Exercise set 2. Subjective probability and utility

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1 Graded exercises

Time and scoring The numbers indicate expected time to complete the exercise. If you take more time than indicated, please note that. A "!" indicates that it may require some extra thought. A "?" indicates that this is an open question. The exercises, including the feedback questionnaire, count towards your grade. Bonus exercises have a 'carry over' effect on the grade.

Exercise 1 (30). *If R is our set of rewards, our utility function is $U : R \to \mathbb{R}$ and we $a \succ^* b$ iff $U(a) > U(b)$, then our preferences are transitive. Give an example of a utility function, not necessarily mapping to $\mathbb{R}$, and a binary relation $>$ such that transitivity can be violated. Back your example with a thought experiment.*

Exercise 2 (10). *In this exercise, we consider the choice somebody would make between two gambles, in two different cases.*

Case 1: Consider a set of two gambles:

1. *The reward is 500,000 with certainty.*
2. *The reward is: 2,500,000 with probability 0.10; 500,000 with probability 0.89, or 0 with probability 0.01.*

Case 2: Choose an alternative set of two gambles:

1. *The reward is 500,000 with probability 0.11, or 0 with probability 0.89.*
2. *The reward is: 2,500,000 with probability 0.1, or 0 with probability 0.9.*

Show that if gamble 1 is preferred in the first example, gamble 1 must also be preferred in the second example.

Exercise 3 (20). *Consider the following experiment:*

1. *You specify an amount a, then observe random value Y.*
2. *If $Y \geq a$, you receive Y currency units!*
3. *If $Y < a$, receive random reward X with known distribution (independent of Y).*
4. *Show that we should choose a s.t. $U(a) = \mathbb{E}[U(X)]$.*

Assume that U is increasing.

Exercise 4 (10). *The usefulness of probability and utility.*

- *Would it be useful to separate randomness from uncertainty? What would be desirable properties of an alternative concept to probability?*
- *Give an example of how the expected utility assumption might be violated.*

2 Bonus exercises

Exercise 5 (90!)**.** *Consider two urns, each containing red and blue balls. The first urn contains an equal number of red and blue balls. The second urn contains a* randomly *chosen proportion X of red balls, i.e. the probability of drawing a red ball from urn 2 is X.*

1. *Suppose that you were to select an urn, and then choose a random ball from that urn. If the ball is red, you win 1 CU, otherwise nothing. Show that: if your utility function is increasing with monetary gain, you should prefer urn 1 iff $\mathbb{E}(X) < \frac{1}{2}$.*
2. *Suppose that you were to select an urn, and then choose n random balls from that urn and that urn only. Each time you draw a red ball, you gain 1 CU. After you draw a ball, you put it back in the urn. Assume that the utility U is strictly concave and suppose that $\mathbb{E}(X) = \frac{1}{2}$. Then you should always select balls from urn 1.*

Hint: Show that for urn 2, $\mathbb{E}(U \mid x)$ is concave for $0 \le x \le 1$ (this can be done by showing $\frac{d^2}{dx^2}\mathbb{E}(U \mid x) < 0$). In fact,

$$\frac{d^2}{dx^2}\mathbb{E}(U \mid x) = n(n-1)\sum_{k=0}^{n-2}[U(k) - 2U(k+1) + U(k+2)]\binom{n-2}{k}x^k(1-x)^{n-2-k}.$$

Then apply Jensen's inequality.

3 Feedback

Finally, some questions about this unit:

1. Did you find the material interesting?
2. Did you find it potentially useful?
3. How much did you already know?
4. How much had you already seen but did not remember in detail?
5. How much have you seen for the first time?
6. Which aspect did you like the most?
7. Which aspect did you like the least?
8. Feel free to add any further comments.