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Data structures

Stacks and queues

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Stacks

- A *stack* stores a sequence of values
- Main operations:
  - `push(x)` – add value `x` to the stack
  - `pop()` – remove the *most-recently-pushed* value from the stack
- LIFO: *last in first out*
  - Value removed by `pop` is always the one that was pushed most recently

Stacks

- Analogy for LIFO: stack of plates
  - Can only add or remove plates at the top!
  - You always take o the most recent plate

Stacks

- More stack operations:
  - `empty()` – is there anything on the stack?
  - `top()` – return most-recently-pushed ("top") value without removing it

Stacks in Haskell

- Stacks in Haskell are just lists

```haskell
type Stack a = [a]

push :: a -> Stack a -> Stack a

pop :: Stack a -> Stack a

top :: Stack a -> a

empty :: Stack a -> Bool
```

Implementing stacks in Haskell

```haskell
type Stack a = [a]

push :: a -> Stack a -> Stack a
push x xs = x:xs

pop :: Stack a -> Stack a
pop (x:xs) = xs

top :: Stack a -> a
top (x:xs) = x

empty :: Stack a -> Bool
empty []       = True
empty (x:xs) = False
```

You don't need a separate stack type if you have Haskell-style lists

Example: balanced brackets

- Given a string:
  - “hello (hello is a greetng **[**sic**]** **{**“sic” is used when quoting a text that contains a typo **(**or archaic **[**and nowadays wrong**]** spelling**)** to show that the mistake was in the original text **(**and not introduced while copying the quote**)}**)”
- Check that all brackets match:
  - Every opening bracket has a closing bracket
  - Every closing bracket has an opening bracket
  - Nested brackets match up: no “**([)]**”!

Algorithm

- Maintain a *stack* of opened brackets
  - Initially stack is empty
  - Go through string one character at a time
  - If we see an opening bracket, push it
  - If we see a closing bracket, pop from the stack and check that it matches
    - e.g., if we see a “**)**”, check that the popped value is a “**(**“
  - When we get to the end of the string, check that the stack is empty

Algorithm

- Maintain a *stack* of opened brackets
  - Initially stack is empty
  - Go through string one character at a time
  - If we see an opening bracket, push it
  - If we see a closing bracket, **pop** from the stack and **check that it matches**
    - e.g., if we see a “**)**”, check that the popped value is a “**(**“
  - When we get to the end of the string, **check that the stack is empty**

Check your understanding: What has gone wrong if each of the steps in bold fails?

(stack can be empty)

Live coding

More uses of stacks

- The *call stack*, which is used by the processor to handle function calls
  - When you call a function, the processor records what it was doing by pushing a record onto the call stack
  - When a function returns, the processor pops a record off the call stack to see what it should carry on doing
- Parsing in compilers
- Lots of uses in algorithms!

Implementing stacks in Java

- Idea: use a dynamic array!
  - Push: add a new element to the end of the array
  - Pop: remove element from the end of the array
- Complexity?
- All operations have amortised $O(1)$ complexity
  - Means: n operations take $O(n)$ time
  - Although a single operation may take $O(n)$ time, an "expensive" operation is always balanced out by a lot of earlier "cheap" operations

Abstract data types

- You should distinguish between:
  - the *abstract data type (ADT)* (a stack) and
  - its *implementation* (e.g. a dynamic array)

- Why?
  - When you *use* a data structure you don't care how it's implemented
  - Most ADTs have many possible implementations

Queues

Queues

- A *queue* also stores a sequence of values
- Main operations:
  - `enqueue(x)` – add value x to the queue
  - `dequeue()` – remove *earliest-added* value
- FIFO: *first in first out*
  - Value dequeued is always the *oldest* one that's still in the queue
- Much like a stack – but FIFO, not LIFO

Queues

- Like a queue in real life!
  - The first to enter the queue is the first to leave

Uses of queues

- Controlling access to shared resources in an operating system, e.g. a printer queue
- A queue of requests in a web server
- Also appears in lots of algorithms
  - (Stacks and queues both appear when an algorithm has to remember a list of things to do)

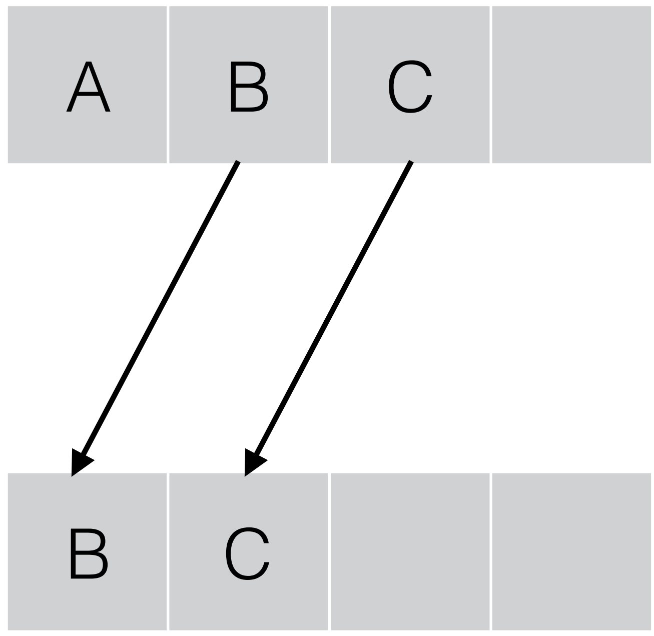
Implementing queues in Java

- What's wrong with this idea?
  - Implement the queue as a dynamic array
  - `enqueue(x)`: add `x` to the end of the dynamic array
  - `dequeue()`: remove and return first element of array

To dequeue, we'd have to copy the entire rest of the array down one place… takes $O(n)$ time

Dynamic arrays are no good

- A queue containing A, B, C:
- Dequeue removes A:
- Moving the rest of the queue into place takes $O(n)$ time!

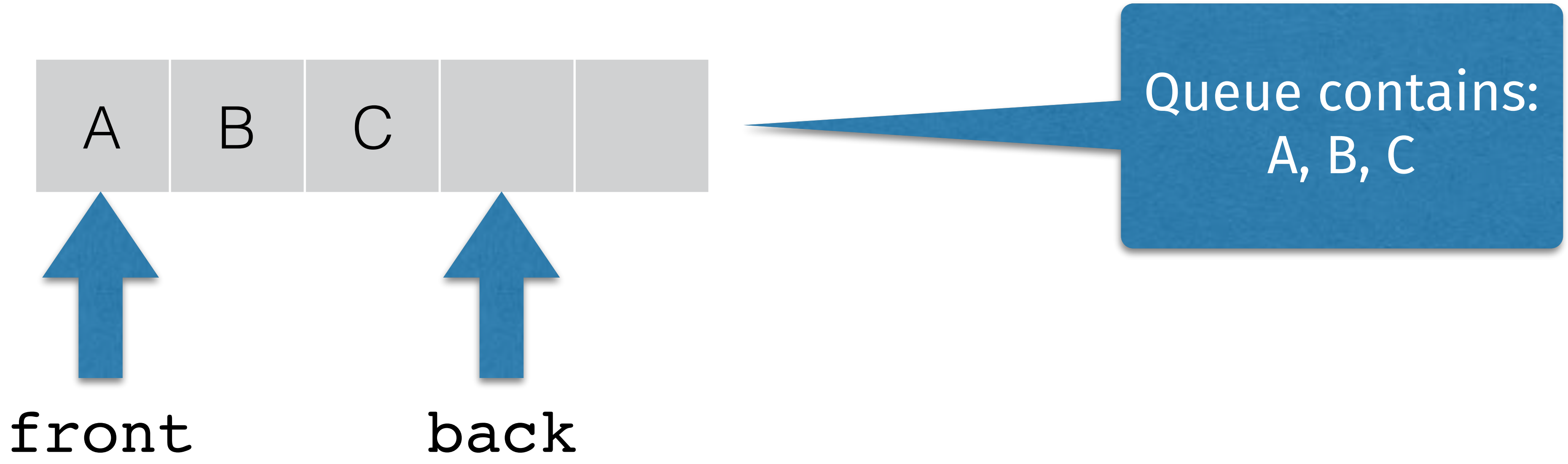


Bounded queues

- Let's solve a simpler problem first: *bounded queues*
- A bounded queue is a queue with a fixed capacity, e.g. 5
  - The queue can't contain more than 5 elements at a time
  - You typically choose the capacity when you create the queue

Bounded queues

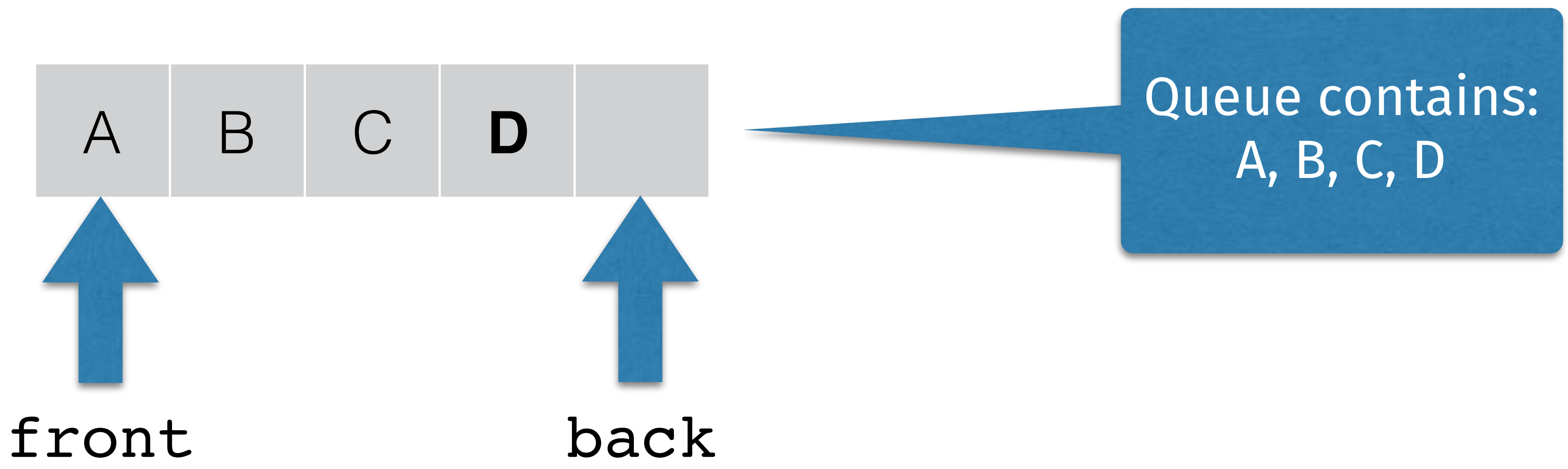
- An array, plus two indices `back` and `front`



- `back`: where we enqueue the next element
- `front`: where we dequeue the next element

Bounded queues

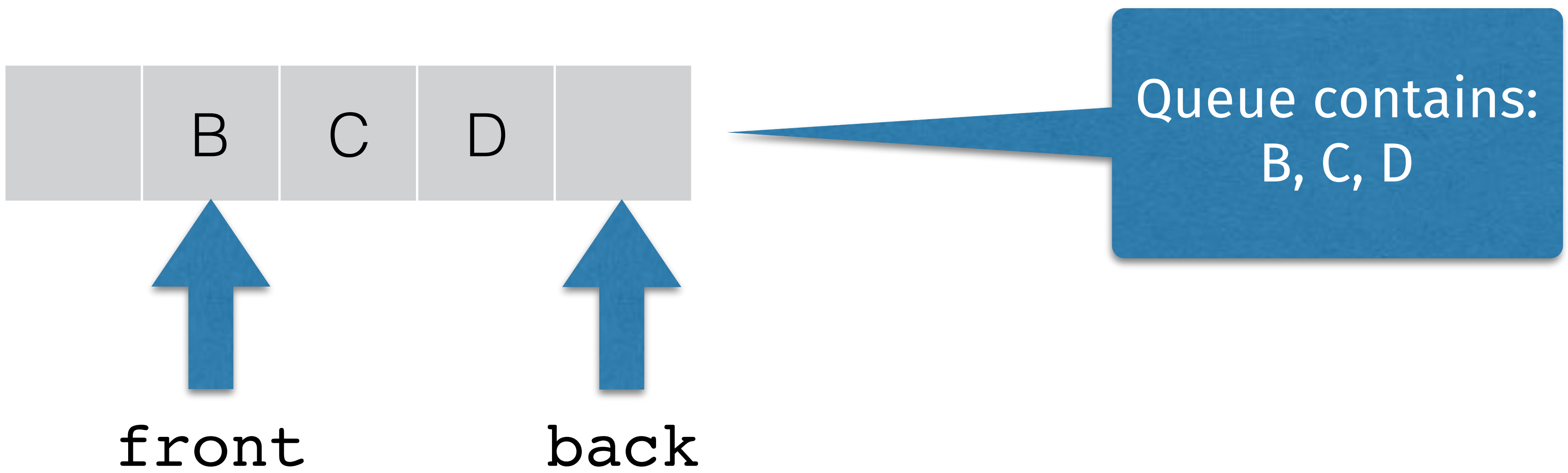
- After enqueueing D



- `array[back] = D; back = back + 1;`

Bounded queues

- After dequeueing (to get A)



- `result = array[front]; front = front + 1;`

Thinking formally about queues

What is the contents of one of our array-queues?

- Everything from index `front` to index `back-1`

If we specify the *meaning* of the array like this, there is only one sensible way to implement `enqueue` and `dequeue`!

- Before dequeue, contents is:
  `array[front], array[front+1], ..., array[back-1]`
- After dequeue: `array[front]` should be gone, contents is
  `array[front+1], ..., array[back-1]`
- Only good way to do this is `front = front + 1`!

Thinking formally about queues

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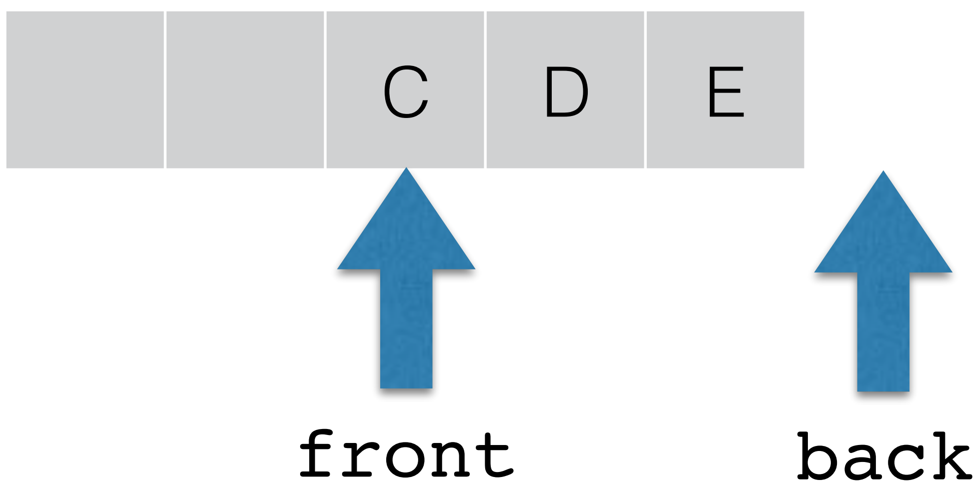
- After dequeue: array[front] should be gone, contents is
  `array[front+1], ..., array[back-1]`

- Only good way to do this is `front = front + 1`!

Data structure design hint:
don't just think what everything should do! Work out the *meaning* of the data structure too.

Bounded queues

- After enqueueing E and dequeueing



- What is the problem here?

Queues as circular buffers

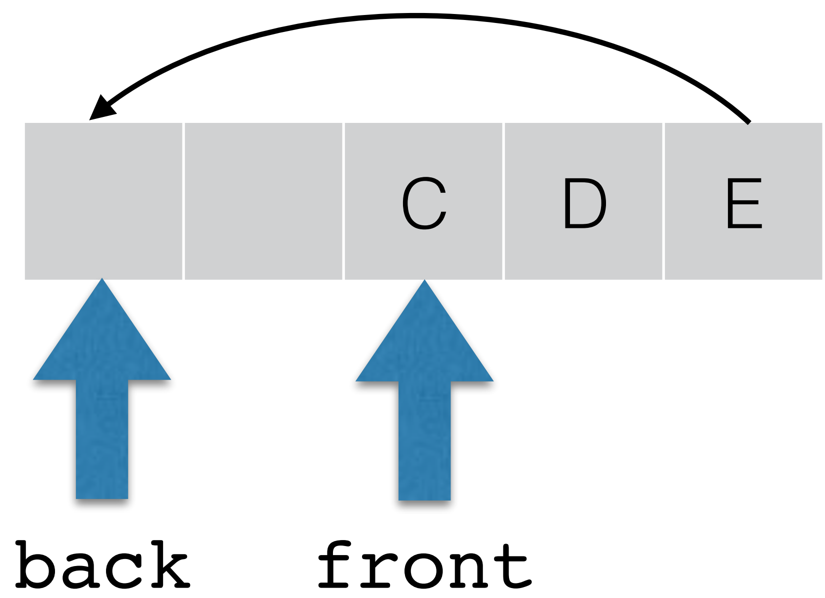
- Problem: when `back` reaches the end of the array, we can't enqueue anything else

- Idea: *circular buffer*
  - When `back` reaches the end of the array, put the next element at index 0 – and set `back` to 0
  - Next after that goes at index 1
  - `front` wraps around in the same way

- Use all the freed space that's left in the beginning of the array after we dequeue!

Bounded queues

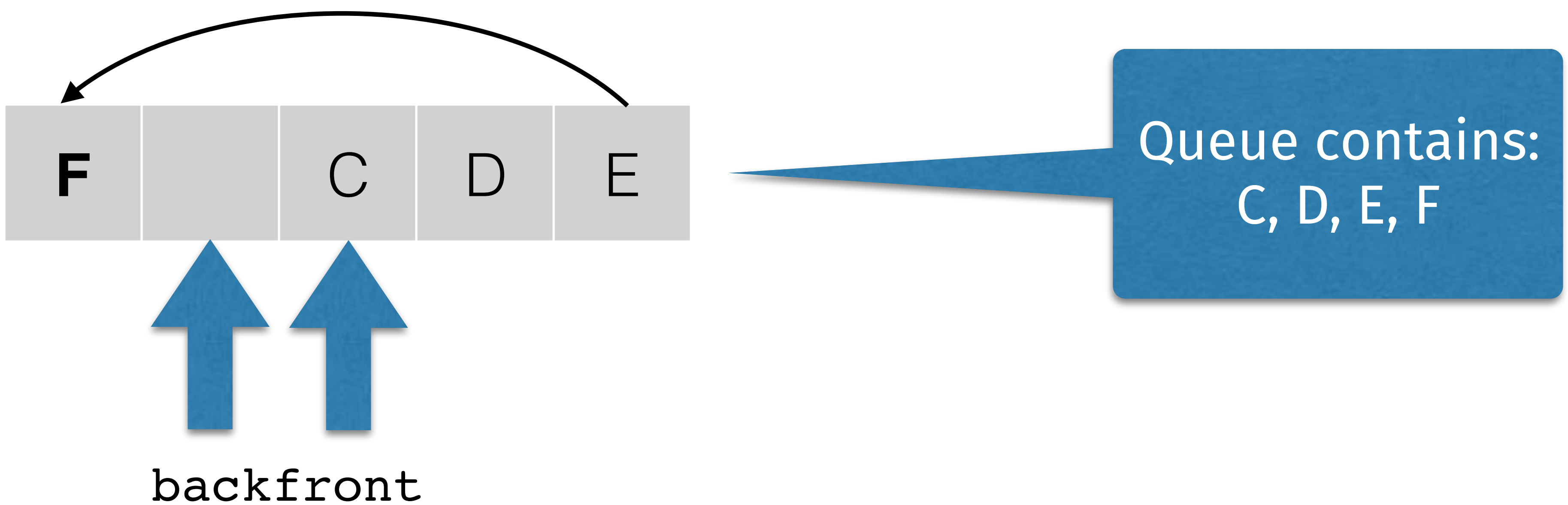
- Try again – after enqueueing E



- `back` wraps around to index 0

Bounded queues

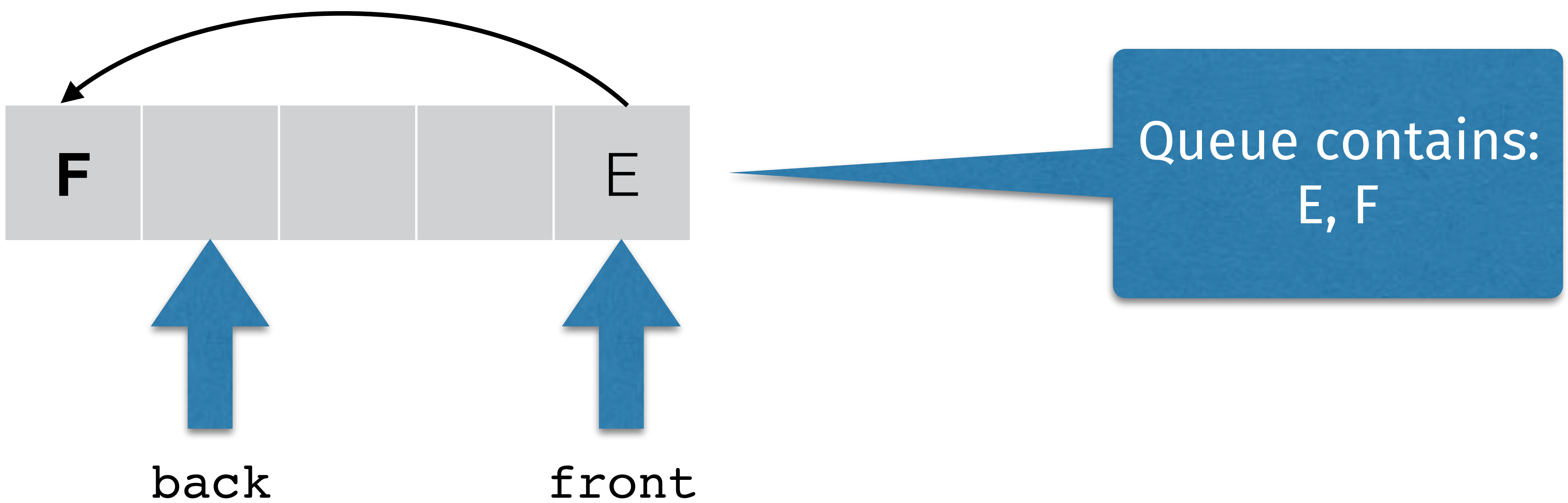
- Now after enqueueing F



- Meaning: queue contains everything from `front` to `back-1` still.
- But wrapping around if `back < front`!
- Exercise: phrase this precisely

Bounded queues

- After dequeueing twice



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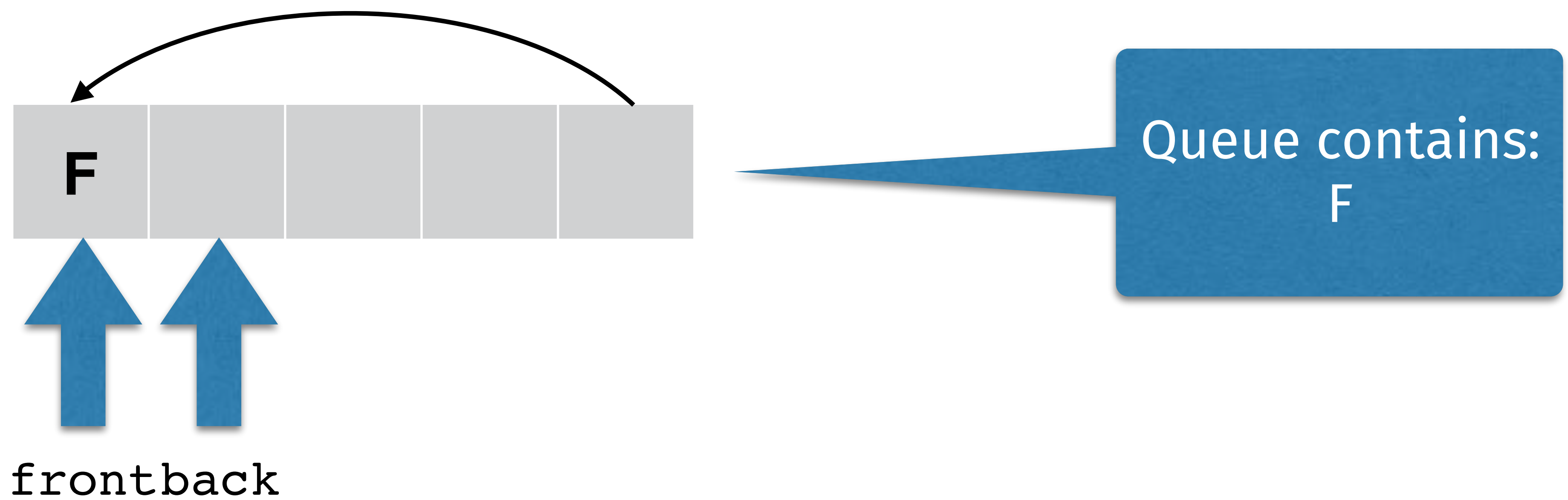
Bounded queues

- After dequeueing again



- `front` wraps too!

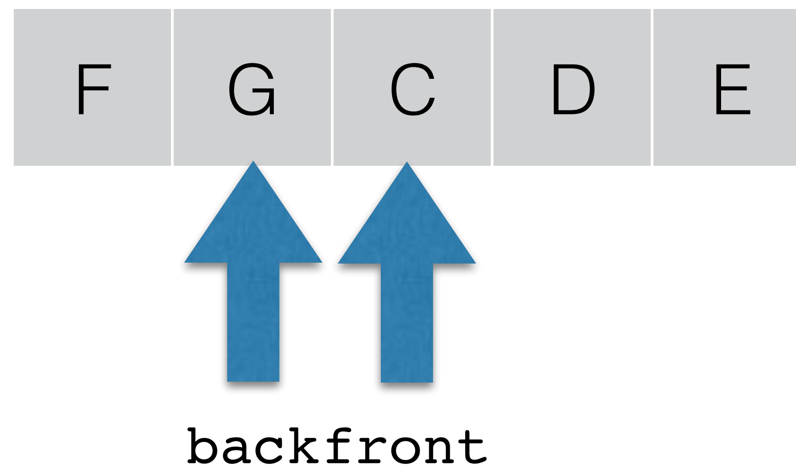
Circular buffers

- Basic idea: an array, plus two indices for the front and back of the queue
  - These indices *wrap around* when reaching the end of the array, which is what makes it work
- The best bounded queue implementation!

Bounded queues

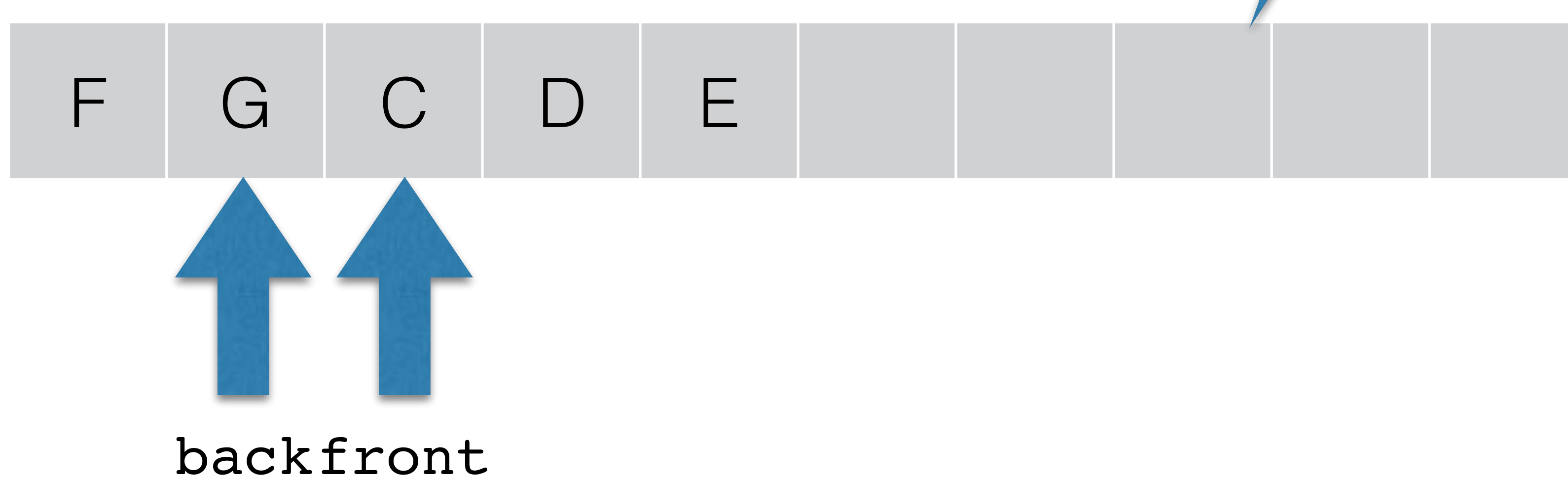
- Circular buffers make a fine *bounded queue*
- To make an unbounded queue, let's be inspired by dynamic arrays
  - Dynamic arrays: fixed-size array, double the size when it gets full
  - Unbounded queues: bounded queue, double the capacity when it gets full
- Whenever the queue gets full, allocate a new queue of double the capacity, and copy the old queue to the new queue

Reallocation, how *not* to do it

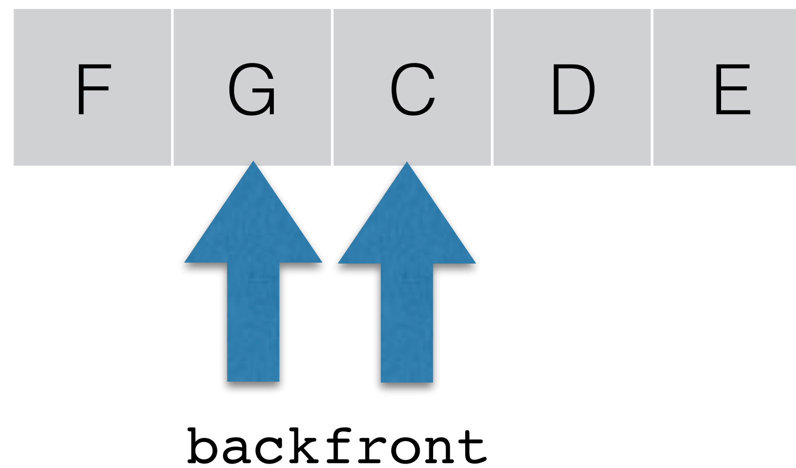


Queue contains:
C, D, E,
five mystery elements,
F, G

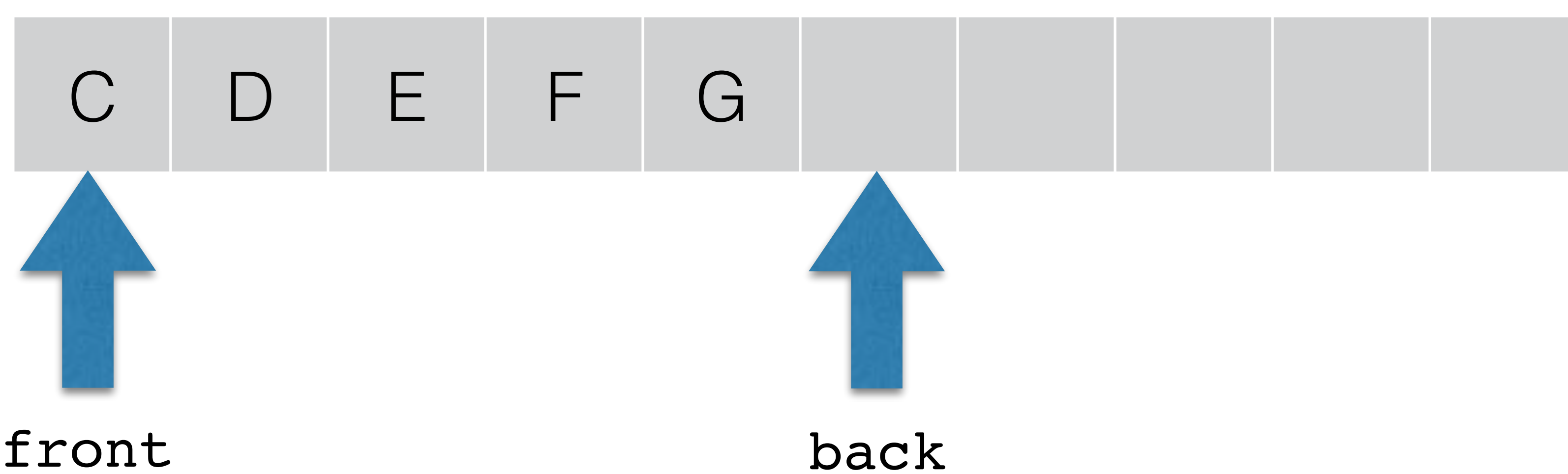
- What is wrong with resizing like this?



Reallocation



- Instead, repeatedly dequeue from the old queue and enqueue into the new queue:



Summary: queues as arrays

- Maintain `front` and `back` indexes
  - Enqueue elements at `back`, remove from front
- Circular array
  - `front` and `back` wrap around when they reach the end
- Idea from dynamic arrays
  - When the queue gets full, allocate a new one of twice the size
  - Don't just resize the array – safer to use the queue operations to copy from the old queue to the new queue
- Important implementation note!
  - To tell when array is full, need an extra variable to hold the current `size` of the queue (exercise: why?)

Queues in Haskell

```haskell
type Queue a = ???

enqueue :: a -> Queue a -> Queue a
dequeue :: Queue a -> (a, Queue a)
empty   :: Queue a -> Bool
```

better API: `dequeue :: Queue a -> Maybe (a, Queue a)`

One possibility: using a list

```
type Queue a = [a]

enqueue :: a -> Queue a -> Queue a
enqueue x xs = xs ++ [x]

dequeue :: Queue a -> (a, Queue a)
dequeue (x:xs) = (x, xs)

empty :: Queue a -> Bool
empty [] = True
empty _  = False
```

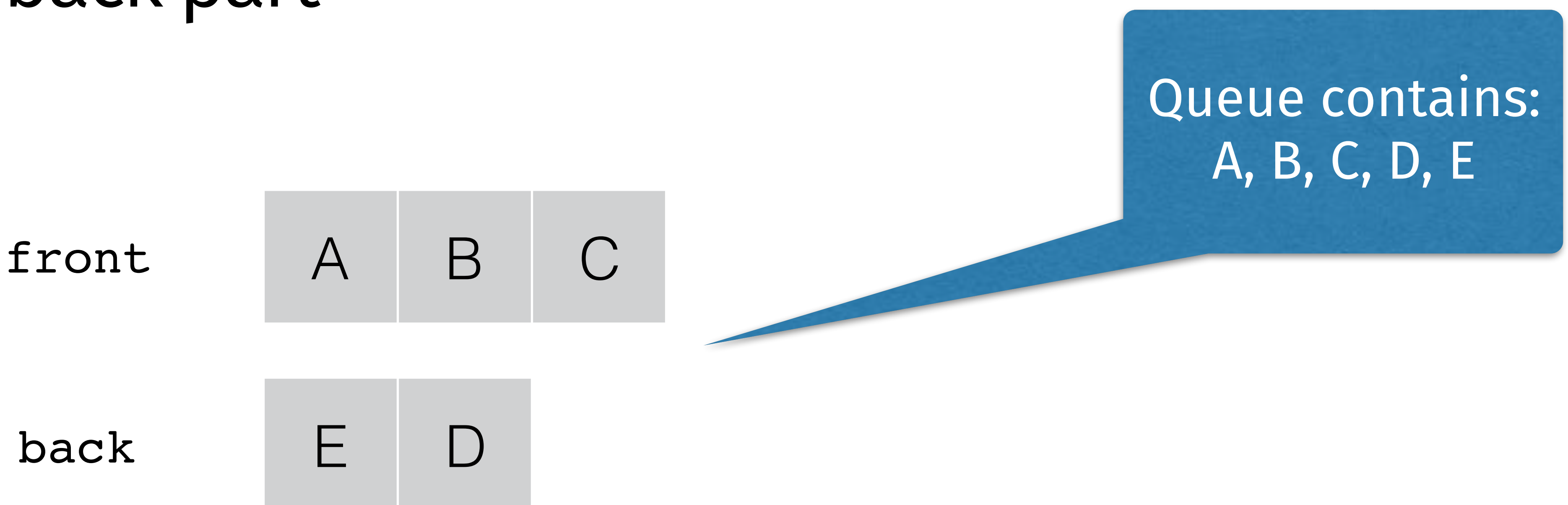
But enqueue takes $O(n)$ time!

An analogy

- Back to the “stack of plates” analogy!
- I am washing plates, you are putting them away
  - You want to put the dishes away in the same order I wash them – FIFO, a queue of plates
- Idea: we both have a stack of plates
  - Me: plates I've washed
  - You: plates you're going to put away
- If you run out of plates, you take my stack of washed plates. But – the oldest plates are at the bottom! So first *turn the stack upside down*!

A cunning plan

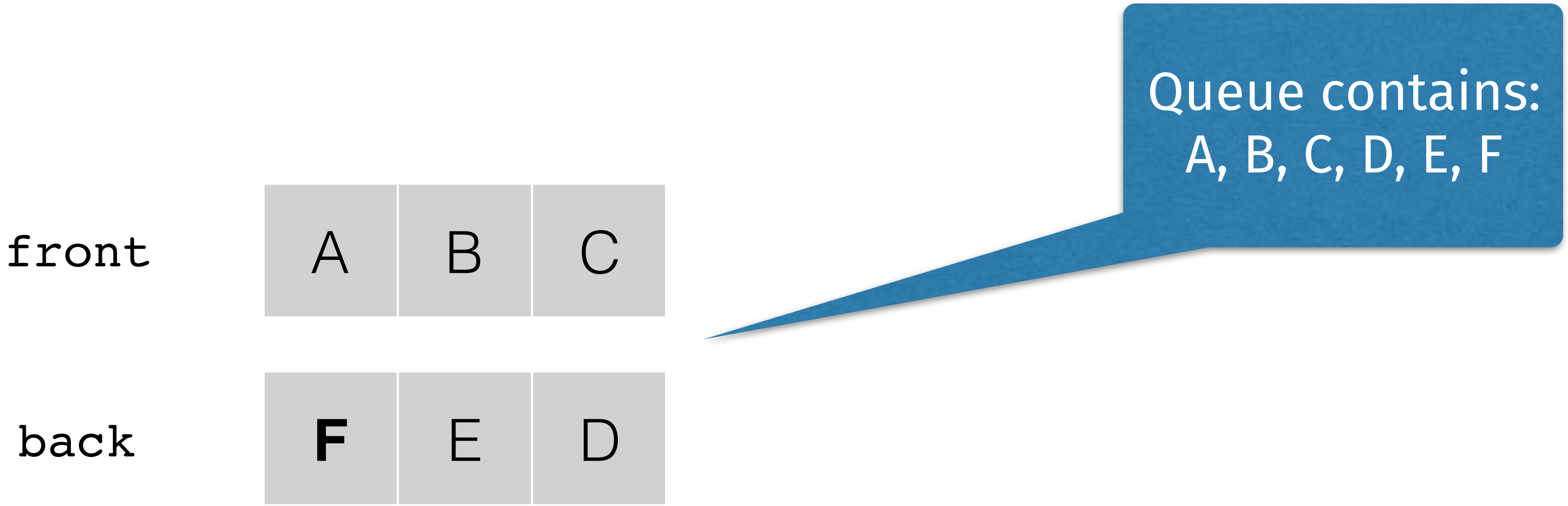
- Implement a queue using *two lists*, the “front part” and the “back part”



- Enqueue into the back part, dequeue from the front part – and *move* items from the back to the front when needed

A cunning plan

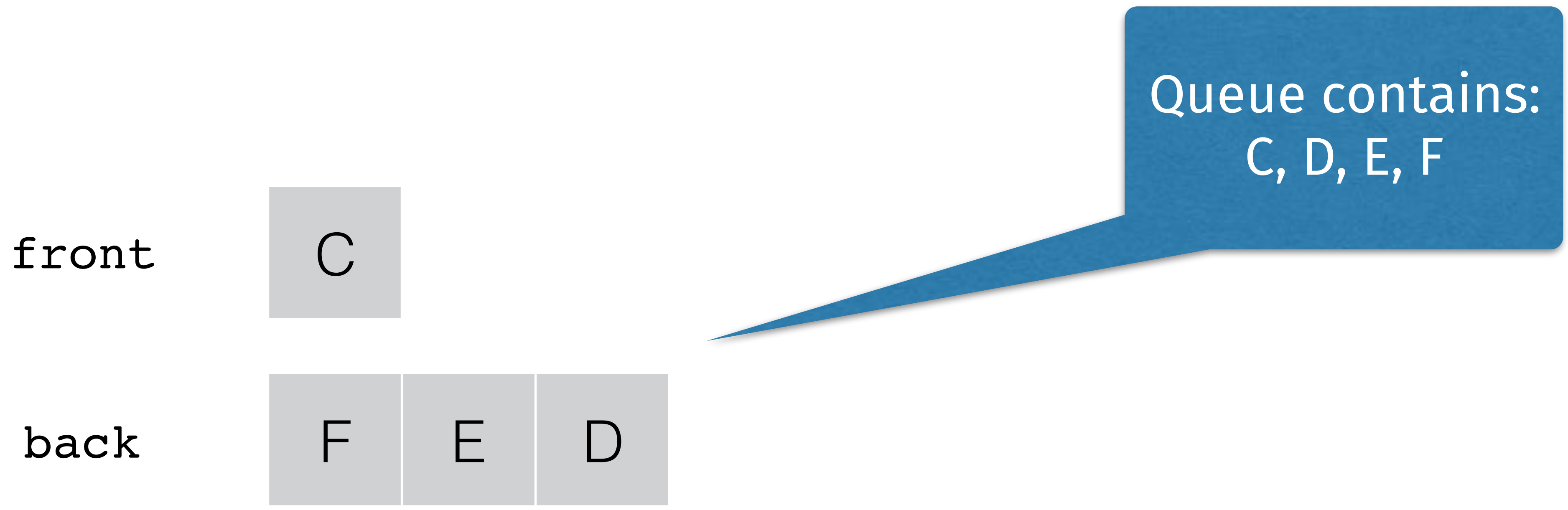
- Enqueuing F:



- Only need to use ’cons’ (:) — constant time

A cunning plan

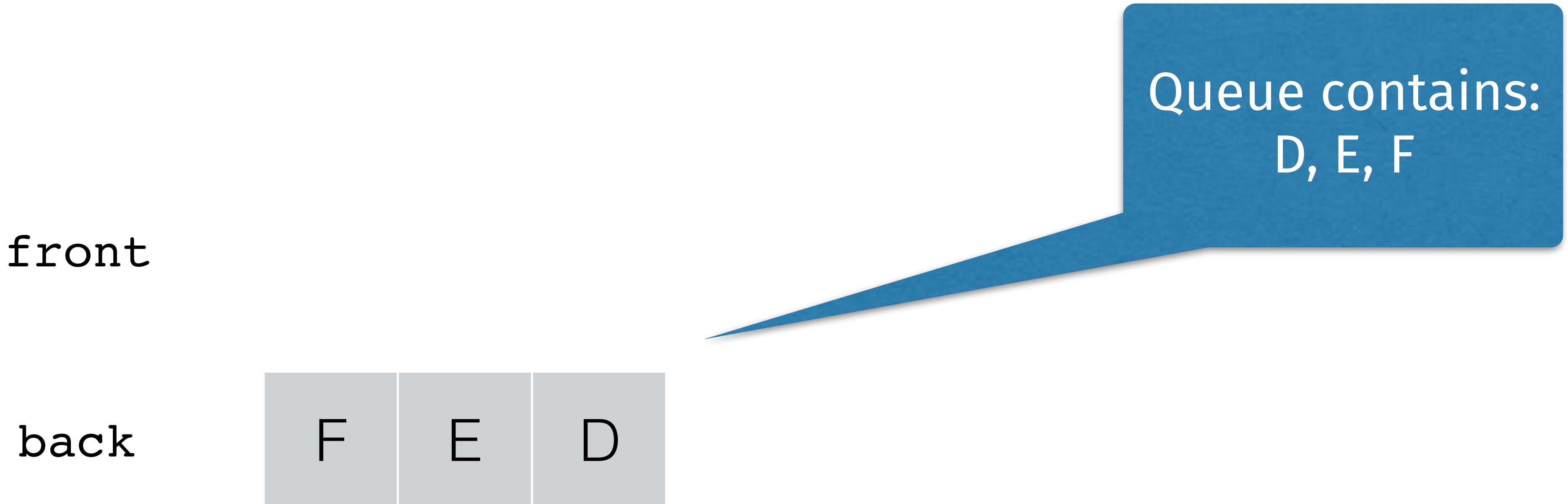
- Dequeuing A, B:



- Only need to look at front list — constant time

A cunning plan

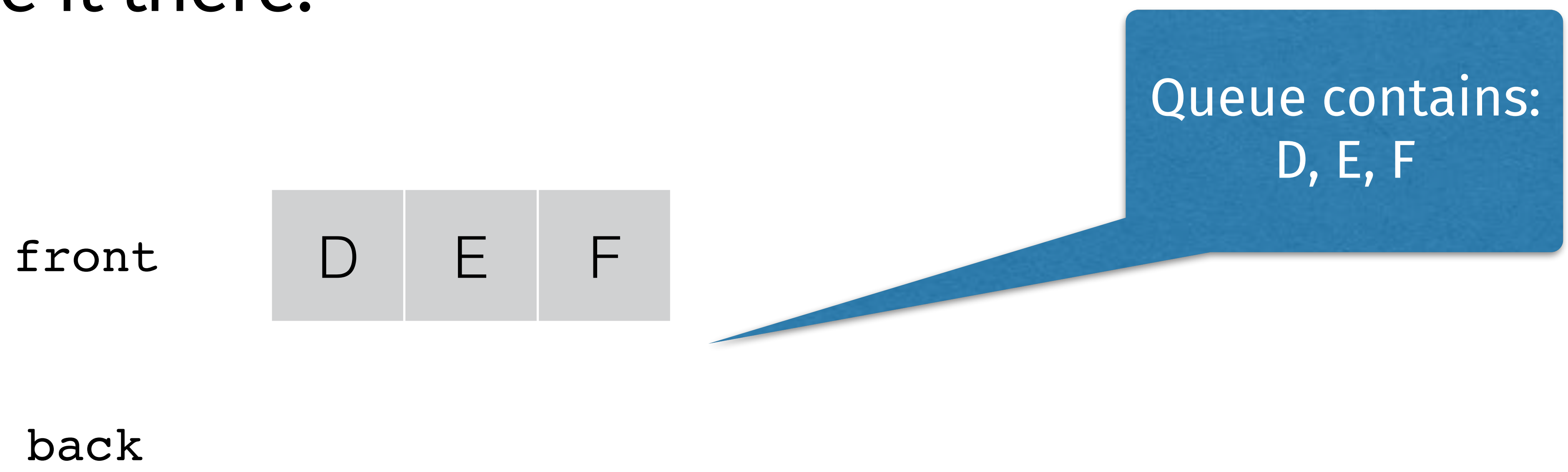
- Dequeuing C:



- *What if we want to dequeue again?*

A cunning plan

- When the front part is empty, *reverse* the back part and move it there!



- *Now we can dequeue again!*

Queues in Haskell

- A queue is a *pair* of lists
  - `data Queue a = Queue {front :: [a], back :: [a]}`
  - To enqueue an element, add it to `back`
  - To dequeue, remove an element from `front`
  - If `front` is empty, replace it with reverse `back`
- The queue `Queue front back` represents the sequence `front ++ reverse back`
  - For example, `Queue [1,2,3] [6,5,4]` represents the queue `1 2 3 4 5 6`
  - By writing this down, we see why we need to reverse when moving `back` to `front`!

Live coding

Is this efficient?

- Isn't this slow? `reverse` takes $O(n)$ time
- No: we get amortised $O(1)$ complexity
- If we enqueue and dequeue n items...
  - We spend some time reversing stuff
  - But only the stuff we enqueue gets reversed, and each item is only added to back once, so the lists we reverse contains n items in total
  - So the reversing takes $O(n)$ time for n items
  - so $O(1)$ time average per item enqueued

Double-ended queues

- So far we have seen:
  - Queues – add elements to one end and remove them from the other end
  - Stacks – add and remove elements from the same end
- In a deque, you can add and remove elements from both ends
  - *add to front, add to rear*
  - *remove from front, remove from rear*
- Good news – circular arrays support this easily
  - For the functional version, have to be a bit careful to get the right complexity – see exercise

In practice

- Your favourite programming language should have a library module for stacks, queues and deques
  - Java: use java.util.Deque<E> – provides addFirst/Last, removeFirst/Last methods
  - The Deque<E> interface is implemented by ArrayDeque (circular, dynamic array) and LinkedList, among others
  - Note: Java also provides a Stack class, b*ut this is deprecated – don't use it*
  - Haskell: instead of a stack, just use a list
  - For queues and deques, use Data.Sequence – a general-purpose sequence data type

Stacks, queues, deques – summary

- All three extremely common
  - Stacks: LIFO, queues: FIFO, deques: generalise both
  - Often used to maintain a set of tasks to do later
  - Imperative language: stacks are dynamic array, queues are circular buffers, $O(1)$ amortised complexity
  - Functional language: stacks are lists, deques can be implemented as a pair of lists with $O(1)$ amortised complexity
- Data structure design hint: always think about what the representation of a data structure *means*!
  - e.g. "what queue does this circular buffer represent?"
  - This is the main design decision you have to make – it drives everything else
  - This lets you design new data structures systematically
  - And also understand existing ones