Software Engineering using Formal Methods

Model Checking with Temporal Logic

Wolfgang Ahrendt

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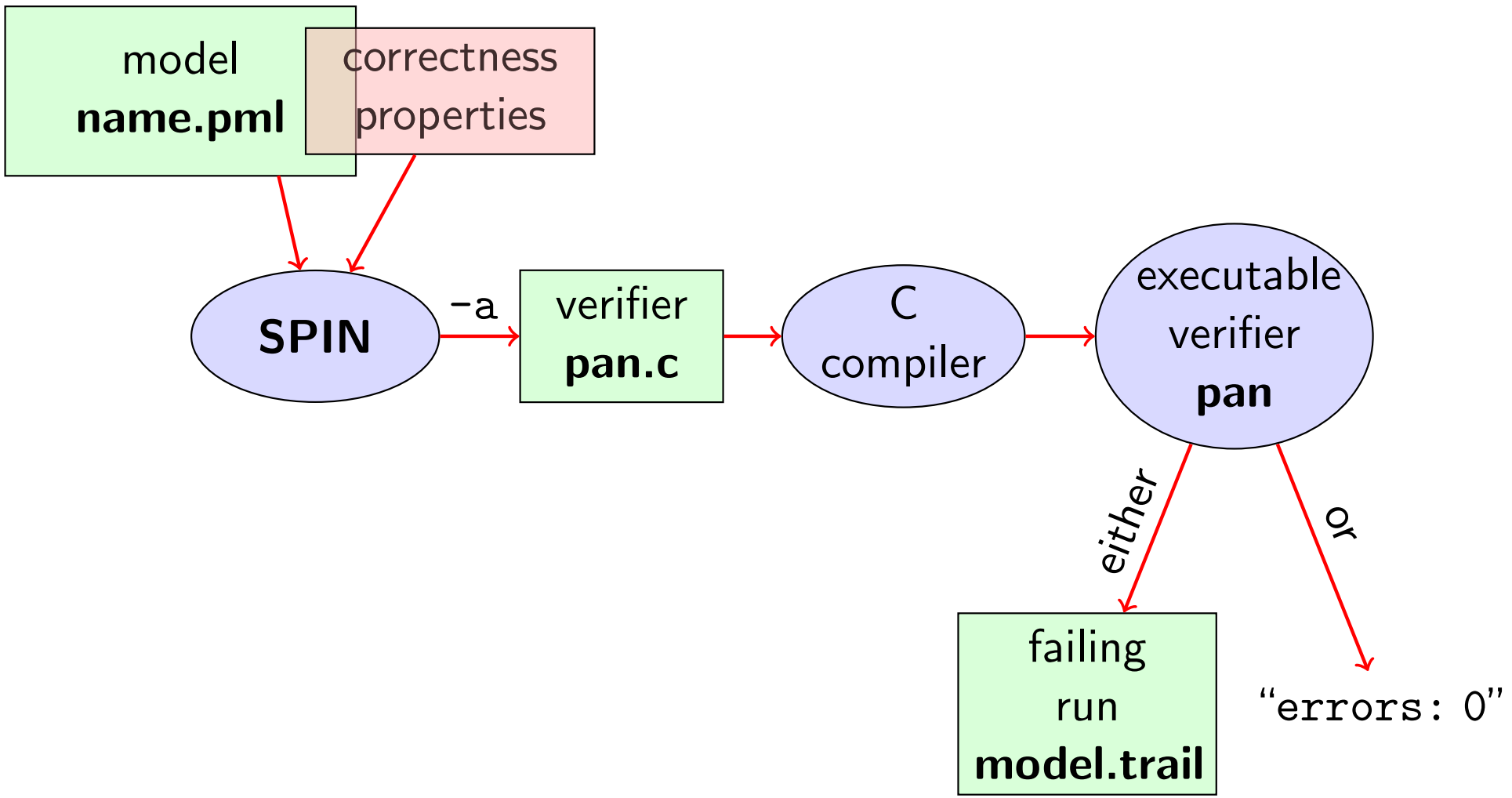
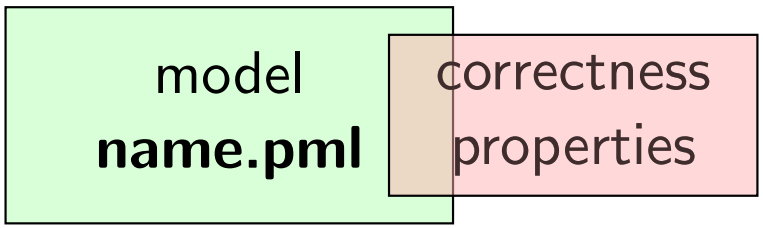
Model Checking with SPIN



Stating Correctness Properties



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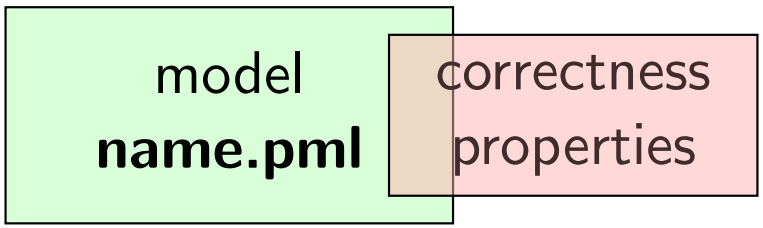

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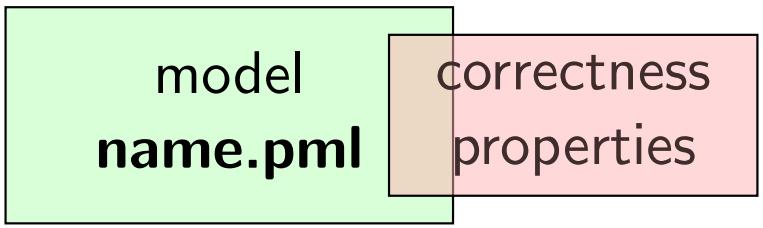

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  - `end` labels ✔
  - `accept` labels
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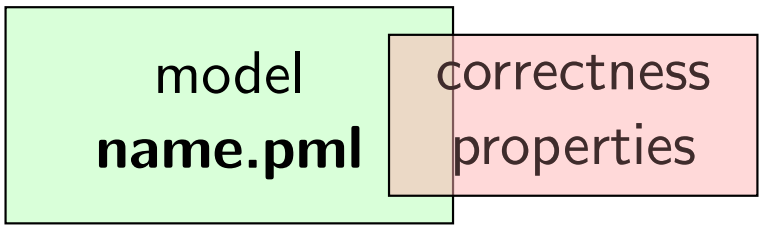

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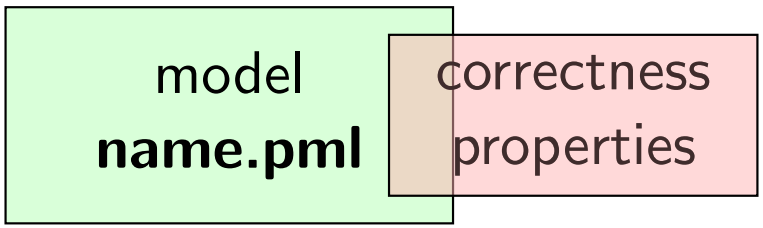

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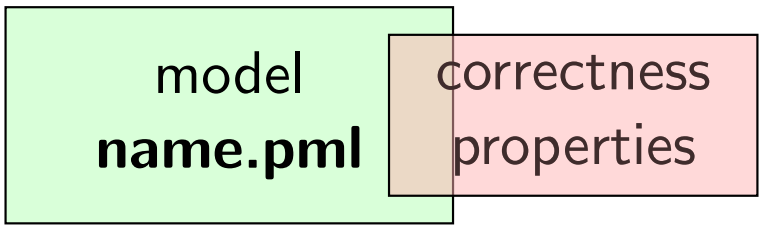

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Preliminaries

1. Accept labels in PROMELA $\leftrightarrow$ Büchi automata
2. Fairness

Preliminaries 1: Acceptance Cycles

Definition (Accept Location)

A location marked with an accept label of the form "`accept`*xxx*`:`" is called an accept location.

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A location marked with an accept label of the form "`accept`*xxx*`:`" is called an accept location.

Accept locations can be used to specify cyclic behavior

Definition (Acceptance Cycle)

A run which infinitely often passes through an accept location is called an acceptance cycle.

Acceptance cycles are mainly used in never claims (see below), to define forbidden infinite behavior

Preliminaries 2: Fairness

Does this model terminate in each run?

Simulate: `start/fair.pml`

```
byte n = 0;
bool flag = false;

active proctype P() {
  do ::  flag -> break
     ::  else -> n = 5 - n
  od
}
active proctype Q() {
  flag = true
}
```

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Termination guaranteed only if scheduling is (weakly) fair!

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Definition (Weak Fairness)

A run is called weakly fair iff the following holds:
each continuously executable statement is executed eventually.

Model Checking of Temporal Properties

Many correctness properties not expressible by assertions

- All properties that involve state changes
- Temporal logic expressive enough to characterize many (but not all) properties

In this course: "temporal logic" synonymous with "linear temporal logic"

Today: model checking of properties formulated in temporal logic

Beyond Assertions

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Mutual exclusion enforced by adding assertion to each critical section

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critical++;
assert( critical <= 1 );
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Drawbacks

- No separation of concerns (model vs. correctness property)
- Changing assertions is error prone (easily out of sync)
- Easy to forget assertions:
  correctness property might be violated at unexpected locations

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Drawbacks

- No separation of concerns (model vs. correctness property)
- Changing assertions is error prone (easily out of sync)
- Easy to forget assertions:
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- Many interesting properties not expressible via assertions

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"`0 <= i <= len-1` holds throughout each run"

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"Whenever several processes try to enter their critical section, eventually one of them does so."

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These are temporal properties $\Rightarrow$ use temporal logic

Boolean Temporal Logic

Numerical variables in expressions

- Expressions such as `i <= len-1` contain numerical variables
- Propositional LTL as introduced so far only knows propositions
- Slight generalisation of LTL required

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In Boolean Temporal Logic, atomic building blocks are *Boolean expressions* over PROMELA variables

Boolean Temporal Logic over PROMELA

Set For_{BTL} of Boolean Temporal Formulas (simplified)

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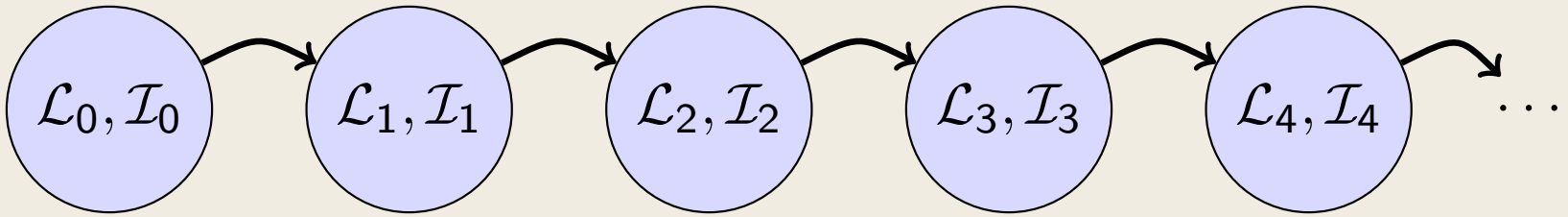
- if `P` is a process and `l` is a label in `P`, then `P@l` is $\in For_{BTL}$ (`P@l` reads "`P` is at `l`")
- if ϕ and ψ are formulas $\in For_{BTL}$, then all of

$$!\,\phi, \quad \phi \;\&\&\; \psi, \quad \phi \;||\; \psi, \quad \phi \;->\; \psi, \quad \phi \;<->\; \psi$$
$$[]\phi, \quad <>\phi, \quad \phi\,\mathtt{U}\,\psi$$

are $\in For_{BTL}$

Semantics of Boolean Temporal Logic

A run σ through a PROMELA model M is a chain of states



- $\mathcal{L}_j$ maps each running process to its current location counter
- From $\mathcal{L}_j$ to $\mathcal{L}_{j+1}$, only one of the location counters has advanced (exception: channel rendezvous)
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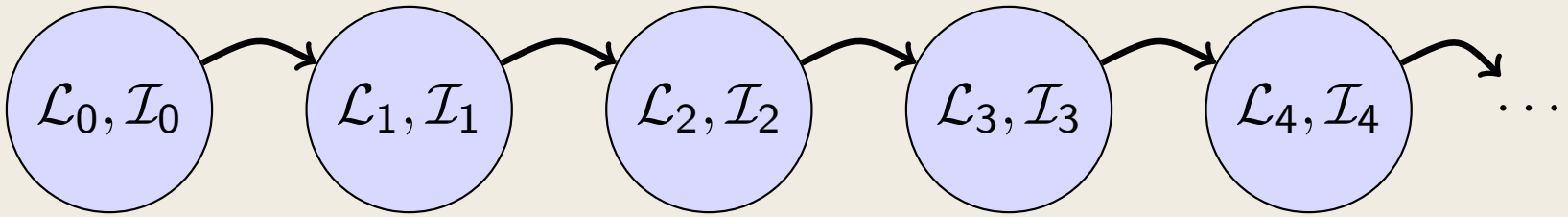

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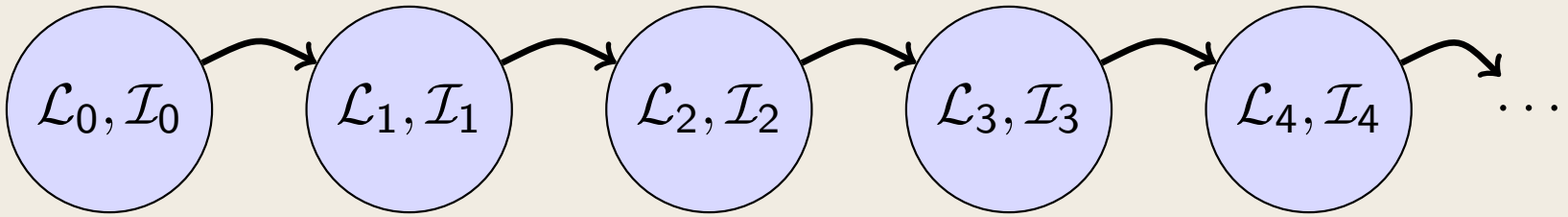

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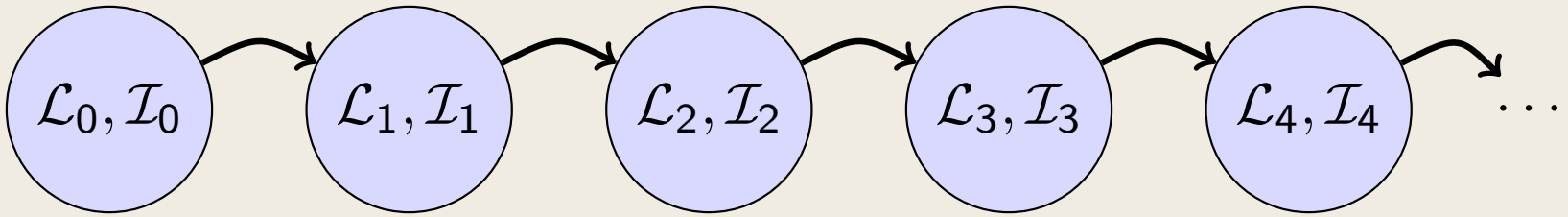

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Evaluating other formulas $\in \mathit{For}_{BTL}$ in runs σ: see previous lecture

Safety Properties

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. . . are formulas of the form $[]\phi$

- state that something 'good', ϕ, is guaranteed throughout each run
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TL formula `[](critical <= 1)`

"It is guaranteed throughout each run that at most one process visits its critical section at any time."

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Example

TL formula `[](critical <= 1)`

"It is guaranteed throughout each run that at most one process visits its critical section at any time."

or, equivalently:
"It will never happen that more than one process visits its critical section."

Applying Temporal Logic to Critical Section Problem

We want to verify `[](critical<=1)` as a correctness property of:

```
active proctype P() {
  do :: /* non-critical activity */
        atomic {
          !inCriticalQ;
          inCriticalP = true
        }
        critical++;
        /* critical activity */
        critical--;
        inCriticalP = false
  od
}

/* similarly for process Q */
```

Model Checking a Safety Property with SPIN

Command Line Execution

Add definition of TL formula to PROMELA file

Example `ltl atMostOne { [](critical <= 1) }`

General `ltl` *name* { *TL-formula* }

can define more than one formula

```
> spin -a file.pml
> gcc -DSAFETY -o pan pan.c
> ./pan -N name
```

Demo: `target/safety1.pml`

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`ltl` definitions not part of Ben Ari's book (SPIN≤ 6): ignore 5.3.2, etc.

Model Checking a Safety Property using Web Interface

1. add definition of TL formula to PROMELA file

   Example `ltl atMostOne { [](critical <= 1) }`

   General `ltl` *`name`* `{` *`TL-formula`* `}`

   can define more than one formula
2. load PROMELA file into web interface
3. ensure `Safety` is selected
4. enter name of LTL formula in according field
5. select `Verify`

Demo: `safety1.pml`

Model Checking a Safety Property using jSpin

1. add definition of TL formula to Promela file

   Example `ltl atMostOne { [](critical <= 1) }`

   General `ltl` *`name`* `{` *`TL-formula`* `}`

   can define more than one formula
2. load Promela file into jSpin
3. write *`name`* in 'LTL formula' field
4. ensure `Safety` is selected
5. select `Verify`
   - (corresponds to command line `./pan -N` *`name`* `...`)
6. (if necessary) select `Stop` to terminate too long verification

Demo: `safety1.pml`

Temporal Model Checking without Ghost Variables

We want to verify mutual exclusion without using ghost variables.

```
bool inCriticalP = false, inCriticalQ = false;

active proctype P() {
  do :: atomic {
          !inCriticalQ;
          inCriticalP = true
        }
cs:     /* critical activity */
        inCriticalP = false
  od
}

/* similar for process Q with same label cs: */

ltl mutualExcl { []!(P@cs && Q@cs) }
```

Demo: `start/noGhost.pml`

Never Claims: Processes trying to show user wrong

Büchi automaton, as PROMELA process, for negated property

1. Negated TL formula translated to '**never**' process
2. accepting locations in Büchi automaton represented with help of accept labels ("accept*xxx*:")
3. If one of these reached infinitely often, the orig. property is violated

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Example (Never claim for <>p, simplified for readability)

```
never {   /* !(<>p) */
  accept_xyz: /* passed ∞ often iff !(<>p) holds */
  do
  :: (!p)
  od
}
```

Model Checking against Temporal Logic Property

Theory behind SPIN

1. Represent the interleaving of all processes as a single automaton (only one process advances in each step), called $\mathcal{M}$

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$$\mathcal{L}^{\omega}(\mathcal{M}) \cap \mathcal{L}^{\omega}(\mathcal{NC}_{\neg\phi}) = \emptyset$$
then ϕ holds in $\mathcal{M}$,
otherwise we have a counterexample
4. To check $\mathcal{L}^{\omega}(\mathcal{M}) \cap \mathcal{L}^{\omega}(\mathcal{NC}_{\neg\phi})$ construct intersection automaton (both automata advance in each step) and search for accepting run

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Example

`<>csp`

(with `csp` a variable only true in the critical section of `P`)

"in each run, process `P` visits its critical section eventually"

Applying Temporal Logic to Starvation Problem

We want to verify <>csp as a correctness property of:

```
active proctype P() {
  do :: /* non-critical activity */
        atomic {
          !inCriticalQ;
          inCriticalP = true
        }
        csp = true;
        /* critical activity */
        csp = false;
        inCriticalP = false
  od
}

/* similarly for process Q */
/* there, using csq        */
```

Model Checking a Liveness Property using jSpin

1. open Promela file `liveness1.pml`
2. write `ltl pWillEnterC { <>csp }` in Promela file
   (as first `ltl` formula)
3. ensure that `Acceptance` is selected
   (Spin will search for *accepting* cycles through the never claim)
4. *for the moment* uncheck `Weak Fairness` (see discussion below)
5. select `Verify`

Verification Fails

Demo: `start/liveness1.pml`

Verification fails!

Why?

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Verification fails!

Why?

The liveness property on one process "had no chance".
Not even weak fairness was switched on!

Model Checking Liveness with Weak Fairness using jSpin

Always check `Weak fairness` when verifying liveness

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2. write `ltl pWillEnterC { <>csp }` in Promela file
   (as first `ltl` formula)
3. ensure that `Acceptance` is selected
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4. ensure `Weak fairness` is checked
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Model Checking Liveness using Web Interface

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   can define more than one formula
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3. ensure `Acceptance` is selected
4. enter name of LTL formula in according field
5. ensure `Weak fairness` is checked
6. select `Verify`

Demo: `liveness1.pml`

Model Checking Liveness using SPIN directly

Command Line Execution

Make sure `ltl` *name* { *TL-formula* } is in *file*.pml

```
> spin -a file.pml
> gcc -o pan pan.c
> ./pan -a -f [-N name]
```

-a acceptance cycles, -f weak fairness

Demo: `start/liveness1.pml`

Limitation of Weak Fairness

Verification fails again!

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Note that `!inCriticalQ` is not continuously executable!

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Note that `!inCriticalQ` is not continuously executable!

Restriction to weak fairness is principal limitation of SPIN

Here, liveness needs strong fairness, which is not supported by SPIN.

Revisit `fair.pml`

- Specify liveness of `fair.pml` using labels

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- Specify liveness of `fair.pml` using labels
- Prove termination

Demo: `target/fair.pml`

Revisit `fair.pml`

- Specify liveness of `fair.pml` using labels
- Prove termination
- Here, weak fairness is needed, *and sufficient*

Demo: `target/fair.pml`

Literature for this Lecture

Ben-Ari Chapter 5
except Sections 5.3.2, 5.3.3, 5.4.2
(`ltl` construct replaces **#define** and `-f` option of SPIN)