

# Software Engineering using Formal Methods

## Reasoning about Programs with Dynamic Logic

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Typed FOL

- ▶ + (JAVA) programs  $p$

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# Dynamic Logic

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### Remark on Hoare Logic and DL

**In Hoare logic**  $\{Pre\} p \{Post\}$

(Pre, Post must be FOL)

**In DL**  $Pre \rightarrow [p]Post$

(Pre, Post any DL formula)

# Proving DL Formulas

## An Example

```
∀ int x;  
(x = n ∧ x ≥ 0 →  
  [ i = 0; r = 0;  
    while(i < n){i = i + 1; r = r + i;}  
    r = r + r - n;  
  ]r = x * x)
```

How can we prove that the above formula is valid  
(i.e. satisfied in all states)?

# Semantics of DL Sequents

$\Gamma = \{\phi_1, \dots, \phi_n\}$  and  $\Delta = \{\psi_1, \dots, \psi_m\}$  sets of DL formulas  
where all logical variables occur bound

Recall:  $\mathcal{S} \models (\Gamma \Rightarrow \Delta)$  iff  $\mathcal{S} \models (\phi_1 \wedge \dots \wedge \phi_n) \rightarrow (\psi_1 \vee \dots \vee \psi_m)$

Define semantics of DL sequents identical to semantics of FOL sequents

## Definition (Validity of Sequents over DL Formulas)

A sequent  $\Gamma \Rightarrow \Delta$  over DL formulas is **valid** iff

$$\mathcal{S} \models (\Gamma \Rightarrow \Delta) \text{ in all states } \mathcal{S}$$

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## Consequence for program variables

Initial value of program variables implicitly “universally quantified”



# Symbolic Execution of Programs

Sequent calculus decomposes top-level operator in formula.  
What is “top-level” in a sequential program  $p; q; r; ?$

## Symbolic Execution

- ▶ Follow the **natural control flow** when analysing a program
- ▶ Values of some variables unknown: **symbolic state representation**

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## Example

Compute the final state after termination of

```
x=x+y; y=x-y; x=x-y;
```

# Symbolic Execution of Programs Cont'd

## General form of rule conclusions in symbolic execution calculus

$$\langle \text{stmt}; \text{rest} \rangle \phi, \quad [\text{stmt}; \text{rest}] \phi$$

- ▶ Rules symbolically execute *first* statement (“**active statement**”)
- ▶ Repeated application of such rules corresponds to **symbolic program execution**

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**Example** (`symbolicExecution/simpleIf.key`,  
**Demo**, active statement only)

```
\programVariables {  
  int x; int y; boolean b;  
}  
\problem {  
  \<{ if (b) { x = 1; } else { x = 2; } y = 3; }\> y > x  
}
```

# Symbolic Execution of Programs Cont'd

## Symbolic execution of conditional

$$\text{if } \frac{\Gamma, b = \text{true} \Rightarrow \langle p; \text{rest} \rangle \phi, \Delta \quad \Gamma, b = \text{false} \Rightarrow \langle q; \text{rest} \rangle \phi, \Delta}{\Gamma \Rightarrow \langle \text{if } (b) \{ p \} \text{ else } \{ q \} ; \text{rest} \rangle \phi, \Delta}$$

Symbolic execution must consider all possible execution branches

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## Symbolic execution of loops: unwind

$$\text{unwindLoop} \frac{\Gamma \Rightarrow \langle \text{if } (b) \{ p; \text{while } (b) p \}; \text{rest} \rangle \phi, \Delta}{\Gamma \Rightarrow \langle \text{while } (b) \{ p \}; \text{rest} \rangle \phi, \Delta}$$

# Updates for KeY-Style Symbolic Execution

## Needed: a Notation for Symbolic State Changes

- ▶ Symbolic execution should “walk” through program in natural **forward** direction
- ▶ Need **succinct representation** of state changes effected by each symbolic execution step
- ▶ Want to **simplify** effects of program execution **early**
- ▶ Want to **apply** state changes **late**  
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We use dedicated notation for state changes: **updates**



# Explicit State Updates

## Definition (Syntax of Updates, Updated Terms/Formulas)

If  $v$  is program variable,  $t$  FOL term type-conformant to  $v$ ,  $t'$  any FOL term, and  $\phi$  any DL formula, then

- ▶  $\{v := t\}$  is an update
- ▶  $\{v := t\}t'$  is DL term
- ▶  $\{v := t\}\phi$  is DL formula

## Definition (Semantics of Updates)

State  $\mathcal{S}$  interprets program variables  $v$  with  $\mathcal{I}_{\mathcal{S}}(v)$

$\beta$  variable assignment for logical variables in  $t$ , define semantics  $\rho$  as:

$$\rho_{\beta}(\{v := t\})(\mathcal{S}) = \mathcal{S}' \text{ where } \mathcal{S}' \text{ identical to } \mathcal{S} \text{ except } \mathcal{I}_{\mathcal{S}'}(v) = \text{val}_{\mathcal{S},\beta}(t)$$

# Explicit State Updates Cont'd

## Facts about updates $\{v := t\}$

- ▶ Update semantics similar to that of assignment
- ▶ Value of update also depends on  $\mathcal{S}$  and **logical** variables in  $t$ , i.e.,  $\beta$
- ▶ Updates are **not assignments**: right-hand side is FOL term
  - $\{x := n\}\phi$  cannot be turned into assignment ( $n$  logical variable)
  - $\{x=i++;\}\phi$  cannot (immediately) be turned into update
- ▶ Updates are **not equations**: they **change** value of  $v$

# Computing Effect of Updates (Automated)

Rewrite rules for update followed by ...

**program variable**  $\left\{ \begin{array}{l} \{x := t\}x \rightsquigarrow t \\ \{x := t\}y \rightsquigarrow y \end{array} \right.$

**logical variable**  $\{x := t\}w \rightsquigarrow w$

**complex term**  $\{x := t\}f(t_1, \dots, t_n) \rightsquigarrow f(\{x := t\}t_1, \dots, \{x := t\}t_n)$   
(because  $f$  is rigid)

**FOL formula**  $\left\{ \begin{array}{l} \{x := t\}(\phi \ \& \ \psi) \rightsquigarrow \{x := t\}\phi \ \& \ \{x := t\}\psi \\ \dots \\ \{x := t\}(\forall \tau y; \phi) \rightsquigarrow \forall \tau y; (\{x := t\}\phi) \end{array} \right.$

**program formula** No rewrite rule for  $\{x := t\}(\langle p \rangle \phi)$  **unchanged!**

Update rewriting delayed until  $p$  symbolically executed

# Assignment Rule Using Updates

## Symbolic execution of assignment using updates

$$\text{assign} \frac{\Gamma \Rightarrow \{x := t\} \langle \text{rest} \rangle \phi, \Delta}{\Gamma \Rightarrow \langle x = t; \text{rest} \rangle \phi, \Delta}$$

- ▶ Simple! No variable renaming, etc.
- ▶ Works as long as  $t$  has no side effects

## Demo

updates/assignmentToUpdate.key

How to apply updates on updates?

## Example

Symbolic execution of

```
t=x; x=y; y=t;
```

yields:

```
{t := x}{x := y}{y := t}
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Need to compose three sequential state changes into a single one:

How to apply updates on updates?

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Need to compose three sequential state changes into a single one:

**parallel updates**

# Parallel Updates Cont'd

## Definition (Parallel Update)

A **parallel update** is an expression of the form  $\{v_1 := r_1 \parallel \dots \parallel v_n := r_n\}$  where each  $\{v_i := r_i\}$  is simple update

- ▶ All  $r_i$  computed in **old state** before update is applied
- ▶ Updates of all program variables  $v_i$  executed **simultaneously**
- ▶ Upon **conflict**  $v_i = v_j, r_i \neq r_j$  later update ( $\max\{i, j\}$ ) wins

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## Definition (Composition Sequential Updates/Conflict Resolution)

$$\{v_1 := r_1\}\{v_2 := r_2\} = \{v_1 := r_1 \parallel v_2 := \{v_1 := r_1\}r_2\}$$



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$$\{v_1 := r_1 \parallel \dots \parallel v_n := r_n\}x = \begin{cases} x & \text{if } x \notin \{v_1, \dots, v_n\} \\ r_k & \text{if } x = v_k, x \notin \{v_{k+1}, \dots, v_n\} \end{cases}$$

# Symbolic Execution with Updates (by Example)

$\Rightarrow x < y \rightarrow \langle \text{int } t=x; x=y; y=t; \rangle y < x$

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$$\begin{aligned}x < y &\implies \{t:=x\}\langle x=y; y=t;\rangle y < x \\ &\quad \vdots \\ \implies x < y &\rightarrow \langle \text{int } t=x; x=y; y=t;\rangle y < x\end{aligned}$$

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## Parallel Updates Cont'd

### Example

symbolic execution of `x=x+y; y=x-y; x=x-y;` gives

$$(\{x := x+y\}\{y := x-y\})\{x := x-y\}$$

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$$\begin{aligned} & (\{x := x+y\}\{y := x-y\})\{x := x-y\} \\ & \{x := x+y \parallel y := (x+y)-y\}\{x := x-y\} \end{aligned}$$

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{x := x+y || y := (x+y)-y || x := (x+y)-((x+y)-y)}
{x := x+y || y := x || x := y}
{y := x || x := y}
```

KeY automatically deletes overwritten (unnecessary) updates

## Demo

updates/swap2.key

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Parallel updates to store intermediate state of symbolic computation

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**Not allowed:**  $\forall \tau \mathbf{i}; \langle \dots \mathbf{i} \dots \rangle \phi$   
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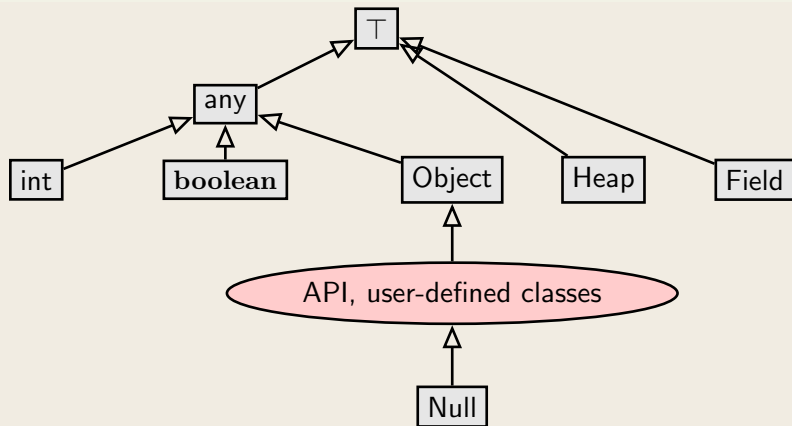
### Instead

Quantify over **value**, and **assign** it to program variable:

$\forall \tau x; \{i := x\} \langle \dots i \dots \rangle \phi$

# Modelling Java in FOL: Fixing a Type Hierarchy

Signature based on Java's type hierarchy



Each interface and class in API and in target program becomes type with appropriate subtype relation

# Modelling the Heap in FOL

## The Java Heap

Objects are stored on (i.e., in) the **heap**.

- ▶ Status of heap changes during execution
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Each element of data type Heap represents a certain heap status.

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- ▶ in  $F_{\Sigma}$ : Heap  $\text{store}(\text{Heap}, \text{Object}, \text{Field}, \text{any})$  ;  
 $\text{store}(h, o, f, v)$  returns heap like  $h$ , but with  $v$  associated to  $(o, f)$

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- ▶ in  $F_{\Sigma}$ : any  $\text{select}(\text{Heap}, \text{Object}, \text{Field})$ ;  
 $\text{select}(h, o, f)$  returns value associated to  $(o, f)$  in  $h$

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## Modelling instance fields

Person
<code>int age</code> <code>int id</code>
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**Key notation** `p.id@h` ( abbreviating `select(h, p, id)` )

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`p.id` ( abbreviating `select(heap, p, id)` )<sup>a</sup>

---

<sup>a</sup>heap is special program variable for “current” heap; mostly implicit in *o.f*

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## Writing to Field id of Person p

**FOL notation** `store(h, p, id, 6238)`

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## Writing to Field id of Person p

**FOL notation**  $\text{store}(h, p, \text{id}, 6238)$

**KeY notation**  $h[p.\text{id} := 6238]$  ( notation for store, not update )

# The Algebra of Heaps

We do *not* formalise the *structure* (implementation) of heaps.  
We formalise the *behaviour*, with an algebra of heap operations:

$$\text{select}(\text{store}(h, o, f, v), o, f) =$$



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## Example

$$\text{select}(\text{store}(h, o, f, 15), o, f) \rightsquigarrow 15$$

$$\text{select}(\text{store}(h, o, f, 15), o, g) \rightsquigarrow \text{select}(h, o, g)$$

$$\text{select}(\text{store}(h, o, f, 15), u, f) \rightsquigarrow$$

$$\text{if } (o = u) \text{ then } (15) \text{ else } (\text{select}(h, u, f))$$

## Shorthand Notations for Heap Operations

<code>o.f@h</code>	is	<code>select(h, o, f)</code>
<code>h[o.f := v]</code>	is	<code>store(h, o, f, v)</code>

## Shorthand Notations for Heap Operations

$o.f@h$  is  $\text{select}(h, o, f)$

$h[o.f := v]$  is  $\text{store}(h, o, f, v)$

*therefore:*

$u.f@h[o.f := v]$  is  $\text{select}(\text{store}(h, o, f, v), u, f)$

$h[o.f := v][o'.f' := v']$  is  $\text{store}(\text{store}(h, o, f, v), o', f', v')$

# Pretty Printing

## Shorthand Notations for Heap Operations

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$u.f@h[o.f := v]$  is `select(store(h, o, f, v), u, f)`

$h[o.f := v][o'.f' := v']$  is `store(store(h, o, f, v), o', f', v')`

## Very-Shorthand Notations for **Current** Heap

Current heap always in special variable `heap`.

$o.f$  is `select(heap, o, f)`

$\{o.f := v\}$  is `update {heap := heap[o.f := x]}`

# Modelling the Heap in FOL—The Full Story

Is formula `select(h, p, id) >= 0` type-safe?



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## Real Field Access

`int::select(h, p, Person::$id) >= 0` is type-safe

- ▶ `int::select` is a function name, not a cast
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## General

For each `T` typed field `f` of class `C`,  $F_\Sigma$  contains

- ▶ a constant declared as `Field C::$f`
- ▶ a function declared as `T T::select(Heap, C, Field)`

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Everything **blue** is a function name

## Writing to Fields

We stick to the above:

Declaration: `Heap store(Heap, Object, Field, any);`

Usage: `store(h, p, Person::$id, 42)`

# Field Update Assignment Rule

## Changing the value of fields

How to translate assignment to field, for example, `p.age=17;` ?

$$\text{assign} \frac{\Gamma \Rightarrow \{o.f := t\} \langle \text{rest} \rangle \phi, \Delta}{\Gamma \Rightarrow \langle o.f = t; \text{rest} \rangle \phi, \Delta}$$

Admit on left-hand side of update **JAVA location expressions**

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## Dynamic Logic: KeY input file

```
\javaSource "path to source code referenced in problem";  
  
\programVariables { Person p; }  
  
\problem {  
    \<{    p.age = 18;    }\> p.age = 18  
}
```

KeY reads in all source files and creates automatically the necessary signature (types, program variables, field constants)

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## Demo

updates/firstAttributeExample.key

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Does abrupt termination count as normal termination?

No! Need to distinguish **normal** and **exceptional** termination

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- ▶  $[p] \phi$ : If  $p$  terminates **normally** then formula  $\phi$  holds in final state (partial correctness)

Abrupt termination on top-level counts as non-termination!

## Example Reconsidered: Exception Handling

```
\javaSource "path to source code";  
  
\programVariables {  
  ...  
}  
  
\problem {  
  p != null -> \<{ p.age = 18; }\> p.age = 18  
}
```

Only provable when no top-level exception thrown

### Demo

updates/secondAttributeExample.key

# The Self Reference

## Modeling reference `this` to the receiving object

Special name for the object whose JAVA code is currently executed:

in **JML**: Object `this`;

in **Java**: Object `this`;

in **KeY**: Object `self`;

Default assumption in JML-KeY translation: `self != null`



# Which Objects do Exist?

How to model **object creation** with **new** ?

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How to model **object creation** with **new** ?

## Constant Domain Assumption

Assume that domain  $\mathcal{D}$  is the same in all states of LTS  $K = (S, \rho)$

**Desirable consequence:**

Validity of **rigid** FOL formulas unaffected by programs containing **new**()

$\models \forall T x; \phi \rightarrow [p](\forall T x; \phi)$  is valid for rigid  $\phi$

# Object Creation

## Realizing Constant Domain Assumption

- ▶ Implicitly declared field `boolean <created>` in class `Object`
- ▶ Equal to `true` iff argument object has been created
- ▶ Object creation modeled as `{heap := create(heap, o)}` for not (yet) created `o` (essentially sets `<created>` field of `o` to `true`)
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ObjectCreation(simplified)

$$\frac{\Gamma, \{u\}(\text{select}(\text{heap}, \text{ob}, \text{Object}::\text{<created>}) = \text{FALSE}) \Rightarrow \{u\}(\{\text{heap} := \text{create}(\text{heap}, \text{ob})\}\{o := \text{ob}\}\langle \pi o.\text{<init>}(\text{param}); \omega \rangle \phi), \Delta}{\Gamma \Rightarrow \{u\}(\langle \pi o = \text{new } T(\text{param}); \omega \rangle \phi), \Delta}$$

`ob` is a fresh program variable

Titlepage

Symbolic Execution

Updates

Parallel Updates

Modeling OO Programs

Self

Object Creation

## **Round Tour**

Java Programs

Arrays

Side Effects

Abrupt Termination

Aliasing

Method Calls

Null Pointers

API

Summary

Literature

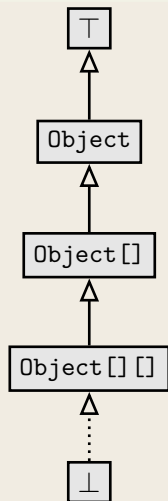
# Dynamic Logic to (almost) full Java

KeY supports full **sequential** Java, with some limitations:

- ▶ Limited concurrency
- ▶ No generics
- ▶ No I/O
- ▶ No floats
- ▶ No dynamic class loading or reflexion
- ▶ API method calls: need either JML contract or implementation

# Java Features in Dynamic Logic: Arrays

## Arrays



- ▶ JAVA type hierarchy includes array types that occur in given program (for finiteness)
- ▶ Types ordered according to JAVA subtyping rules
- ▶ Value of entry in array  $T[]$   $a$ ; defined in class  $C$  depends on reference  $a$  to array in  $C$  and index
- ▶ Function  $\text{arr} : \text{int} \rightarrow \text{Field}$  injective mapping from indices to fields
- ▶ Store array elements on heap, e.g., the value of  $a[i]$  on the heap  $\text{store}(\text{heap}, a, \text{arr}(i), 17)$  is 17
- ▶ Arrays  $a$  and  $b$  can refer to same object (aliases)
- ▶ KeY implements simplification and evaluation rules for array locations

# Java Features in Dynamic Logic: Complex Expressions

## Complex expressions with side effects

- ▶ JAVA expressions may contain assignment operator with **side effect**
- ▶ JAVA expressions can be complex, nested, have method calls
- ▶ FOL terms have **no** side effect on the state

## Example (Complex expression with side effects in Java)

```
int i = 0; if ((i=2)>= 2) i++;    value of i ?
```



# Complex Expressions Cont'd

## Decomposition of complex terms by symbolic execution

Follow the rules laid down in JAVA Language Specification

### Local code transformations

$$\text{evalOrderIteratedAssgnmt} \frac{\Gamma \Rightarrow \langle y = t; x = y; \omega \rangle \phi, \Delta}{\Gamma \Rightarrow \langle x = y = t; \omega \rangle \phi, \Delta} \quad t \text{ simple}$$

### Temporary variables store result of evaluating subexpression

$$\text{ifEval} \frac{\Gamma \Rightarrow \langle \text{boolean } v0; v0 = b; \text{if } (v0) \text{ p}; \omega \rangle \phi, \Delta}{\Gamma \Rightarrow \langle \text{if } (b) \text{ p}; \omega \rangle \phi, \Delta} \quad b \text{ complex}$$

Guards of conditionals/loops always evaluated (hence: side effect-free)  
before conditional/unwind rules applied

# Java Features in Dynamic Logic: Abrupt Termination

## Abrupt Termination: Exceptions and Jumps

Redirection of control flow via return, break, continue, **exceptions**

$$\langle \pi \text{ try } \{p\} \text{ catch}(e) \{q\} \text{ finally } \{r\} \omega \rangle \phi$$

Rules ignore inactive **prefix**, work on **active statement**, leave **postfix**

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Rule **tryThrow** matches **try-catch** in pre-/postfix and active throw

$$\Rightarrow \langle \pi \text{ if}(e \text{ instanceof } T) \{ \text{try}\{x=e; q\} \text{ finally } \{r\} \} \text{ else } \{r; \text{throw } e; \} \omega \rangle \phi$$

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### Demo

exceptions/try-catch.key

# Java Features in Dynamic Logic: Aliasing

Demo

aliasing/attributeAlias1.key

# Java Features in Dynamic Logic: Aliasing

## Demo

aliasing/attributeAlias1.key

## Reference Aliasing

Naive alias resolution causes **proof split** (on  $o = u$ ) at each access

$$\Rightarrow o.age = 1 \rightarrow \langle u.age = 2; \rangle o.age = u.age$$

# Java Features in Dynamic Logic: Method Calls

## Method Call

First evaluate arguments, leading to:

$$\{arg_0 := t_0 \parallel \dots \parallel arg_n := t_n \parallel c := t_c\} \langle c.m(arg_0, \dots, arg_n); \rangle \phi$$

## Actions of rule **methodCall**

- ▶ for each **formal parameter**  $p_i$  of  $m$ :  
declare and initialize new local variable  $\tau_i \ p\#i = arg_i$ ;
- ▶ look up **implementation** class  $C$  of  $m$  and split proof  
if implementation cannot be uniquely determined
- ▶ create concrete **method invocation**  $c.m(p\#0, \dots, p\#n)@C$

## Method Body Expand

1. Execute code that binds actual to formal parameters  $\tau_i \text{ p\#i} = \text{arg}_i;$
2. Call rule `methodBodyExpand`

$$\frac{\Gamma \Rightarrow \langle \pi \text{ method-frame}(\text{source}=\text{C}, \text{this}=\text{c})\{\text{ body } \} \omega \rangle \phi, \Delta}{\Gamma \Rightarrow \langle \pi \text{ c.m}(\text{p\#0}, \dots, \text{p\#n})@C; \omega \rangle \phi, \Delta}$$



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## Demo

methods/instanceMethodInlineSimple.key

## Localisation of Fields and Method Implementation

JAVA has complex rules for **localisation** of fields and method implementations

- ▶ Polymorphism
- ▶ Late binding
- ▶ Scoping (class vs. instance)
- ▶ Context (static vs. runtime)
- ▶ Visibility (private, protected, public)

**Proof split into cases when implementation not statically determined**

## Null pointer exceptions

There are no “exceptions” in FOL:  $\mathcal{I}$  total on FSym

Need to model possibility that  $o = \mathbf{null}$  in  $o.a$

- ▶ KeY branches over  $o \neq \mathbf{null}$  upon each field access

# A Round Tour of Java Features in DL Cont'd

## Formal specification of Java API

How to perform symbolic execution when `JAVA` API method is called?

1. API method has reference implementation in `JAVA`

Call method and execute symbolically

**Problem** Reference implementation not always available

**Problem** Breaks modularity

2. Use JML contract of API method:

**2.1** Show that **requires** clause is satisfied

**2.2** Obtain postcondition from **ensures** clause

**2.3** Delete updates with **modifiable** locations from symbolic state

# A Round Tour of Java Features in DL Cont'd

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## Java Card API in JML or DL

DL version available in KeY, JML work in progress See W. Mostowski

<http://limerick.cost-ic0701.org/home/verifying-java-card-programs-with-key>

# Summary

- ▶ Most JAVA features covered in KeY
- ▶ Several of remaining features available in experimental version
  - ▶ Simplified multi-threaded JMM
  - ▶ Floats
- ▶ Degree of automation for loop-free programs is very high
- ▶ Proving loops requires user to provide invariant
  - ▶ Automatic invariant generation sometimes possible
- ▶ Symbolic execution paradigm lets you use KeY w/o understanding details of logic

# Literature for this Lecture

## Essential

**KeY Book** Verification of Object-Oriented Software (see course web page), Chapter 10: **Using KeY**

**KeY Book** Verification of Object-Oriented Software (see course web page), Chapter 3: **Dynamic Logic**, Sections 3.1, 3.2, 3.4, 3.5, 3.6.1, 3.6.2, 3.6.3, 3.6.4, 3.6.5, 3.6.7