# Monads

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# Monads seen so far: IO vs Gen

| IO A | Gen A |
| --- | --- |
| • Instructions to build a value of type A by interacting with the operating system | • Instructions to create a random value of type A |
| • Run by the ghc runtime system | • Run by the QuickCheck library functions to perform random tests |

# Terminology

- A *"monadic value"* is just an expression whose type is an instance of class Monad
- *"t is a monad"* means t is an instance of the class Monad
- We have often called a monadic value an *"instruction".* This is not standard terminology
  - but sometimes they are called "actions"

# Monads seen so far: IO vs Gen

## IO A

- Instructions to build a value of type A by interacting with the operating system
- Run by the ghc runtime system

## Gen A

- Instructions to create a random value of type A
- Run by the QuickCheck library functions to perform random tests

# Monads = Instructions

- What is the type of doTwice?

```
Main> :i doTwice
doTwice :: Monad m => m a -> m (a,a)
```

Even the *kind of instructions* can vary!
Different kinds of instructions, depending on who obeys them.

Whatever kind of result argument produces, we get a pair of them

# Monads and do notation

- To be an instance of class Monad you need (as a minimal definition) two operations: **>>=** and **return**

```
class Monad m where
  (>>=) :: m a -> (a -> m b) -> m b
  return :: a -> m a

  (>>) :: m a -> m b -> m b
  x >> y = x >>= \_ -> y

  fail :: String -> m a
  fail msg = error msg
```

Default implementations

# The truth about Do

- Do syntax is just a shorthand:

```
do act1
   act2
```

==

```
act1 >> act2
```

==

```
act1 >>= \_ -> act2
```

```
do v <- act1
   act2
```

==

```
act1 >>= \v -> act2
```

# The truth about Do

## Full translation (I)

```
do act1
   …
   actn
```

==

```
act1 >> do …
           actn
```

```
do v <- act1
   …
   actn
```

==

```
act1 >>= \v -> do …
                  actn
```

```
do actn
```

==

```
actn
```

# Example

```
foo :: IO ()
foo = do
    filename <- getLine
    contents <- readFile filename
    putStrLn contents
```

```
do v <- act1
   …
   actn
```

=

```
act1 >>= \v -> do …
                  actn
```

# Example

```haskell
foo :: IO ()
foo = do
    filename <- getLine
    contents <- readFile filename
    putStrLn contents
```

```haskell
foo' = getLine >>= \filename ->
       do
        contents <- readFile filename
        putStrLn $ take 100 contents
```

# Example

```
foo' = getLine >>= \filename ->
       do
        contents <- readFile filename
        putStrLn $ take 100 contents
```

```
do v <- act1
   …
   actn
```

=

```
act1 >>= \v -> do …
                  actn
```

```
do actn
```

=

```
actn
```

# Equivalent to

```haskell
foo' = getLine >>= \filename ->
       do
        contents <- readFile filename
        putStrLn $ take 100 contents
```

```haskell
foo'' = getLine >>= \filename ->
        readFile filename >>= \contents ->
        putStrLn $ take 100 contents
```

# Eta conversion & Monad Law

```
foo'' = getLine >>= \filename ->
         readFile filename >>= \contents ->
         putStrLn $ take 100 contents
```

```
f (g x) ==  (f . g) x
```

```
\v -> f v  ==  f
```

```
act1 >>= \v -> f v >>= g
```

=

```
act1 >>= f >>= g
```

# Eta conversion & Monad Law

```
foo'' = getLine >>= \filename ->
          readFile filename >>= \contents ->
          putStrLn $ take 100 contents
```

```
foo''' = getLine >>= readFile >>= putStrLn . take 100
```

# The truth about Do

## Full Translation (II): Let and pattern matching

```
do let p = e
   …
   actn
```

==

```
let p = e in
do …
   actn
```

```
do pattern <- act1
   …
   actn
```

==

```
let f pattern = do …
                   actn
    f _         = fail “Error”
in  act1 >>= f
```

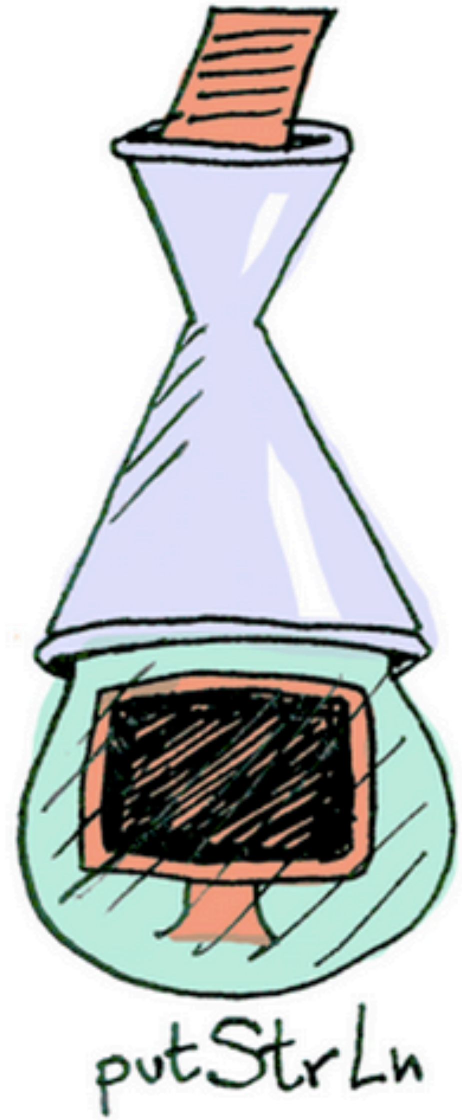
# Pictures from a blog post about functors, applicatives and monads

http://adit.io/posts/2013-04-17-functors,_applicatives,_and_monads_in_pictures.html

Aditya Y. Bhargava


```
getLine :: IO String
```


```
readFile :: FilePath -> IO String
```

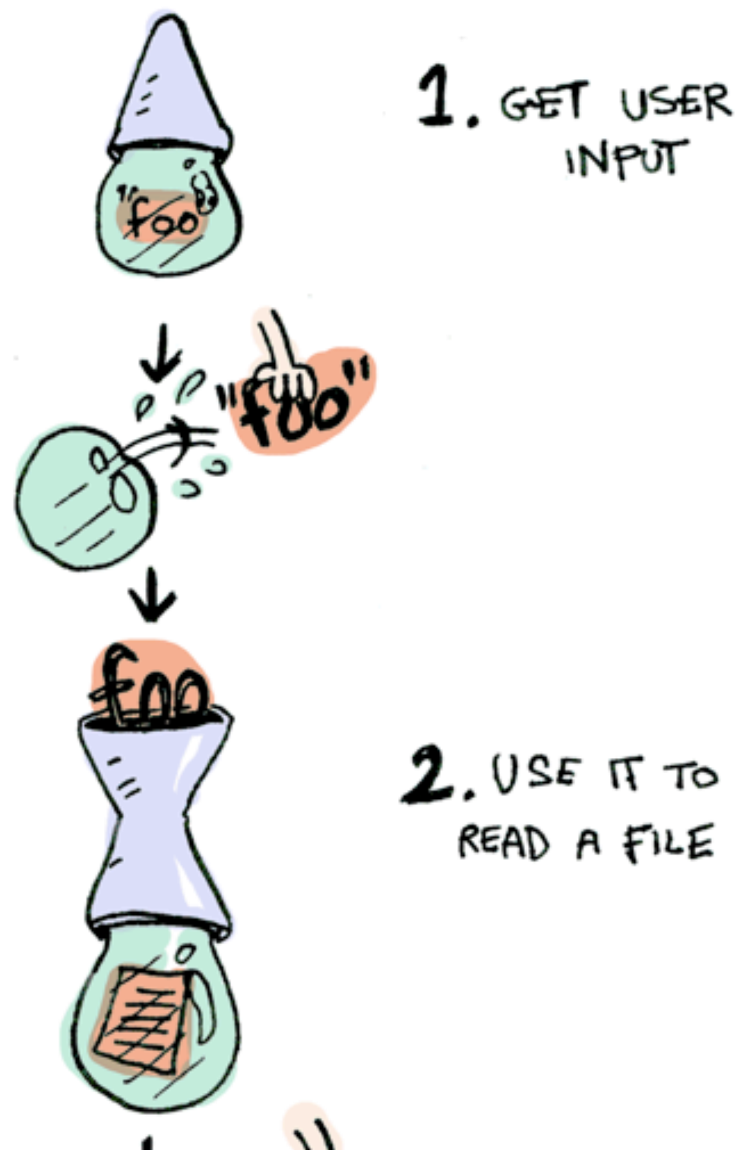



```
putStrLn :: String -> IO ()
```

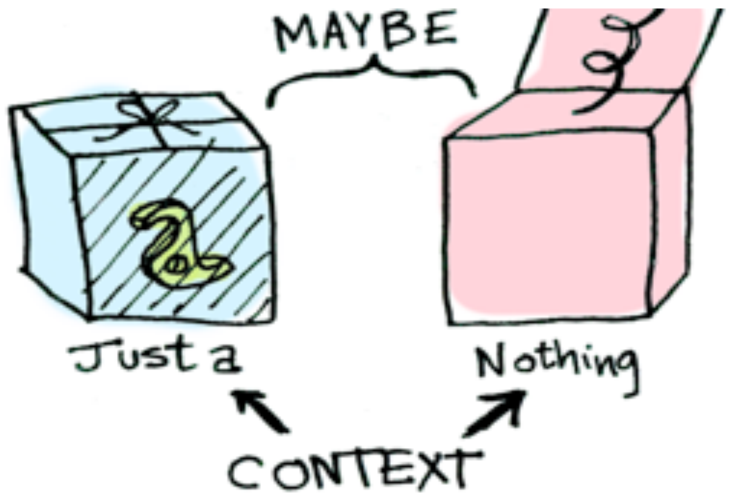
All three functions take a value (or no value) and produce an IO “wrapped” value

The function >>= allows us to join them together

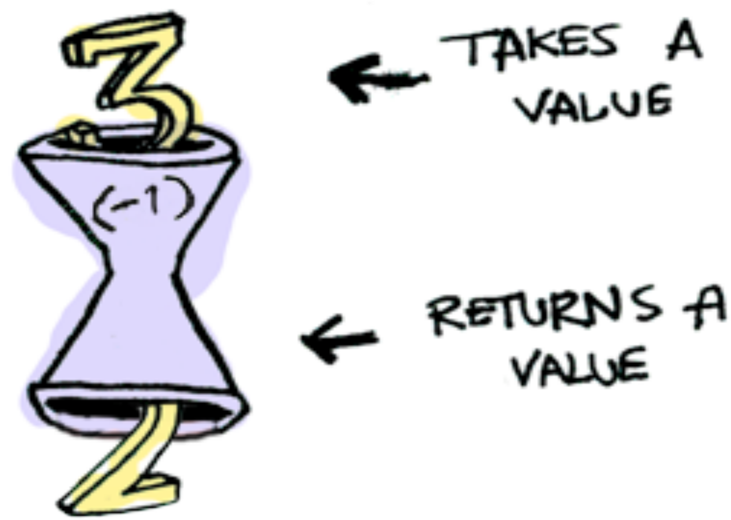
getLine >>= readFile >>= putStrLn


1. GET USER INPUT
"foo"
"foo"
foo
2. USE IT TO READ A FILE

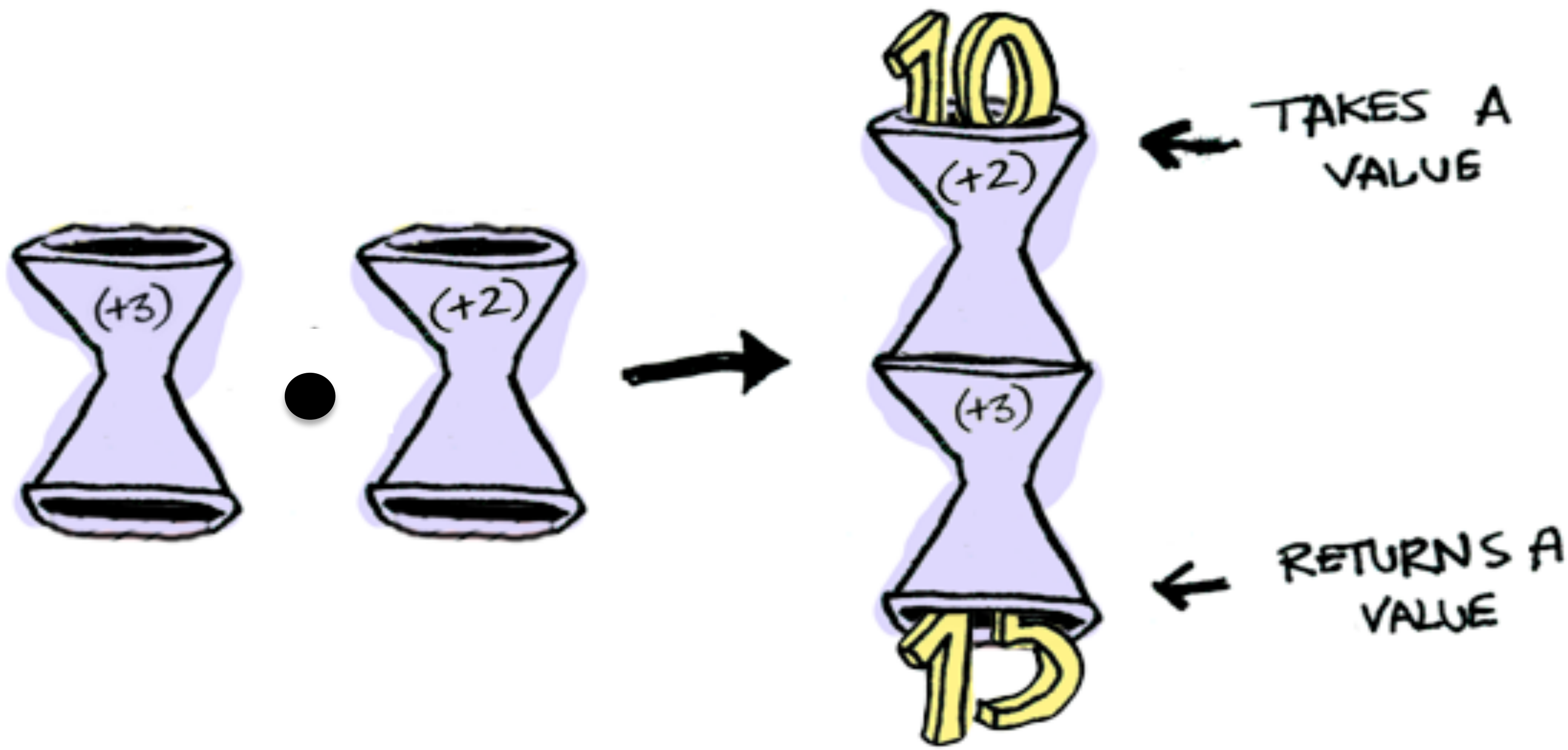
# Maybe



# Here is a function

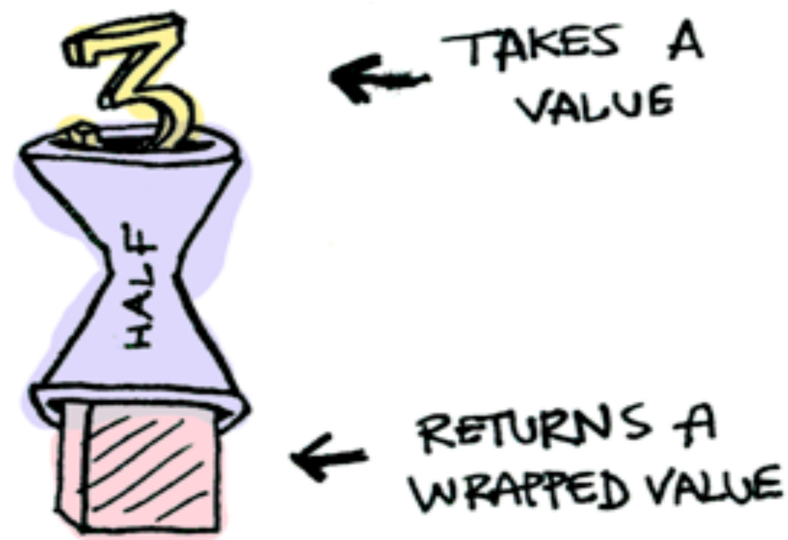


# They can be composed



# Here is a function

```
half x = if even x
            then Just (x `div` 2)
            else Nothing
```



# What if we feed it a wrapped value?


We need to use >>= to shove our wrapped value into the function

# >>=

# >>=

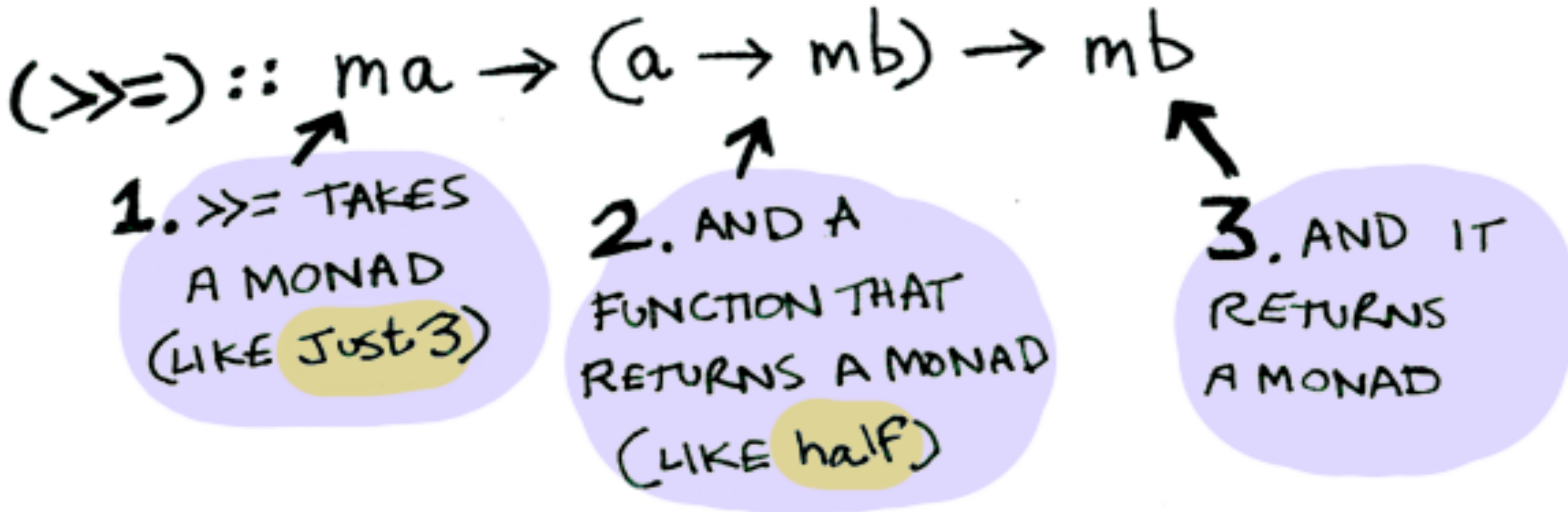
Here's how it works:

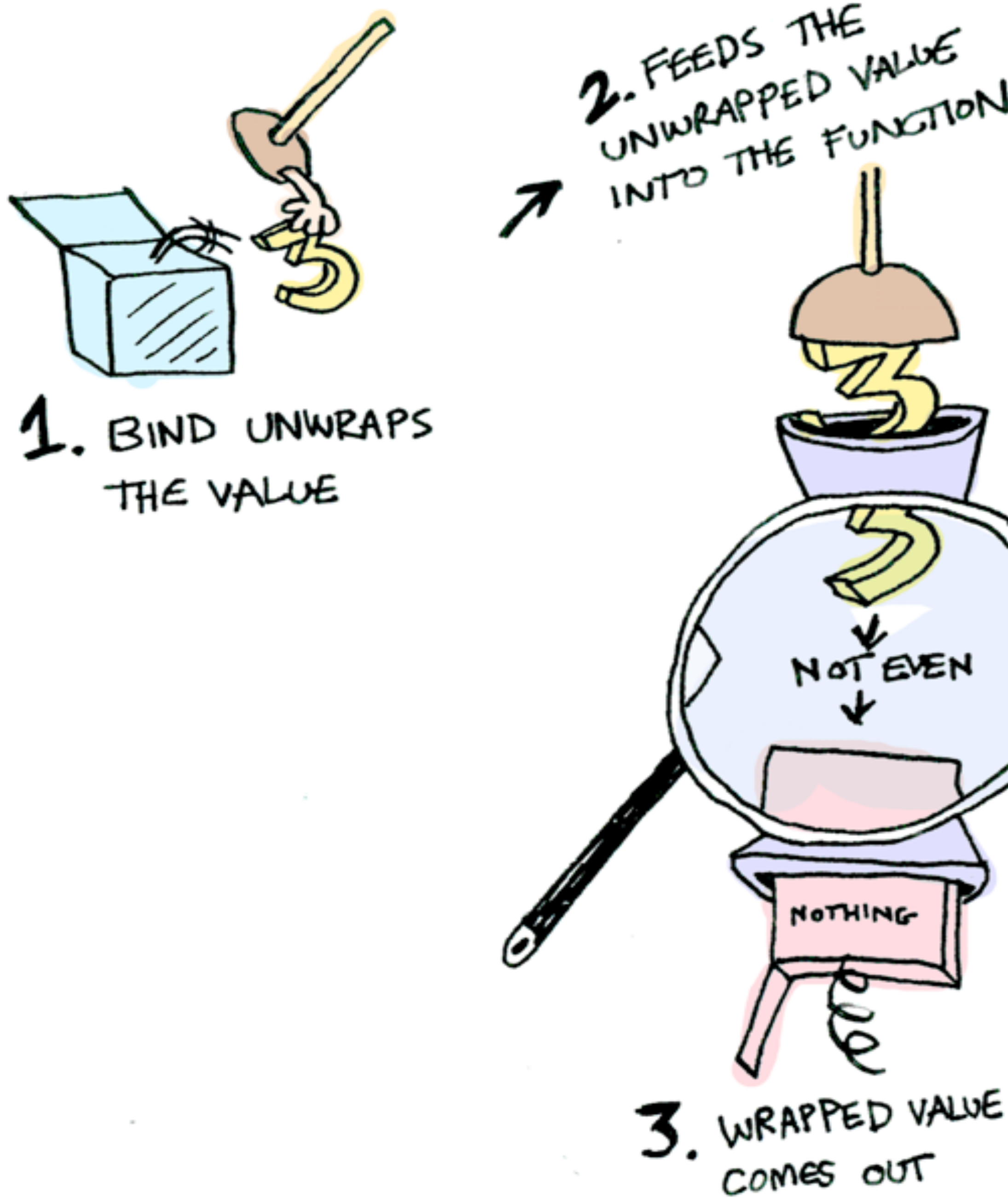
```
> Just 3 >>= half
Nothing
> Just 4 >>= half
Just 2
> Nothing >>= half
Nothing
```

What's happening inside? `Monad` is another typeclass. Here's a partial definition:

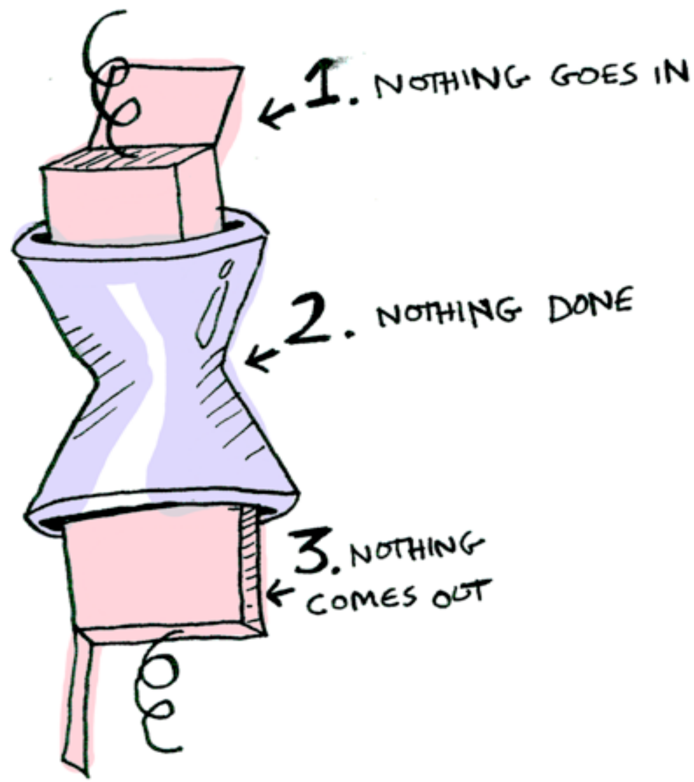
```
class Monad m where
    (>>=) :: m a -> (a -> m b) -> m b
```

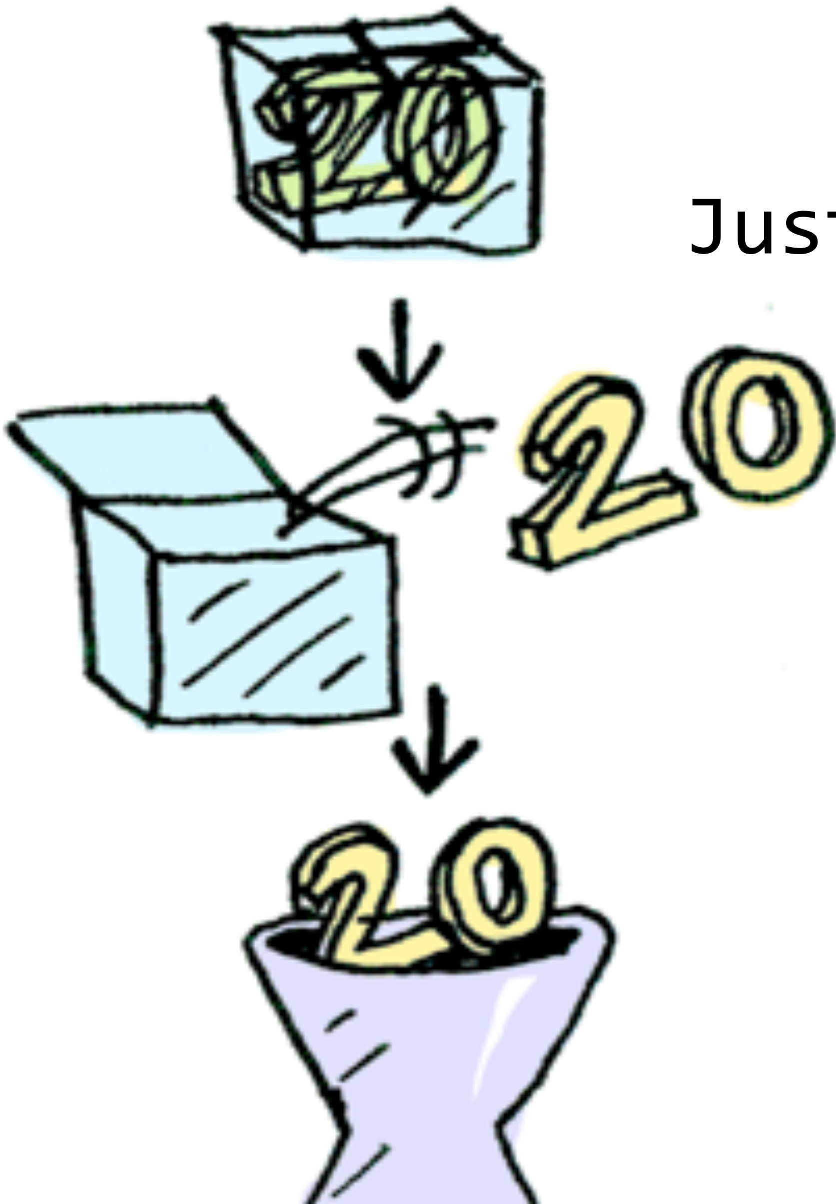
# >>=


(>>=) :: ma → (a → mb) → mb
1. >>= TAKES A MONAD (LIKE Just 3)
2. AND A FUNCTION THAT RETURNS A MONAD (LIKE half)
3. AND IT RETURNS A MONAD

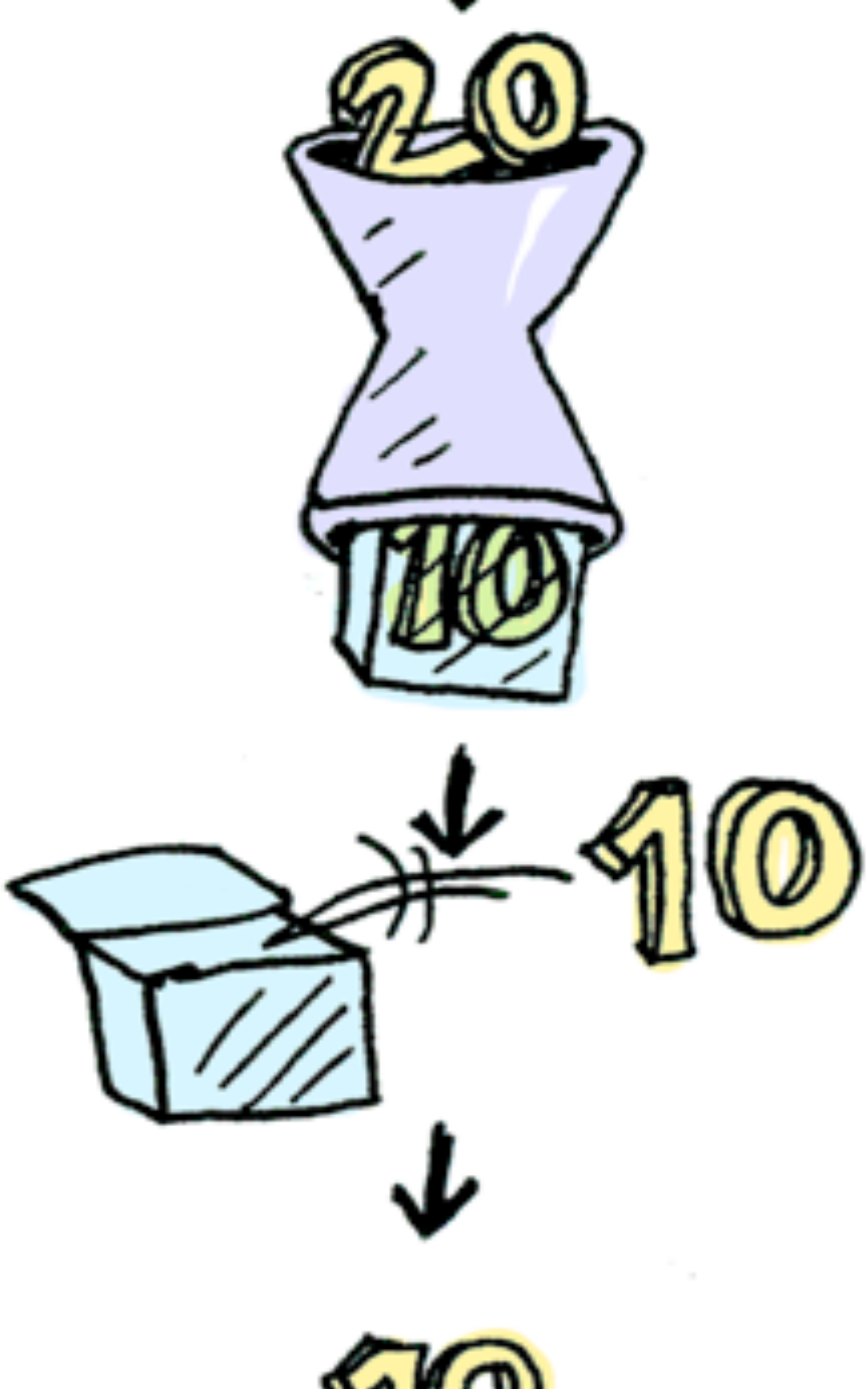

1. BIND UNWRAPS THE VALUE
2. FEEDS THE UNWRAPPED VALUE INTO THE FUNCTION
NOT EVEN
NOTHING
3. WRAPPED VALUE COMES OUT

# >>=


1. NOTHING GOES IN
2. NOTHING DONE
3. NOTHING COMES OUT




```
Just 20 >>= half >>= half
        >>= half
```


20
10
10


10
5
5

# Instance Monad Maybe

- Maybe is a very simple monad

```
instance Monad Maybe where
   Just x  >>= k = k x
   Nothing >>= _ = Nothing

   return        = Just
   fail s        = Nothing
```

Although simple it can be useful…

# Congestion Charge Billing

# Congestion Charge Billing

Registration number used to find the Personnummer of the owner

```
carRegister :: [(RegNr,PNr)]
```

Personnummer used to find the name of the owner

```
nameRegister :: [(PNr,Name)]
```

Name used to find the address of the owner

```
addressRegister :: [(Name,Address)]
```

# Example:
# Congestion Charge Billing

```
type CarReg = String ; type PNr = String
type Name = String ; type Address = String

carRegister :: [(CarReg,PNr)]
carRegister
 = [("JBD 007","750408-0909"), ...]

nameRegister :: [(PNr,Name)]
nameRegister
 = [("750408-0909","Dave“), ... ]

addressRegister :: [((Name,PNr),Address)]
addressRegister =
  [(("Dave","750408-0909"),"42 Streetgatan\n Askim")
  , ... ]
```

# Example: Congestion Charge Billing

With the help of

```
lookup :: Eq a => a -> [(a,b)] -> Maybe b
```

we can return the address of car owners

```
billingAddress :: CarReg -> Maybe (Name, Address)
billingAddress car =
 case lookup car carRegister of
   Nothing -> Nothing
   Just pnr -> case lookup pnr nameRegister of
        Nothing -> Nothing
        Just name ->
           case lookup (name,pnr) addressRegister of
                Nothing -> Nothing
                Just addr -> Just (name,addr)
```

# Example:
# Congestion Charge Billing

Using the fact that Maybe is a member of class Monad we can avoid the spaghetti and write:

```
billingAddress car = do
  pnr  <- lookup car carRegister
  name <- lookup pnr nameRegister
  addr <- lookup (name,pnr) addressRegister
  return (name,addr)
```

# Example:
# Congestion Charge Billing

Unrolling one layer of the do syntactic sugar:

```
billingAddress car ==
 lookup car carRegister >>= \pnr ->
 do
  name <- lookup pnr nameRegister
  addr <- lookup (name,pnr) addressRegister
  return (name,addr)
```

- `lookup car carRegister` gives `Nothing` then the definition of `>>=` ensures that the whole result is `Nothing`
- `return` is `Just`

# The Big Picture

Refactor/generalise

`ReadExpr.hs`

- “Brute force” parser.
- Big ugly case expressions.
- Minimal reuse.

Refactor

`RefactoredParser`

- Few basic bulding blocks (datatype dependent)
- Parser “Combinators”

`RefactoredReadExpr`

- A few lines of code

Alternative approach

`Parsing.hs`

- Parser as an instance of Monad

`ReadExprMonadic`

- A few lines of code

# Monad

- To be an instance of class Monad you need two operations: **>>=** and **return**

```
instance Monad Parser where
  return = succeed
  (>>=)  = (>*>)
   -- (>->) is equivalent to (>>)
```

- Why bother?

> •First example of a home-grown monad
> •Can understand and use do notation

# Summary

- We can use higher-order functions to build Parsers from other more basic Parsers.
- Parsers can be viewed as an instance of Monad
- We can build our own Monads!
  – A lot of ”plumbing” is nicely hidden away
  – The implementation of the Monad is not visible and can thus be changed or extended

# Parsing

- So far: how to write

```
readExpr :: String -> Maybe Expr
```

- Key idea:

```
type Parser = String -> Maybe (a, String)
```

- This lecture: Building Parsers; Parsers as a new type of ”instructions” – i.e. a monad.

# Recall some key building blocks

```
succeed :: a -> Parser a
succeed a = P $ \s -> Just(a,s)

sat :: (Char -> Bool) -> Parser Char

(>->) :: Parser a -> Parser b -> Parser b
(>*>) :: Parser a -> (a -> Parser b) -> Parser b
```

```
Main> parse (digit >*> \a -> sat (==a)) "22xx"
Just ('2',"xxx")
Main> parse (digit >*> \a -> sat (==a)) "12xx"
Nothing
```

# The Parser Monad

- Using these building blocks we can make Parser an instance of the class Monad
  - We get a language of “Parsing Instructions”
  - Another way to write Parsers using do notation
  - Deeper understanding of Monads

# Example

- recall doTwice

```
doTwice :: Monad m => m a -> m (a,a)
doTwice cmd =
  do a <- cmd
     b <- cmd
     return (a,b)
```

**Main>** parse (*doTwice digit) "9876"*
Just (('9','8'), "76")

# Example revisited: Parsing Expressions

```
expr :: Parser Expr
expr s1 = case parse num s1 of
             Just (a,s2) -> case s2 of
                              '+':s3 -> case parse expr s3 of
                                          Just (b,s4) -> Just (Add a b, s4)
                                          Nothing     -> Just (a,s2)
                              _       -> Just (a,s2)
             Nothing     -> Nothing
```

modified to use the new version of Parser type. Otherwise as before

Monadic style abstracts away from implementation of the Parser type

```
expr :: Parser Expr
expr  = do a <- num
           do char '+'
              b <- expr
              return (Add a b)
            +++ return a
```

# Parser Combinators

```
zeroOrMore, oneOrMore :: Parser a -> Parser [a]

zeroOrMore p = oneOrMore p +++ return []

oneOrMore p = do v <- p
                 vs <- zeroOrMore p
                 return(v:vs)
```

**Main>** parse (*oneOrMore digit*) *"9876+"*
Just ("9876","+")

**Combinator**: a function which take functions as arguments and produces a function as a result

# Parser Combinators

```
nat :: Parser Int -- Parses a non negative integer
nat =  do xs <- oneOrMore number
          return (read xs)

int :: Parser Int
int =  nat +++
       do char '-'
          n <- nat
          return (-n)
```

# Chain

```
chain p op f = P $ \s1 ->
    case parse p s1 of
     Just (a,s2) -> case s2 of
                      c:s3 | c == op -> case chain p op f s3 of
                                          Just (b,s4) -> Just (f a b, s4)
                                          Nothing     -> Just (a,s2)
                      _                    -> Just (a,s2)
     Nothing     -> Nothing
```

Old definition (modified to work with the new type)

```
chain' p op f = do v <- p
                   vs <- zeroOrMore (char op >> p)
                   return (foldr1 f (v:vs))
```

**Prelude.foldr1** : fold operation for lists with at least one element (no ”nil” case)

# Factor

```
factor :: Parser Expr
factor ('(':s) =
   case expr s of
      Just (a, ')':s1) -> Just (a, s1)
      _                    -> Nothing

factor s = num s
```

```
factor :: Parser Expr
factor = num +++
             do char '('
                e <- expr
                char ')'
                return e
```

**IO t**

- Instructions for interacting with operating system
- Run by GHC runtime system produce value of type t

**Gen t**

- Instructions for building random values
- Run by **quickCheck** to generate random values of type t

**Parser t**

- Instructions for parsing
- Run by **parse** to parse a string and **Maybe** produce a value of type t

# Three Monads

# Code

- Parsing.hs
  - module containing the parser monad and simple parser combinators.

- ReadExprMonadic.hs
  - A reworking of Read

See course home page

- We can build our own Monads!
  - A lot of ”plumbing” is nicely hidden away
  - A powerful pattern, used widely in Haskell
  - A pattern that can be used in other languages, but syntax support helps
    - F# computation expressions
    - Scala

# More examples

- http://adit.io/posts/2013-06-10-three-useful-monads.html
- stack (slides/video from last year)

# Another Example: A Stack

- A Stack is a stateful object
- Stack operations can push values on, pop values off, add the top elements

```
type Stack = [Int]
newtype StackOp t = StackOp (Stack -> (t,Stack))

-- the type of a stack operation that produces
-- a value of type t
pop :: StackOp Int
push :: Int -> StackOp ()
add :: StackOp ()
```

# Running a StackOp

```
type Stack = [Int]
newtype StackOp t = StackOp (Stack -> (t,Stack))

run (StackOp f) = f

-- run (StackOp f) state = f state
```

# Operations

```haskell
pop :: StackOp Int
pop = StackOp $ \(x:xs) -> (x,xs) -- can fail

push :: Int -> StackOp ()
push i = StackOp $ \s -> ((),i:s)

add :: StackOp ()
add = StackOp $ \(x:y:xs) -> ((),x+y:xs) -- can fail
```

# Building a new StackOp…

```
swap :: StackOp ()
swap = StackOp $ \s ->
          let (x,s') = run pop s
              (y,s'') = run pop s'
              (_,s''') = run (push x) s''
              (_,s'''') = run (push y) s''’
           in (_, s'''')
```

No thanks!

# StackOp is a Monad

- Stack instructions for producing a value

```haskell
-- (>>=) :: StackOp a -> (a -> StackOp b) -> StackOp b
instance Monad StackOp
  where return n = StackOp $ \s -> (n,s)
        sop >>= f  = StackOp $ \s ->
                         let (i,s') = run sop s
                         in run (f i) s'
```

# So now we can write…

```
swap = do
     a <- pop
     b <- pop
     push a
     push b
```

**Stack t**

- Stack instructions producing a value of type t
- Run by **run**

**Maybe t**

- Instructions for either producing a value or nothing
- Run by ?? (not an abstract data type)

# Two More Monads