Logic in Computer Science

CTL: some corrections

We define $M \models \psi$ to mean $s \models \psi$ for all states s of $M = (S, \rightarrow, L)$.

For a model M we don't have that $M \models \varphi \leftrightarrow \psi$ is equivalent to $M \models \varphi \leftrightarrow M \models \psi$ (exercise: find a counter-example). What we have is that $M \models \varphi \leftrightarrow \psi$ is equivalent to $s \models \varphi \leftrightarrow s \models \psi$ for all states of M.

Similarly to have $M \models \varphi \rightarrow \psi$ is the same as having $s \models \varphi \rightarrow s \models \psi$ for all states of M which is *not* the same (exercise) as $M \models \varphi \rightarrow M \models \psi$.

Because of this, we don't have in general

$$\models (\varphi \to EX\varphi) \to (\varphi \to EG\varphi)$$

but what we have is that, if $M\models\varphi\to EX\varphi$ then $M\models\varphi\to EG\varphi$