Verification of concurrent programs

Lecture 14 of TDA384/DIT391
(Principles of Concurrent Programming)

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Finite-state models of concurrency: recap

Specification

Verification
  Testing
  Model checking
Finite-state models of concurrency: recap
State/transition diagrams

We capture the essential elements of concurrent programs using state/transition diagrams (also called: (finite) state automata, (finite) state machines, or transition systems).

- **states** in a diagram capture possible program states
- **transitions** connect states according to execution order

**Structural properties** of a diagram capture semantic properties of the corresponding program.
A state captures the shared and local states of a concurrent program:

```plaintext
int counter = 0;
```

**Thread t**

```plaintext
int cnt;
1 cnt = counter;
2 counter = cnt + 1;
3 // terminates
```

**Thread u**

```plaintext
int cnt;
cnt = counter;
counter = cnt + 1;
// terminates
```

**Shared state**

<table>
<thead>
<tr>
<th></th>
<th>cnt: 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>counter: 0</td>
<td></td>
</tr>
</tbody>
</table>

**Local state of thread t**, including pc:

<table>
<thead>
<tr>
<th></th>
<th>cnt: 0</th>
</tr>
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<tbody>
<tr>
<td>counter: 0</td>
<td></td>
</tr>
</tbody>
</table>

**Local state of thread u**, including pc:
A **state** captures the shared and local states of a concurrent program:

<table>
<thead>
<tr>
<th>counter: 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>▷2</td>
</tr>
<tr>
<td>cnt: 0</td>
</tr>
</tbody>
</table>

When unambiguous, we simplify a state with only the **essential information**:
The initial state of a computation is marked with an incoming arrow:

```
int counter = 0;
```

```
<table>
<thead>
<tr>
<th>thread t</th>
<th>thread u</th>
</tr>
</thead>
<tbody>
<tr>
<td>int cnt;</td>
<td>int cnt;</td>
</tr>
<tr>
<td>1 cnt = counter;</td>
<td>cnt = counter;</td>
</tr>
<tr>
<td>2 counter = cnt + 1;</td>
<td>counter = cnt + 1;</td>
</tr>
<tr>
<td>3 // terminates</td>
<td>// terminates</td>
</tr>
</tbody>
</table>
```
The final states of a computation – where the program terminates – are marked with double-line edges:

\[
\begin{array}{c|c}
\text{counter: 2} & \\
\hline
\text{3} & \text{6} \\
\text{done} & \text{done} \\
\end{array}
\]

\[
\text{int \ counter} = 0;
\]

\[
\begin{align*}
\text{thread t} & \quad \text{thread u} \\
\text{int \ cnt;} & \quad \text{int \ cnt;} \\
1 \ \ \text{cnt} & = \text{counter;} \quad 4 \ \ \text{cnt} & = \text{counter;} \\
2 \ \ \text{counter} & = \text{cnt} + 1; \quad 5 \ \ \text{counter} & = \text{cnt} + 1; \\
3 \ // \ terminal & \text{ates} \quad 6 \ // \ terminal & \text{ates}
\end{align*}
\]
A transition corresponds to the execution of one atomic instruction, and it is an arrow connecting two states (or a state to itself):

```
int counter = 0;
thread t
  int cnt;
  1  cnt = counter;
  2  counter = cnt + 1;
  3  // terminates
thread u
  int cnt;
  4  cnt = counter;
  5  counter = cnt + 1;
  6  // terminates
```
A complete state/transition diagram

The complete state/transition diagram for the concurrent counter example explicitly shows all possible interleavings:
Counter with locks: state/transition diagram

The state/transition diagrams of the concurrent counter example using locks shows the absence of race conditions:
The **structural properties** of a diagram capture semantic properties of the corresponding program:

**mutual exclusion:** there are no states where two threads are in their critical region

**deadlock freedom:** for every (non-final) state, there is an outgoing transition

**starvation freedom:** there is no (looping) path such that a thread never enters its critical region while trying to do so

**no race conditions:** all the final states have the same result

Building and analyzing state/transition diagrams quickly becomes tedious. That's where **formal verification** techniques such as model checking can help.
Transition tables are equivalent representations of the information of state/transition diagrams.

<table>
<thead>
<tr>
<th>CURRENT</th>
<th>NEXT WITH $t$</th>
<th>NEXT WITH $u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle 0, ▶1, \bot, ▶4, \bot \rangle$</td>
<td>$\langle 0, ▶2, 0, ▶4, \bot \rangle$</td>
<td>$\langle 0, ▶1, \bot, ▶5, 0 \rangle$</td>
</tr>
<tr>
<td>$\langle 0, ▶2, 0, ▶4, \bot \rangle$</td>
<td>$\langle 1, ▶3, --, ▶4, \bot \rangle$</td>
<td>—</td>
</tr>
<tr>
<td>$\langle 0, ▶1, \bot, ▶5, 0 \rangle$</td>
<td>—</td>
<td>$\langle 1, ▶1, \bot, ▶6, -- \rangle$</td>
</tr>
<tr>
<td>$\langle 1, ▶3, --, ▶4, \bot \rangle$</td>
<td>—</td>
<td>$\langle 1, ▶3, --, ▶5, 1 \rangle$</td>
</tr>
<tr>
<td>$\langle 1, ▶1, \bot, ▶6, -- \rangle$</td>
<td>$\langle 1, ▶2, 1, ▶6, -- \rangle$</td>
<td>—</td>
</tr>
<tr>
<td>$\langle 1, ▶3, --, ▶5, 1 \rangle$</td>
<td>—</td>
<td>$\langle 2, ▶3, --, ▶6, -- \rangle$</td>
</tr>
<tr>
<td>$\langle 1, ▶2, 1, ▶6, -- \rangle$</td>
<td>$\langle 2, ▶3, --, ▶6, -- \rangle$</td>
<td>—</td>
</tr>
<tr>
<td>$\langle 2, ▶3, --, ▶6, -- \rangle$</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>
Specification
Writing correct programs

Programming means writing instructions that achieve a certain functionality. How do we know if a program is correct? And what does it even mean that a program is correct?

To this end, we distinguish between implementation and specification:

- The implementation is the code that is written, compiled, and executed
- The specification is a description of what the program should do, usually at a more abstract level than the implementation

**Implementation:**
```c
void withdraw(int amount) {
    balance -= amount;
}
```

**Specification:**
```
method withdraw takes a positive integer amount not exceeding balance, and decreases balance by amount
```
Functional specifications

In sequential programming, we are mainly interested in functional – or input/output – specifications of individual methods. Such specifications consist of two parts:

1. **precondition**: a constraint that defines the method’s valid inputs
2. **postcondition**: a functional description of the expected output after executing the method

In object-oriented programs, the input and output of a method also include the object state before and after executing the method.

Implementation:
```c
void withdraw(int amount) {
    balance -= amount;
}
```

Specification:
1. **precondition**: 
   
   \[ 0 < amount \land amount \leq balance \]

2. **postcondition**: 
   
   \[ \text{“after” } balance = \text{“before” } balance - amount \]
Java does not have support for writing pre/postcondition specifications in the source file.

**JML** (Java Modeling Language) is a system for annotating Java programs in special comments.

class BankAccount {
    int balance;

    //@ requires 0 < amount && amount <= balance;
    //@ ensures balance == old(balance) - amount;
    void withdraw(int amount) {
        balance -= amount;
    }
}
In addition to pre- and postconditions of individual methods, functional specifications include class invariants, which specify properties of the state of objects of that class that should always hold between method calls.

```java
class BankAccount {
    int balance;
    invariant { balance >= 0 } // balance never negative
        // (holds if withdraw is called with amount <= balance)
    void withdraw(int amount) {
        balance -= amount;
    }
    void deposit(int amount) {
        balance += amount;
    }
}
```
The specification of concurrent programs should cover two parts:

- a **functional** specification defines the correct **input/output** behavior
- a **temporal** specification defines the **absence** of undesired behavior, such as no race conditions, deadlock, and starvation

Functional specification techniques such as pre- and postconditions, and class invariants are also applicable to concurrent programs. Class invariants are particularly useful for **shared-memory** concurrency, where invariants characterize the **valid states** of shared objects.

**Temporal specifications** require **new notations** and techniques.
Temporal logic was invented by philosophers and later brought to computer science by Pnueli in the 1970s.

Temporal logic is a notation to specify behavior over time. More precisely, it formally defines properties of traces of states, like those that originate from the execution of a (concurrent) program.

Out of the many variants of temporal logic that have been developed, we present the widely used LTL (Linear Temporal Logic).
LTL includes all the usual **Boolean operators** of propositional logic:

<table>
<thead>
<tr>
<th>FORMULA</th>
<th>MEANING</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>$p$ is true</td>
</tr>
<tr>
<td>$\neg p$</td>
<td>$p$ is <strong>not</strong> true (i.e., false)</td>
</tr>
<tr>
<td>$p \land q$</td>
<td>$p$ and $q$ are true</td>
</tr>
<tr>
<td>$p \lor q$</td>
<td>$p$ or $q$ is true (or both)</td>
</tr>
<tr>
<td>$p \Rightarrow q$</td>
<td>$p$ true <strong>implies</strong> that $q$ true (if $p$ then $q$ too)</td>
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LTL operators

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In addition, it has a few **temporal operators**:

<table>
<thead>
<tr>
<th>FORMULA</th>
<th>MEANING</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Diamond p$</td>
<td>$p$ is <em>eventually</em> true (from now on)</td>
</tr>
<tr>
<td>$\Box p$</td>
<td>$p$ is <em>always</em> true (from now on)</td>
</tr>
<tr>
<td>$p \mathcal{U} q$</td>
<td>$p$ is true (from now on) <em>until</em> $q$ is true</td>
</tr>
<tr>
<td>$Xp$</td>
<td>$p$ is true in the <em>next</em> step</td>
</tr>
</tbody>
</table>
When we use LTL to specify properties of concurrent programs, propositions (like \( p \) and \( q \)) represent properties of a program’s global state – including shared memory, and threads’ local memory and program counters. For example:

<table>
<thead>
<tr>
<th>PROPOSITION</th>
<th>STATE PROPERTY</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_t )</td>
<td>thread ( t ) is in its critical section</td>
</tr>
<tr>
<td>( c_u )</td>
<td>thread ( u ) is in its critical section</td>
</tr>
<tr>
<td>( e_t )</td>
<td>thread ( t ) is trying to enter its critical section</td>
</tr>
<tr>
<td>( n_t )</td>
<td>thread ( t ) has terminated</td>
</tr>
</tbody>
</table>

With this convention, we can rigorously specify temporal properties.
LTL specifications: example

In our running example of concurrent increment of counter:

- each thread’s critical section is the whole code it executes
- the global state includes: the value of counter, the values of the local cnt, and the program counter of each thread

```c
int counter = 0;

thread t
  int cnt;
  1 cnt = counter;
  2 counter = cnt + 1;
  3 // terminates

thread u
  int cnt;
  4 cnt = counter;
  5 counter = cnt + 1;
  6 // terminates
```
LTL specifications: example

In our running example of concurrent increment of counter:

- each thread’s **critical section** is the whole code it executes
- the **global state** includes: the value of `counter`, the values of the local `cnt`, and the program counter of each thread

```c
int counter = 0;
```

<table>
<thead>
<tr>
<th>PROPOSITION</th>
<th>MEANING</th>
<th>FORMULA</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t \triangleright k$</td>
<td>thread $t$ is at line $k$</td>
<td>$e_t$</td>
<td>$t \triangleright 1$</td>
</tr>
<tr>
<td>$u \triangleright k$</td>
<td>thread $u$ is at line $k$</td>
<td>$c_t$</td>
<td>$t \triangleright 2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$n_t$</td>
<td>$t \triangleright 3$</td>
</tr>
</tbody>
</table>
Mutual exclusion means that no two threads are in the critical section at the same time. For a program with two threads $t$ and $u$:

$$\Box \neg (c_t \land c_u)$$

“Always (in every state), it is not the case that both $t$ and $u$ are in their critical section.”
A deadlock occurs when no thread makes progress; thus **deadlock freedom** is when some thread makes progress. For a program with two threads *t* and *u*:

<table>
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<tr>
<th>PROPOSITION</th>
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</thead>
<tbody>
<tr>
<td><em>e</em>&lt;sub&gt;t&lt;/sub&gt;</td>
<td>thread <em>t</em> is trying to enter its critical section</td>
</tr>
<tr>
<td><em>e</em>&lt;sub&gt;u&lt;/sub&gt;</td>
<td>thread <em>u</em> is trying to enter its critical section</td>
</tr>
</tbody>
</table>

\[ \Box \left( (e_t \land e_u) \Rightarrow \Diamond (c_t \lor c_u) \right) \]

“Always, if both *t* and *u* are trying to enter their critical sections, then *t* or *u* will eventually (in some future state) be inside its critical section.”

Or, equivalently: “Not all threads get stuck out forever.”
Starvation freedom in LTL

Starvation occurs when one thread does not make progress; thus starvation freedom is when all threads make progress. For a program with two threads $t$ and $u$, using the same propositions as before:

$$\Box \left( e_t \Rightarrow \Diamond c_t \right) \land \Box \left( e_u \Rightarrow \Diamond c_u \right)$$

"Always, if $t$ is trying to enter its critical sections, then $t$ will eventually be inside its critical section; and the same holds for $u$.”

Or, equivalently: “No threads get stuck out forever.”
Counter without locks: mutual exclusion

Mutual exclusion in writing to counter: $\square \neg (c_t \land c_u)$, with $n_t$ denoting that $t$ is not in its critical section, and $n_u$ denoting that $u$ is not in its critical section.
Counter with locks: mutual exclusion

Mutual exclusion in writing to counter: $\square \neg (c_t \land c_u)$, with $n_t$ denoting that $t$ is not in its critical section, and $n_u$ denoting that $u$ is not in its critical section.
Counter with locks: deadlock and starvation freedom

Deadlock freedom: $\ □ \left( (e_t \land e_u) \Rightarrow \Diamond (c_t \lor c_u) \right) \n
Starvation freedom: $\ □ \left( e_t \Rightarrow \Diamond c_t \right) \land □ \left( e_u \Rightarrow \Diamond c_u \right)$"
Verification
Verification is the process of checking that a program is correct. This means that, in addition to the implementation, there is also some form of specification (possibly only informal).

Two main techniques to do verification:

- **testing**: run the program using many different inputs and check that every run satisfies the specification
- **formal verification**: mathematically prove that every possible run of the program satisfies the specification
Verification

Testing
Testing in a nutshell:

- run the program using many different inputs
- check that every run satisfies the specification

Method deposit under test:

```java
class BankAccount {
    int balance;

    void deposit(int amount);
    void withdraw(int amount);
}
```

Testing code:

```java
BankAccount ba = new BankAccount();
ba.deposit(100);
check(ba.balance == 100);
ba.deposit(20);
check(ba.balance == 100 + 20);
ba.withdraw(11);
check(ba.balance == 100 + 20 - 11);
// ...
```
Testing is unreliable to find error in concurrent programs because of nondeterminism: a correct run does not guarantee that some other run with the same input will also be correct!

```java
public class Counter
    implements Runnable
{
    // thread’s computation:
    public void run() {
        int cnt = counter;
        counter = cnt + 1;
    }
}
```

```java
Counter c = new Counter();
Thread t = new Thread(c);
Thread u = new Thread(c);
t.start();
u.start();
t.join();
u.join();
check(c.count == 2);
```

sometimes it holds, sometimes it fails!
Besides nondeterminism, there is another problem that occurs if we try to test temporal properties.

- Testing mutual exclusion: if we run the program and detect that two threads are in their critical section at the same time, we know that there is a bug.
- Testing deadlock freedom: if we run the program and detect that all threads are blocked for, say, one hour, we still cannot be sure that they will be blocked there forever.

In simple examples, setting an arbitrary timeout may be enough, but in large systems with massive workloads it may be hard to figure out how much waiting time is to be expected.
Verification

Model checking
The difference between properties such as mutual exclusion and deadlock freedom is captured by two classes of temporal properties:

**Safety properties** are violated by a finite trace:

- informally: “nothing bad ever happens”
- example: mutual exclusion – a trace where, at some given time, two threads are both in their critical section shows that mutual exclusion does not hold

**Liveness properties** are violated only by an infinite trace:

- informally: “something good eventually happens”
- example: deadlock freedom – a trace where, from some time on, all threads are in the same state forever shows that deadlock freedom does not hold
Since testing is inadequate to reliably verify concurrent programs, **formal verification** is more widely used even if it is generally more difficult and expensive.

Remember that the specification of concurrent programs consists of two parts: **functional** and **temporal**. Verification proceeds as follows:

- first, prove that the **temporal** specification is always satisfied
- then, **assume** the temporal specification and prove that the **functional** specification is always satisfied

The advantages of this approach include:

- verifying a temporal specification alone often is feasible on **abstract models** of programs which ignore details such as the precise value of all variables
- if a strong temporal specification holds, we can often verify the functional specification as if the program were **sequential** (because concurrent executions are free from race conditions!)
Formal verification of the counter

Verifying the concurrent counter:

• to prove mutual exclusion, we only analyze the locking behavior and ignore the exact value of counter
• if mutual exclusion holds, the two threads execute run sequentially, thus we analyze the program as if it were sequential

```java
class Counter implements Runnable {
    // thread’s computation:
    public void run() {
        lock();
        int cnt = counter;
        counter = cnt + 1;
        unlock();
    }
}
```

Counter c = new Counter();
Thread t = new Thread(c);
Thread u = new Thread(c);
t.start();
u.start();
t.join();
u.join();
check(c.count == 2);
Model checking is an effective technique to verify concurrent programs, first developed in the 1980s.

Model checking mainly targets the verification of temporal specifications – expressed in temporal logic – about the behavior of state/transition diagrams – also called transition systems or finite-state automata:

1. given a concurrent program, build a state/transition diagram using finitely many states that captures its concurrent behavior
2. model checking algorithms analyze all infinitely many traces of the state/transition diagram and check whether a given temporal logic specification holds:
   - if model checking is successful, we have verified that all executions of the program satisfy the temporal specification
   - if model checking is unsuccessful, it returns a counterexample – a concrete trace that shows that the temporal specification is violated
Building a state/transition diagram that correctly captures the behavior of a concurrent program is something that cannot always be done automatically. Model checking tools provide convenient languages to formalize concisely complex state/transition diagrams.

For example, this is a model of the concurrent behavior of the shared counter in ProMeLa – the input language of the Spin model checker:

```plaintext
int count = 0;

proctype IncThread() {
    int tmp; tmp = count; count = tmp + 1;
}

init { // spawn two threads running in parallel
    run IncThread(); run IncThread();
}
```
There are two large families of model-checking techniques and tools:

- **Explicit-state** model checking works by explicitly exploring the state space generated by a given state/transition diagram. **Spin** is the most popular explicit-state model checker.

- **Symbolic** model checking works by encoding a given state/transition diagram using logic formulas (or other specialized data structures), and then expressing the temporal properties as logic properties of the encoding. **NuSMV** is a state-of-the-art symbolic model checker.

To know more about model checking: course “Formal methods in software development”