Kurs: MAN321/TMV026 Ändliga automater
Plats: M-huset
Tid: 08.30-12.30
Datum: 2007-05-31 No help documents
Telefonvakt: Thierry Coquand, 7721030
The questions can be answered in english or in swedish.
total 30; ≥13: 3, ≥19: 4, ≥25: 5
total 30; ≥13: G, ≥21: VG

1. What is, mathematically, a context-free Language (1p)? Give, with motivation, an example of a language which is context-free, but not regular (1p) and an example of a language which is not context-free (1p)

2. Let $A = (Q, \Sigma, \delta, q_0, F)$ be a DFA. We recall that we define $q.x$ for $x \in \Sigma^*$ by recursion

$$q.\epsilon = q, \qquad q.(ax) = (q.a).x$$

   Explain why we have $q.(xy) = (q.x).y$ for all $x, y \in \Sigma^*$ (2p)

3. Let Σ be $\{0, 1\}$. We recall that the regular expressions are on Σ are given by the grammar

$$E \ ::= \ 0 \mid 1 \mid \emptyset \mid \epsilon \mid E + E \mid EE \mid E^*$$

   Give a regular expression E such that

$$L(E) = \Sigma^* - L((0+1)^*01) \ \ (2p)$$

4. Minimize the following automaton (2p)

	a	b
→1	2	3
2	5	6
∗3	1	4
∗4	6	3
5	2	1
6	5	4

5. Build a DFA that recognizes exactly the word in $\{0, 1\}^*$ ending with the string 1010. (2p)

6. Consider the regular expression $E = a^*b^* + b^*a^*$. Build the minimal DFA for E (2p).

7. Is it true that if L is regular then so is $L^2 - L$? Explain why (2p)

8. Is the following grammar ambiguous? Why (2p)?

$$S \rightarrow SS \mid aSb \mid ab \mid ba$$

9. Give an example of two languages L_1, L_2 such that

   (a) L_1 is regular, L_2 is not regular and $L_1 \cup L_2$ is regular (1p)

   (b) L_1 is regular, L_2 is not regular and $L_1 \cap L_2$ is regular and nonempty (1p)

10. Give a grammar in Chomsky normal form for $\{a^n b^{2n} c^k \mid k, n > 0\}$ (1p) and $\{a^n b^k a^n \mid k, n > 0\}$ (1p).

11. Is the following true of false. Motivate.

    (a) Any subset of a regular language is regular (1p)

    (b) If L_n is a family of regular language then $\cup_n L_n$ is regular (1p)

12. Explain why $\{a^n \mid n \geq 0\} \cup \{b^n c^n \mid n \geq 0\}$ is *not* regular (2p).

13. We recall that if $L \subseteq \Sigma^*$ is a language then and $a \in \Sigma$ then L/a denotes the language $L/a = \{u \in \Sigma^* \mid au \in L\}$. Explain why if L is regular then so is L/a for any $a \in \Sigma$ (2p). It follows from this that $(L/a)a = \{ua \mid au \in L\}$ is also regular. Explain why (1p). If w is the word $a_1 \ldots a_n$ we denote by $shift(w)$ the word $a_2 \ldots a_n a_1$. (If w is the empty word ϵ then $shift(w)$ is ϵ.) Deduce from the regularity of $(L/a)a$ and $(L/b)b$ that if $L \subseteq \{a, b\}^*$ is regular then so is $\{shift(w) \mid w \in L\}$ (2p).

Kurs: MAN321/TMV026 Ändliga automater och formella språk
Plats: M-huset
Tid: 08.30-12.30
Datum: 2007-08-27 No help documents
Telefonvakt: Thierry Coquand, 7721030
The questions can be answered in english or in swedish.
total 30; ≥13: 3, ≥19: 4, ≥25: 5
total 30; ≥13: G, ≥21: VG

1. Let Σ be an alphabet. What is, mathematically, a deterministic finite automaton on Σ (1p)? Explain what is the language determined by such a finite automaton (1p). Explain why such a language is a context-free language (1p).

2. Minimize the following automaton (2p)

	a	b
→0	1	3
1	0	3
2	1	4
∗3	5	5
4	3	3
∗5	5	5

3. Build a NFA with 3 states that accepts the language $\{ab, abc\}^*$. (2p)

4. Build a DFA corresponding to the regular expression $(ab)^* + a^*$. (3p)

5. Let Σ be $\{0, 1\}$. We recall that the regular expressions are on Σ are given by the grammar

$$E \ ::= \ 0 \mid 1 \mid \emptyset \mid \epsilon \mid E + E \mid EE \mid E^*$$

Give a regular expression E such that

$$L(E) = \Sigma^* - L(10(0 + 1)^*) \quad (2p)$$

6. Build a DFA that recognizes exactly the word in $\{0, 1\}^*$ ending with the string 1110. (2p)

7. Is the following grammar ambiguous? Why (2p)?

$$S \to AB \mid aaB, \qquad A \ \to \ a \mid Aa, \qquad B \to b$$

Construct an unambiguous grammar which is equivalent to this grammar (2p).

8. Consider the grammar

$$S \rightarrow aaB, \qquad A \rightarrow bBb \mid \epsilon, \qquad B \rightarrow Aa$$

Show that the string *aabbabba* is *not* in the language generated by this grammar (3p).

9. Find context-free grammars for the languages

   (a) $L = \{ a^n b^n c^k \mid n \leq k \}$ (1p)

   (b) $L = \{ a^n b^m \mid n \neq m \}$ (1p)

10. Let L, M, N be languages on an alphabet Σ^* (that is, we have L, M, N subsets of Σ^*). Explain why we have $L(M \cap N) \subseteq LM \cap LN$ (2p). Give an example showing that we do not have $LM \cap LN \subseteq L(M \cap N)$ in general (2p).

11. Let Σ be an alphabet and let L_1, L_2 be subsets of Σ^*. Assume that $L_1 \cap L_2 = \emptyset$ and L_1 is finite and that $L_1 \cup L_2$ is regular. Can we deduce that L_2 is regular (3p)?