Testing, Debugging, Program Verification

Formal Verification, Part I

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Recap: Functions and Predicates

- Method calls are not allowed in specifications.
  - May have side effects - bad for proofs
- Functions and Predicates are allowed in specifications
  - No side effects, cannot manipulate memory.
  - Only allowed in specifications: Not present in compiled code only for verification.
  - `function method` compiled, allowed both in code and specification.
  - Can write inefficient but mathematically simpler function to specify more efficient iterative code.

Recap: Loop Invariants and Variants

Loops are difficult to reason about.

- Don't know how many times we go around.
- But Dafny needs to consider all paths! How?

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An invariant is an property which is true before entering loop and after each execution of loop body.

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Solution: Loop Invariants

An invariant is an property which is true before entering loop and after each execution of loop body.

But what about termination?

Solution: Loop Variants

An variant is an expression which decrease with each iteration of the loop, and is bounded from below by 0. Dafny can often guess variants automatically.

Consequence of Sortedness for Implementations

method `Find`

- Can employ binary search (logarithmic complexity)
- It assumes sortedness in pre-state

Exercise: Specifying Sortedness

Recall:

```
var limit : int;
var arr : array<int>;
var size : int;

predicate Valid()
  reads this, this.arr;
   {arr != null && 0 <= size <= limit && limit == arr.Length &&
   forall i,j :: 0 <= i < j < size ==> arr[i] != arr[j]}
```

Write down a predicate `Sorted()`.
It should hold when `arr` is sorted in increasing order in the filled up range (i.e. from 0 up to `size`).

Specifying Sortedness

```
predicate Sorted()
 reads this, this.arr;
 {forall i, j :: 0 <= i < j < size ==> arr[i] <= arr[j]}
```

- What's the value of `Sorted()` if `size < 2`?

Specifying Sortedness

```
predicate Sorted()
 reads this, this.arr;
 {forall i, j :: 0 <= i < j < size ==> arr[i] <= arr[j]}
```

- What's the value of `Sorted()` if `size < 2`?
- Where in the specifications do we use the `Sorted` predicate?

Specifying BSearch

Can assume sortedness of pre-state

```
method BSearch(a : array<int>, e : int) returns (r : int)
requires a != null && Sorted(a)
ensures if (exists i :: 0 <= i < a.Length && a[i] == elem)
        then 0 <= r < a.Length && a[r] == elem else r < 0
```

Same postcondition as Find. Difference is input is sorted.

Specifying BSearch

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requires a != null && Sorted(a)
ensures if (exists i :: 0 <= i < a.Length && a[i] == elem)
        then 0 <= r < a.Length && a[r] == elem else r < 0
{
    var low, high := 0 , arr.Length;
    while(low < high)
    invariant ?
    {
        var mid := (low + high) / 2;
        if e < a[mid]          { high := mid; }
        else if e > a[mid]  { low := mid + 1; }
        else                        { return mid; }
    }
    return -1;
}
```

Specifying BSearch

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{
    var low, high := 0 , arr.Length;
    while(low < high)
    invariant 0 <= low <= high <= arr.Length
    invariant forall i :: (0 <= i < low ||
          high <= i < arr.Length) ==> arr[i] != elem
    {
        var mid := (low + high) / 2;
        if e < a[mid]       { high := mid; }
        else if e > a[mid]  { low := mid + 1; }
        else                { return mid; }
    }
    return -1;
}
```

Block 4: Formal Verification

- Three lectures
- One assignment to hand in.

Todays main topics:

- Dafny behind the scenes: How does it prove programs correct?
- Weakest Precondition Calculus

Formal Software Verification: Motivation

Limitations of Testing

- Testing ALL inputs is usually impossible.
- Even strongest coverage criteria cannot guarantee abcence of further defects.

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Goal of Formal Verification

Given a formal specification S of the behaviour of a program P:
Give a mathematically rigorous proof that each run of P conforms to S

P is correct with respect to S

Formal Software Verification: Motivation

The Main Steps towards Formal Verification

1. Write a specification of a given program that can be proven
2. Device a correctness proof method without exhaustive case analysis
3. Design mathematically rigorous proof rules: “calculus”
4. Implement an automated theorem prover for your calculus.

Formal Software Verification: Limitations

- No absolute notion of program correctness!
  - Correctness always relative to a given specification
  - Example: forgot to specify permutation property for `sort()`
- Hard and expensive to develop provable formal specifications
  - In practice, no attempt to specify full functionality.
  - Safety properties e.g.
    - Well-formed data
    - Exception freedom, ...
- Some properties may be difficult or impossible to specify. e.g.
  - Time and memory (possible, but not done here)
  - User behaviour, the environment in general

Formal Software Verification: Limitations cont.

- Requires lots of expertise and expenses (so far...)
- Disclaimer: Even fully specified & verified programs can have runtime failures
  - Defects in the compiler
  - Defects in the runtime environment
  - Defects in the hardware

Possible & desirable: Exclude defects in source code wrt. a given spec

Dafny: Behind the Scenes

What happens when we ask Dafny to compile our program?
How does it prove that it is correct according to its specification?

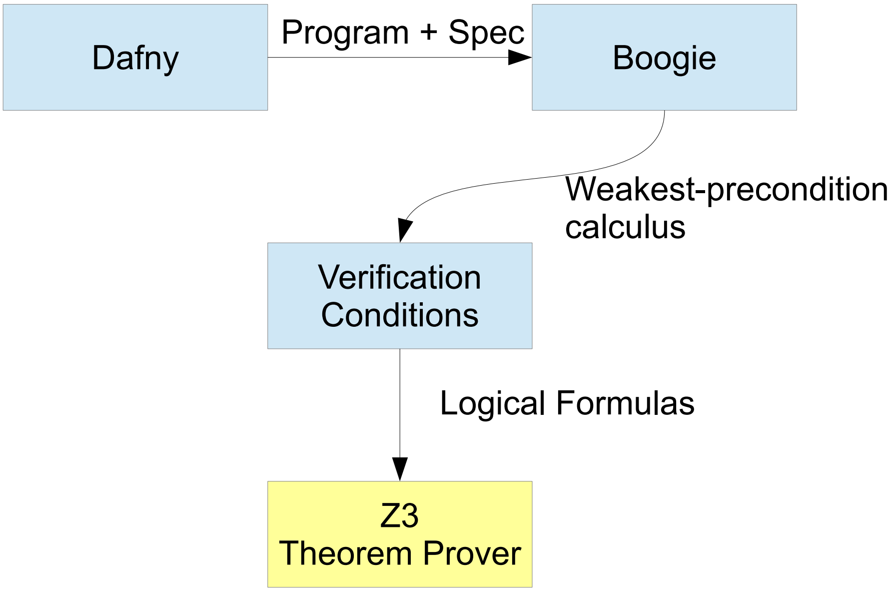
Dafny: Behind the Scenes

What happens when we ask Dafny to compile our program?
How does it prove that it is correct according to its specification?

More than just the programming language:

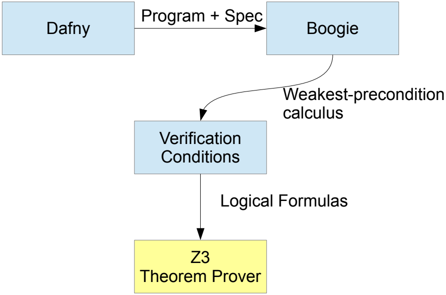
- The Programming Language and its Specification constructs.
- The Program Verifier
  - Formalisation of program semantics and proof obligations (from spec).
  - Generator of logical formulas: verification conditions (VCs).
  - An automatic theorem prover, which proves the VCs

Dafny: Behind the Scenes



Dafny: Behind the Scenes

- Boogie: Intermediate Language (c.f. Java Byte Code)
- "Simpler" than Dafny.
- **Our focus:** How do we extract verification conditions?
- **This module**: Weakest precondition calculus.
- Won't deal with full Dafny/Boogie, but simplified subset involving assignments, if-statements, while loops.



What do we Need to Prove and How?

```
method MyMethod(. . .)
  requires Q
  ensures R
  {
   S: program statements
  }
```

In literature, often expressed as a Hoare Triple: {Q} S {R}

Hoare Triple: {Q} S {R}

If execution of program S starts in a state satisfying pre-condition Q, the is is guaranteed to terminate in a state satisfying the post-condition R.

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Weakest Precondition:

- Assuming that R holds after executing S,

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  - i.e. does Q imply the weakest pre-condition?

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  - Formally: $wp(S, R)$
- Does Q satisfy at least these restrictions?
  - i.e. does Q imply the weakest pre-condition?
  - To prove: $Q \rightarrow wp(S, R)$
  - Proving Hoare triple {Q} S {R} amounts to showing that $Q \rightarrow wp(S, R)$.

Weakest Precondition

Weakest Precondition: $wp(S, R)$

The weakest precondition of a program S and post-condition R represents the set of all states such that execution of S started in any of these is guaranteed to terminate in a state satisfying R.

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Why weakest?

- **Recall:** Dafny won't let you call a method if preconditions not satisfied.
- No need to restrict use of method more than necessary.

Example

- Program statement S: `i := i + 1`
- Post-condition R: `i <= 1`

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- Reason backwards: $wp(i := i + 1, i <= 1) = i <= 0$

Example

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What is the weakest precondition, $wp(S, R)$?

- Reason backwards: $wp(i := i + 1, i <= 1) = i <= 0$
- Executing `i := i + 1` in any state satisfying `i <= 0` will end in a state satisfying `i <= 1`.

Example

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- Post-condition R: `i <= 1`

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- Reason backwards: $wp(i := i + 1, i <= 1) = i <= 0$
- Executing `i := i + 1` in any state satisfying `i <= 0` will end in a state satisfying `i <= 1`.
- **Note:** Taking Q: `i < -5` does also satisfy R. But overly restrictive, excludes initial states where `-5 <= i <=0`. Weakest precondition can help us find a suitable contract.

First-Order Formulas and Program States

First-order formulas define sets of program states

What do we mean by $wp(S, R)$ defining a set of program states?

$wp(S, R)$ is a logical formula, F, involving some with program locations (a.k.a. variables).

Then F is true in some states and not true in others.

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Then F is true in some states and not true in others.

Example

- `(i>j & j>=0)` is true in exactly those states S where $\mathtt{i}^s > \mathtt{j}^s$ and $\mathtt{j}^s$ is non-negative.
- `exists i :: i == j`

  is true in any state S, because the value of `i` can be chosen to be $\mathtt{j}^s$

Mini Quiz: Guess the Weakest Precondition

Write down $wp(S, R)$ for the following S and R:

	S	R
a)	`i := i+1`	`i > 0`
b)	`i := i+2; j := j-2`	`i + j == 0`
c)	`a[i] := 1`	`a[i] == a[j]`
d)	`i := i+1; j := j-1`	`i * j == 0`

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c)	`a[i] := 1`	`a[i] == a[j]`
d)	`i := i+1; j := j-1`	`i * j == 0`

Solution:

a)	`i >= 0`
b)	`i + j == 0`
c)	`a[j] == 1`
d)	`i == -1 || j == 1`

Proving Validity of Programs

To prove that a program satisfies its contract we need a calculus.

A calculus is a set of (schematic) rules.

> The rules of the weakest precondition calculus provide semantics, a logical meaning, for the statements in our programming language.

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A calculus is a set of (schematic) rules.

The rules of the weakest precondition calculus provide semantics, a logical meaning, for the statements in our programming language.

Our Verification Algorithm

- Have a program S, with precondition Q and postcondition R
- Compute $wp(S, R)$
- Prove that $Q \to wp(S, R)$

Proving Validity of Programs

We will prove validity of programs written in a slightly simplified subset of Dafny/Boogie featuring:

Assignment: `x := e`

Sequentials: `S1; S2`

Assertions: `assert B`

If-statements: `if B then S1 else S2`

While-loops: `while B S`

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Semantics

We will define the weakest precondition for each of these program constructs.

Weakest Precondition Calculus: Assignment

Assignment

$wp(x := e, R) = R[x \mapsto e]$

Note: $R[x \mapsto e]$ means "R with all occurrences of x replaced by e".

Example

Let S:

```
i := i + 1;
```

Let R: $i > 0$

Weakest Precondition Calculus: Assignment

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Note: $R[x \mapsto e]$ means "R with all occurrences of x replaced by e".

Example

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```

Let R: $i > 0$

$wp(i := i + 1, i > 0) =$

Weakest Precondition Calculus: Assignment

Assignment

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Note: $R[x \mapsto e]$ means "R with all occurrences of x replaced by e".

Example

Let S:

```
i := i + 1;
```

Let R: $i > 0$

$wp(i := i + 1, i > 0) =$
(By Assignment rule)
$i + 1 > 0$

This program satisfies its postcondition if started in any state where $i \geq 0$.

Weakest Precondition Calculus: Sequential Composition

Sequential Composition

$wp(S1; S2, R) = wp(S1, wp(S2, R))$

Example

Let S:

```
x := i;
i := i + 1;
```

Let R: $x < i$

Weakest Precondition Calculus: Sequential Composition

Sequential Composition

$wp(S1; S2, R) = wp(S1, wp(S2, R))$

Example

$$wp(x := i; i := i + 1, x < i) =$$

Let S:

```
x := i;
i := i + 1;
```

Let R: $x < i$

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Example

Let S:

```
x := i;
i := i + 1;
```

Let R: $x < i$

$wp(x := i; i := i + 1, x < i) =$
(By Sequential rule)
$wp(x := i, wp(i := i + 1, x < i)) =$

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Example

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Let R: $x < i$

$wp(x := i; i := i + 1, x < i) =$
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$wp(x := i, wp(i := i + 1, x < i)) =$
(By Assignment rule)
$wp(x := i, x < i + 1) =$

Weakest Precondition Calculus: Sequential Composition

Sequential Composition

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Example

Let S:

```
x := i;
i := i + 1;
```

Let R: $x < i$

$wp(x := i; i := i + 1, x < i) =$
(By Sequential rule)
$wp(x := i, wp(i := i + 1, x < i)) =$
(By Assignment rule)
$wp(x := i, x < i + 1) =$
(By Assignment rule)
$i < i + 1$
(trivially true)

This program satisfies its postcondition in any initial state.

Weakest Precondition Calculus: Assertion

Assertion

$wp(assert\ B, R) = B \wedge R$

Example

Let S:

```
x := y;
assert x > 0;
```

Let R: $x < 20$

Weakest Precondition Calculus: Assertion

Assertion

$wp(\mathit{assert}\ B,\ R) = B \wedge R$

Example

$$wp(x := y; \mathit{assert}\ x > 0, x < 20) =$$

Let S:

```
x := y;
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```

Let R: $x < 20$

Weakest Precondition Calculus: Assertion

Assertion

$wp(assert\ B,\ R) = B \land R$

Example

Let S:

```
x := y;
assert x > 0;
```

Let R: $x < 20$

$wp(x := y; assert\ x > 0, x < 20) =$

(By Sequential rule)

$wp(x := y, wp(assert\ x > 0, x < 20)) =$

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(By Sequential rule)

$wp(x := y, wp(assert\ x > 0, x < 20)) =$

(By Assertion rule)

$wp(x := y, x > 0 \wedge x < 20) =$

Weakest Precondition Calculus: Assertion

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$wp(assert\ B,\ R) = B \wedge R$

Example

Let S:

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x := y;
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Let R: $x < 20$

$wp(x := y; assert\ x > 0, x < 20) =$
(By Sequential rule)
$wp(x := y, wp(assert\ x > 0, x < 20)) =$
(By Assertion rule)
$wp(x := y, x > 0 \wedge x < 20) =$
(By Assignment rule)
$y > 0 \wedge y < 20$

This program satisfies its postcondition in those initial states where `y` is a number between 1 and 19 (inclusive).

Weakest Precondition Calculus: Conditional

Conditional

$wp(\mathit{if}\ B\ \mathit{then}\ S1\ \mathit{else}\ S2,\ R) = (B \to wp(S1, R)) \wedge (\neg B \to wp(S2, R))$

Example

Let S:

```
if (i >= 0) then x := i
 else x := -i
```

Abbreviate:

S1: `x := i`

S2 `x := -i`

Let R: $x \geq 0$

Weakest Precondition Calculus: Conditional

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S1: `x := i`

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Let R: $x \geq 0$

$wp(\textit{if } (i \geq 0) \textit{ then } S1 \textit{ else } S2, x \geq 0) =$

Weakest Precondition Calculus: Conditional

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S1: `x := i`

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Let R: $x \geq 0$

$wp(\textit{if } (i \geq 0) \textit{ then } S1 \textit{ else } S2, x \geq 0) =$

(By Conditional rule)

$i \geq 0 \rightarrow wp(x := i, x \geq 0) \wedge$

$\neg(i \geq 0) \rightarrow wp(x := -i, x \geq 0) =$

Weakest Precondition Calculus: Conditional

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$wp(\textit{if } (i \geq 0) \textit{ then } S1 \textit{ else } S2, x \geq 0) =$

(By Conditional rule)

$i \geq 0 \rightarrow wp(x := i, x \geq 0) \wedge$

$\neg(i \geq 0) \rightarrow wp(x := -i, x \geq 0) =$

(By Assignment rule)

$(i \geq 0 \rightarrow i \geq 0) \wedge (\neg(i \geq 0) \rightarrow -i \geq 0) =$

Weakest Precondition Calculus: Conditional

Conditional

$wp(\mathit{if}\ B\ \mathit{then}\ S1\ \mathit{else}\ S2,\ R) = (B \to wp(S1, R)) \land (\neg B \to wp(S2, R))$

Example

Let S:

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if (i >= 0) then x := i
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```

Abbreviate:

S1: `x := i`

S2 `x := -i`

Let R: $x \geq 0$

$wp(\mathit{if}\ (i \geq 0)\ \mathit{then}\ S1\ \mathit{else}\ S2, x \geq 0) =$

(By Conditional rule)

$i \geq 0 \to wp(x := i, x \geq 0)\ \land$

$\neg(i \geq 0) \to wp(x := -i, x \geq 0) =$

(By Assignment rule)

$(i \geq 0 \to i \geq 0) \land (\neg(i \geq 0) \to -i \geq 0) =$

true

This program satisfies its postcondition in any initial state.

Weakest Precondition Calculus: Conditional

Conditional, empty `else` branch

$wp(\mathit{if}\ B\ \mathit{then}\ S1,\ R) = (B \rightarrow wp(S1, R)) \wedge (\neg B \rightarrow R)$

If `else` is empty, need to show that R follows just from negated guard.

Mini Quiz: Derive the weakest precondition

The Rules

$wp(x := e, R) = R[x \mapsto e]$
$wp(S1; S2, R) = wp(S1, wp(S2, R))$
$wp(\mathit{assert}\ B,\ R) = B \wedge R$
$wp(\mathit{if}\ B\ \mathit{then}\ S1\ \mathit{else}\ S2,\ R) = (B \rightarrow wp(S1, R)) \wedge (\neg B \rightarrow wp(S2, R))$

This time, you should derive the weakest precondition, stating which rules you use in each step. In each case, consider if there exist an initial state which could satisfy the post-condition.

	S	R
a)	`i := i+2; j := j-2`	`i + j == 0`
b)	`i := i+1; assert i > 0`	`i <= 0`
c)	`if isEven(x) then y:=x/2 else y:=(x-1)/2`	`isEven(y)`

Mini Quiz: Derive the weakest precondition

Solution:
(full derivation on next slides)

a) `i + j == 0`
(apply seq. rule followed by assignment rule, simplify)

b) `i+1 > 0 && i+1 <= 0`
(apply seq rule, assert rule, assignment)
Simplifies to `i => 0 && i <= -1` which is false! No initial state can satisfy this postcondition.

c)
`isEven(x) ==> isEven(x/2) && !isEven(x) ==> isEven((x-1)/2)`
(apply cond. rule, followed by assignment.)

$wp(\texttt{i := i+2; j := j-2}, \texttt{i + j == 0})$

Seq. rule

$= wp(\texttt{i := i+2}, wp(\texttt{j := j-2}, \texttt{i + j == 0}))$

Assignment rule

$= wp(\texttt{i := i+2}, \texttt{i + j - 2 == 0})$

Assignment rule

$= \texttt{i + 2 + j - 2 == 0}$

By elemental algebra

$= \texttt{i + j == 0}$

$wp(\texttt{i := i+1; assert i > 0}, \texttt{i <= 0})$

Seq. rule

$= wp(\texttt{i := i+1}, wp(\texttt{assert i > 0}, \texttt{i <= 0}))$

Assert rule

$= wp(\texttt{i := i+1}, \texttt{i > 0} \wedge \texttt{i <= 0})$

Assignment rule

$= \texttt{i + 1 > 0} \wedge \texttt{i + 1 <= 0}$

By elemental algebra

$= \texttt{i > -1} \wedge \texttt{i <= -1}$

By elemental algebra

$= \texttt{i > -1} \wedge \texttt{i <= -1}$

By elemental algebra

$= \texttt{false}$

$wp(\texttt{if isEven(x) then y:=x/2 else y:=(x-1)/2}, \texttt{isEven(y)})$

If rule

$= (\texttt{isEven(x)} \rightarrow wp(\texttt{y:=x/2}, \texttt{isEven(y)}))$

$\wedge(\neg\texttt{isEven(x)} \rightarrow wp(\texttt{y:=(x-1)/2}, \texttt{isEven(y)}))$

Assignment rule (2x)

$= (\texttt{isEven(x)} \rightarrow \texttt{isEven(x/2)}$

$\wedge(\neg\texttt{isEven(x)} \rightarrow \texttt{isEven(x-1)/2)}$

What Next?

While loops!

Difficulties of While Loops

- Need to “unwind” loop body one by one
- In general, no fixed loop bound known (depends on input)
- How the loop invariants and variants are used in proofs.

Summary

- Testing cannot replace verification
- Formal verification can prove properties for all runs,
  ...but has inherent limitations, too.
- Dafny is compile to intermediate language Boogie.
- Verification conditions (VCs) extracted, using weakest precondition calculus rule.
- VCs are logical formulas, which can be passed to a theorem prover.
- Prove that precondition imply *wp*.

Reading: *The Science of Programming* by David Gries. Chapters 6-10, bearing in mind that the notation and language differ slightly from ours. Available as E-book from Chalmers library.