heapAtPre} := \texttt{heap}\}$$
$$\langle \texttt{exc=}\mathbf{null}\texttt{;}\ \mathbf{try}\ \texttt{\{self.m(args)\}}\ \mathbf{catch}\ \ldots\ \rangle$$
$$(post \ \land \ frame)$$

If *mod* is a location set, then *frame* is defined as:

$$\forall o; \forall f; \big(\quad (o, f) \in \{\texttt{heap} := \texttt{heapAtPre}\} mod$$
$$\lor\ o.\texttt{<created>@heaptAtPre} = \texttt{FALSE}$$
$$\lor\ o.f = o.f\texttt{@heapAtPre} \quad \big)$$

Says that every location (o, f) either

- belongs to the modifies set (evaluated in the pre-state), or
- was not (yet) created before the method invocation, or
- holds the same value before and after the method execution

Generating a PO from Intermediate Format: Result Value

$$(\mathit{genPre} \land \mathit{pre}) \rightarrow \{\texttt{heapAtPre} := \texttt{heap}\}$$
$$\langle \texttt{exc=}\textbf{null}\texttt{;}\ \textbf{try}\ \texttt{\{self.m(args)\}}\ \textbf{catch}\ \ldots\ \rangle$$
$$(\mathit{post}\ \land\ \mathit{frame})$$

is still not complete.

- For non-void methods, need to refer to result in *post*

$$(\mathit{genPre} \land \mathit{pre}) \rightarrow \{\texttt{heapAtPre} := \texttt{heap}\}$$
$$\langle \texttt{exc=}\textbf{null}\texttt{;}\ \textbf{try}\ \texttt{\{result = self.m(args)\}}\ \textbf{catch}\ \ldots\ \rangle$$
$$(\mathit{post}\ \land\ \mathit{frame})$$

Recall: `\result` was translated to program variable `result`, see p.25

Examples

Demo