Solution

Solution 2a

```
(1)  star, name           → * (all other attributes)
(2)  star, position       → * (all other attributes)
(3)  star, distance       → * (all other attributes)

(4)  radius               → area
(5)  area                 → radius

(6)  water                → land
(7)  land                 → water

(8)  mass, radius         → gravity
(9)  mass, gravity        → radius
(10) gravity, radius      → mass

(11) atmosphere, oxygen   → otherGas
(12) atmosphere, otherGas → oxygen
(13) oxygen, otherGas     → atmosphere
```

Solution 2b

These 3 subsets of attributes of the Planets relation are keys:

```
{star, name}
{star, position}
{star, distance}
```

A key is a minimal superkey. Each of these subsets is a superkey of the relation Planets because their closure is the full set of attributes of Planets. In addition, each of these superkeys is minimal because there is no subset of attributes that is also a superkey.

Solution 2c

Functional dependencies 4-13 violate BCNF (all FDs except the first 3), because the left-hand side of each of these FDs is not a superkey of the Planets relation.

Solution 2d

Step 1

FD 4 violates BCNF, so we create a new relation **Areas** and remove the **area** attribute from the **Planets** relation. After this step, the violation of FD 5 is automatically resolved.

```
Planets(* - area)
    remaining FDs: 1-3, 6-13
Areas(radius, area)
    radius -> Planets.radius
    (4) radius → area
    (5) area   → radius
```

Step 2

FD 6 violates BCNF, so we create a new relation **Surfaces** and remove the **land** attribute from the **Planets** relation. After this step, the violation of FD 7 is automatically resolved.

```
Planets(* - area - land)
    remaining FDs: 1-3, 8-13
Areas(radius, area)
    radius -> Planets.radius
    (4) radius → area
    (5) area   → radius
Surfaces(water, land)
    water -> Planets.water
    (6) water → land
    (7) land  → water
```

Step 3

FD 8 violates BCNF, so we create a new relation **Gravities** and remove the **gravity** attribute from the **Planets** relation. After this step, the violation of FD 9 and 10 is automatically resolved.

```
Planets(* - area - land - gravity)
    remaining FDs: 1-3, 11-13
Areas(radius, area)
    radius -> Planets.radius
    (4) radius → area
    (5) area   → radius
Surfaces(water, land)
    water -> Planets.water
    (6) water → land
    (7) land  → water
Gravities(mass, radius, gravity)
    mass, radius -> Planets.(mass, radius)
    (8)  mass, radius    → gravity
    (9)  mass, gravity   → radius
    (10) gravity, radius → mass
```

Step 4

FD 11 violates BCNF, so we create a new relation **Atmospheres** and remove the **otherGas** attribute from the **Planets** relation. After this step, the violation of FD 11 and 12 is automatically resolved.

```
Planets(name, star, position, distance, radius, water, mass, atmosphere,
   oxygen)
    (1) star, name     → * (all other attributes)
    (2) star, position → * (all other attributes)
    (3) star, distance → * (all other attributes)
Areas(radius, area)
    radius -> Planets.radius
    (4) radius → area
    (5) area   → radius
Surfaces(water, land)
    water -> Planets.water
    (6) water → land
    (7) land  → water
Gravities(mass, radius, gravity)
    mass, radius -> Planets.(mass, radius)
    (8)  mass, radius    → gravity
    (9)  mass, gravity   → radius
    (10) gravity, radius → mass
Atmospheres(atmosphere, oxygen, otherGas)
    atmosphere, oxygen -> Planets.(atmosphere, oxygen)
    (11) atmosphere, oxygen   → otherGas
    (12) atmosphere, otherGas → oxygen
    (13) oxygen, otherGas     → atmosphere
```