Software Engineering using Formal Methods

Reasoning about Programs with Loops and Method Calls

Wolfgang Ahrendt

17 October 2014

Program Logic Calculus – Repetition

Calculus realises symbolic interpreter:

- works on first active statement

$$\Gamma \Longrightarrow \langle \texttt{i=j++;}\mathtt{\textbf{if}}\texttt{(isValid)\{ok=}\mathtt{\textbf{true}}\texttt{;\}...}\rangle\phi$$

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\hline
\dots \\
\hline
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\end{array} \\
\hline
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- decomposition of complex statements into simpler ones
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- control flow branching induces proof splitting
- application of update computes weakest precondition of $\mathcal{U}'$ wrt. ϕ

$$
\begin{array}{c}
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\ldots \qquad\qquad \ldots \\
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- all components of an array `arr` of length 2 are set to 0?

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  $\{\mathtt{arr[0]} := 0 \parallel \mathtt{arr[1]} := 0\}\phi$
- all components of an array `arr` of length `n` are set to 0?

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  $$\{\mathtt{arr[0]} := 0 \parallel \mathtt{arr[1]} := 0\}\phi$$
- all components of an array `arr` of length `n` are set to 0?

For example to deal with things like

$$\langle \mathtt{a = \textbf{new} \ \textbf{int}[n];} \rangle \, \forall\, \mathbf{int}\ x; (0 \leq x < \mathtt{n} \rightarrow \mathtt{a}[x] \doteq 0)$$

Quantified Updates

Definition (Quantified Update)

For T well-ordered type (no ∞ descending chains): quantified update:

$$\{\texttt{\textbackslash for}\ T\ x;\ \texttt{\textbackslash if}\ \phi(x);\ l(x) := r(x)\}$$

- For all objects d in T such that $\phi(d)$ perform the updates $\{l(d) := r(d)\}$ in parallel
- If there are several l with conflicting d then choose T-minimal one

- The conditional expression is optional
- Typically, x occurs in ϕ, l, and r (but doesn't need to)
- There is a normal form for updates computed efficiently by KeY

Quantified Updates Cont'd

Example (Initialization of field `a` for all objects in class `C`)

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Example (Initialization of components of array `a`)

$$\{\texttt{\textbackslash for}\ \texttt{int}\ i;\ \mathtt{a}[i] := 0\}$$

Example (Integer types are well-ordered in KeY)

$$\{\texttt{\textbackslash for}\ \texttt{int}\ i;\ \mathtt{a}[0] := i\}(\mathtt{a}[0] \doteq 0)$$

- Non-standard order for $\mathbb{Z}$ (with 0 smallest and preserving $<$ for arguments of same sign)
- Proven automatically by update simplifier

Loop Invariants

Symbolic execution of loops: unwind

$$\text{unwindLoop}\ \frac{\Gamma \Longrightarrow \mathcal{U}[\pi\, \texttt{\textbf{if}(b)\{p; \textbf{while}(b) p\}}\, \omega]\phi, \Delta}{\Gamma \Longrightarrow \mathcal{U}[\pi\, \texttt{\textbf{while}(b) p}\, \omega]\phi, \Delta}$$

(We omitted $\mathcal{U}$ previous lectures, for simplicity.)

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How to handle a loop with. . .

- 0 iterations?

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How to handle a loop with...

- 0 iterations? Unwind 1×

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How to handle a loop with...

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- 10 iterations?

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- 0 iterations? Unwind 1×
- 10 iterations? Unwind 11×

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- 10000 iterations?

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We need an invariant rule (or some form of induction)

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Idea behind loop invariants

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- Relevant context information must be added to *Inv* ☹

Example

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int i = 0;
while(i < a.length) {
    a[i] = 1;
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(Implicit) Class Invariant: $a \doteq\!\!\!\!/\ \mathtt{\textbf{null}}$

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$\mathcal{V} = \{\mathtt{i} := c \parallel \mathtt{\backslash for}\ x;\ \mathtt{a}[x] := f(x)\}$
(c, f fresh constant resp. function symbol)

Loop Invariants Cont'd

Improved Invariant Rule

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Loop Invariants Cont'd

Improved Invariant Rule

$$\frac{\begin{array}{c}\Gamma \Longrightarrow \mathcal{U}\mathit{Inv}, \Delta \\ \Gamma \Longrightarrow \mathcal{U}\mathcal{V}(\mathit{Inv}\ \&\ b \doteq \mathtt{TRUE} \rightarrow [\mathrm{p}]\mathit{Inv}), \Delta \\ \Gamma \Longrightarrow \mathcal{U}\mathcal{V}(\mathit{Inv}\ \&\ b \doteq \mathtt{FALSE} \rightarrow [\pi\ \omega]\phi), \Delta\end{array}}{\Gamma \Longrightarrow \mathcal{U}[\pi\, \mathtt{\textbf{while}}\,\mathtt{(b)\ p}\,\omega]\phi, \Delta} \quad \begin{array}{l}\text{(valid when entering loop)} \\ \text{(preserved by p)} \\ \text{(assumed after exit)}\end{array}$$

Loop Invariants Cont'd

Improved Invariant Rule

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- Context is kept as far as possible
- Invariant not 'responsible' for un-assignable locations
- Missing assignable clause (equiv. to **`assignable \everything`**):
  - $\mathcal{V} = \{* := *\}$ wipes out **all** information
  - Equivalent to basic invariant rule
  - Avoid this! Always give a specific **`assignable`** clause

Example with Improved Invariant Rule

```
int i = 0;
while(i < a.length) {
    a[i] = 1;
    i++;
}
```

Example with Improved Invariant Rule

(Implicit) Class Invariant: $\mathtt{a} \doteq\!\!\!\!/\ \mathbf{null}$

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int i = 0;
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Example with Improved Invariant Rule

(Implicit) Class Invariant: $\mathtt{a} \mathrel{\dot{\neq}} \mathbf{null}$

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int i = 0;
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Postcondition: $\forall\, \mathbf{int}\ x;\ (0 \leq x < \mathtt{a.length} \rightarrow \mathtt{a}[x] \doteq 1)$

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Loop invariant: $0 \leq \texttt{i}\ \ \&\ \ \texttt{i} \leq \texttt{a.length}$

$\&\ \ \forall\,\textbf{int}\ x;\ (0 \leq x < \texttt{i} \rightarrow \texttt{a[}x\texttt{]} \doteq 1)$

Example with Improved Invariant Rule

(Implicit) Class Invariant: $\mathtt{a} \not\doteq \mathbf{null}$

```
int i = 0;
while(i < a.length) {
    a[i] = 1;
    i++;
}
```

Postcondition: $\forall\,\mathbf{int}\ x;\ (0 \leq x < \mathtt{a.length} \rightarrow \mathtt{a}[x] \doteq 1)$

Loop invariant: $0 \leq \mathtt{i}\ \ \&\ \ \mathtt{i} \leq \mathtt{a.length}$
$\&\ \ \forall\,\mathbf{int}\ x;\ (0 \leq x < \mathtt{i} \rightarrow \mathtt{a}[x] \doteq 1)$

Example with Improved Invariant Rule

(Implicit) Class Invariant: $\mathtt{a} \not\doteq \mathbf{null}$ not needed for loop invariant

```
int i = 0;
while(i < a.length) {
    a[i] = 1;
    i++;
}
```

Postcondition: $\forall\, \mathbf{int}\ x;\ (0 \leq x < \mathtt{a.length} \rightarrow \mathtt{a[}x\mathtt{]} \doteq 1)$

Loop invariant: $0 \leq \mathtt{i}\ \ \&\ \ \mathtt{i} \leq \mathtt{a.length}$

$\&\ \ \forall\, \mathbf{int}\ x;\ (0 \leq x < \mathtt{i} \rightarrow \mathtt{a[}x\mathtt{]} \doteq 1)$

Example in JML/JAVA – `Loop.java`

```
public int[] a;
/*@ public normal_behavior
  @  ensures (\forall int x; 0<=x && x<a.length; a[x]==1);
  @  diverges true;
  @*/
public void m() {
  int i = 0;
  /*@ loop_invariant
    @  (0 <= i && i <= a.length &&
    @   (\forall int x; 0<=x && x<i; a[x]==1));
    @ assignable i, a[*];
    @*/
  while(i < a.length) {
    a[i] = 1;
    i++;
  }
}
```

Example from last week

$\forall$ `int` x;
  $(x \doteq \mathtt{n} \land x >= 0 \rightarrow$
    $[$ `i = 0; r = 0;`
      `while` `(i<n) { i = i + 1; r = r + i;}`
      `r=r+r-n;`
    $]\mathtt{r} \doteq ?)$

How can we prove that the above formula is valid
(i.e. satisfied in all states)?

Example from last week

$$\begin{array}{l}
\forall\ \mathtt{int}\ x; \\
\quad (x \doteq \mathtt{n} \land x >= 0 \rightarrow \\
\qquad [\ \mathtt{i\ =\ 0;\ r\ =\ 0;} \\
\qquad\ \ \mathbf{\mathtt{while}}\ \mathtt{(i<n)\ \{\ i\ =\ i\ +\ 1;\ r\ =\ r\ +\ i;\}} \\
\qquad\ \ \mathtt{r=r+r-n;} \\
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How can we prove that the above formula is valid
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Solution:

```
@ loop_invariant
@     i>=0 && 2*r == i*(i + 1) && i <= n;
@ assignable i, r;
```

Example from last week

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Solution:

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@ loop_invariant
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File: Loop2.java

Hints

Proving `assignable`

- The invariant rule assumes that **`assignable`** is correct
  E.g., with **`assignable \nothing;`** one can prove nonsense
- Invariant rule of KeY generates proof obligation that ensures correctness of **`assignable`**

Hints

Proving `assignable`

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Setting in the KeY Prover when proving loops

- Loop treatment: Invariant
- Quantifier treatment: No Splits with Progs
- If program contains `*`, `/`:
  Arithmetic treatment: DefOps
- Is search limit high enough (time out, rule apps.)?
- When proving partial correctness, add **`diverges true;`**

Total Correctness

Find a decreasing integer term v (called variant)

Add the following premisses to the invariant rule:

- $v \geq 0$ is initially valid
- $v \geq 0$ is preserved by the loop body
- v is strictly decreased by the loop body

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Proving termination in JML/JAVA

- Remove directive `diverges true;`
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- KeY uses suitable invariant rule and PO (with $\langle \ldots \rangle \phi$)

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Example (The `array` loop)

```
@ decreasing
```

Total Correctness

Find a decreasing integer term v (called variant)

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Example (The `array` loop)

```
@ decreasing  a.length - i;
```

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Example (The `array` loop)

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Files:

- `LoopT.java`
- `Loop2T.java`

Method Calls – Repetition

Method Call with actual parameters $arg_0, \ldots, arg_n$

$$\{arg_0 := t_0 \,||\, \ldots \,||\, arg_n := t_n \,||\, c := t_c\}\langle c.\mathtt{m}(arg_0, \ldots, arg_n);\rangle\phi$$

where $\mathtt{m}$ declared as $\mathtt{\textbf{void}}\ \mathtt{m}(\mathrm{T}_0\ \mathrm{p}_0, \ldots, \mathrm{T}_\mathrm{n}\ \mathrm{p}_\mathrm{n})$

Actions of rule methodCall

- for each formal parameter $\mathtt{p_i}$ of `m`:
  declare and initialize new local variable $\mathtt{T_i}\ \mathtt{p\#i} = arg_i;$

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  if implementation cannot be uniquely determined

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Actions of rule methodCall

- for each formal parameter $\mathtt{p_i}$ of `m`:
  declare and initialize new local variable $\mathtt{T_i}\ \mathtt{p\#i} = arg_i;$
- look up implementation class C of `m`, or split proof
  if implementation cannot be uniquely determined
- create method invocation $c.\mathtt{m}(\mathtt{p\#0}, \ldots, \mathtt{p\#n})@C$

Method Calls Cont'd

Method Body Expand

1. Execute code that binds actual to formal parameters $\mathtt{T_i}$ `p#i` $=arg_i$;
2. Apply rule methodBodyExpand

$$\frac{\Gamma \Longrightarrow \langle \pi\ \mathtt{method\text{-}frame(source{=}C,\ this{=}c)\{}\ \mathtt{body}\ \mathtt{\}}\ \omega\rangle\phi, \Delta}{\Gamma \Longrightarrow \langle \pi\ \mathtt{c.m(p\#0,\ldots,p\#n)@C;}\ \omega\rangle\phi, \Delta}$$

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Symbolic Execution

Only static information available, proof splitting;

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File: `inlineDynamicDispatch.key`

Problem

Formal specification of JAVA API and other called methods

How to perform symbolic execution when JAVA API method is called?

1. Method has reference implementation in JAVA
   Inline method body and execute symbolically

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   Problems Reference implementation not always available
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   Impossible to deal with recursion

2. Use method contract instead of method implementation

Method Contract Rule – Normal Behavior Case

Warning: Simplified version

```
/*@ public normal_behavior
  @ requires normalPre;
  @ ensures normalPost;
  @ assignable mod;
  @*/
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- $\mathcal{F}(\cdot)$: translation to Java DL (see last lecture)

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- $\mathcal{V}_{mod}$: anonymising update (similar to loops)

Method Contract Rule – Exceptional Behavior Case

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```
/*@ public exceptional_behavior
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KeY uses actually only one rule for both kinds of cases.

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Therefore translation of postcondition ϕ_{post} as follows (simplified):

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\phi_{post\_e} &\equiv \mathcal{F}(\texttt{\textbackslash old}(\texttt{excPre})) \wedge \mathcal{F}(\texttt{excPost})
\end{aligned}
$$

$$
\frac{\begin{array}{ll}
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\Gamma \Longrightarrow \mathcal{U}\mathcal{V}_{mod_{normal}}(\phi_{post\_n} \rightarrow \langle \pi\, \omega \rangle \phi), \Delta & \text{(normal)}
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Understanding Proof Situations

Reasons why a proof may not close

- bug or incomplete specification
- bug in program
- maximal number of steps reached: restart or increase # of steps
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Understanding open proof goals

- follow the taken control-flow from the root to the open goal
- branch labels may give useful hints
- identify (part of) the post-condition or invariant that cannot be proven
- sequent remains always in "pre-state". I.e., constraints like $\mathtt{i} \geq 0$ refer to the value of `i` before executing the program (exception: sub-formulae prefixed by update or modality)
- remember: $\Gamma \Longrightarrow \mathtt{o} \doteq \mathbf{\mathtt{null}}, \Delta$ is equivalent to $\Gamma, \mathtt{o} \not\doteq \mathbf{\mathtt{null}} \Longrightarrow \Delta$

Summary

- Most JAVA features covered in KeY
- Degree of automation for loop-free programs is high
- Proving loops requires user to provide invariant

Literature for this Lecture

Essential

KeY Book	Verification of Object-Oriented Software (see course web page), Chapter 10: Using KeY
KeY Book	Verification of Object-Oriented Software (see course web page), Chapter 3: Dynamic Logic, Sections 3.1, 3.2, 3.4, 3.5, 3.6.1, 3.6.2, 3.6.3, 3.6.4, 3.6.5, 3.6.7, 3.7