Finite Automata Theory and Formal Languages
TMV027/DIT321 – LP4 2013

Turing Machines

Week 8

1. Given the alphabet $\Sigma = \{I\}$, the Natural number n can be represented in a tape as n consecutive occurrences of I.

   Give both a high-level description and a state-transition diagram of a Turing machine computing the following operations on Natural numbers:

   (a) Successor and predecessor;

   (b) Addition and subtraction;

   (c) Multiplication.

   Consider the input on the tape of the form $\sharp n$ or $\sharp n \sharp m$, with $n, m \in \Sigma^*$, depending on the problem.

2. For each of the below languages L_i, give both a high-level description and a state-transition diagram of a Turing machine for L_i.

   In each case, state whether your Turing machine is also a Turing decider or not.

   (a) $L_1 = \{\sharp w_1 \sharp w_2 \mid w_1, w_2 \in \{0,1\}^* \text{ and } w_1 \neq w_2\}$;

   (b) $L_2 = \{\sharp w_1 \sharp w_2 \mid w_1, w_2 \in \{0,1\}^* \text{ and } \text{length}(w_1) < \text{length}(w_2)\}$;

   (c) $L_3 = \{\sharp w_1 \sharp w_2 \mid w_1, w_2 \in \{0,1\}^* \text{ and } \text{length}(w_1) = \text{length}(w_2)\}$;

   (d) $L_4 = \{\sharp w \sharp w^{\mathsf{r}} \mid w \in \{0,1\}^*\}$, where w^{r} stands for the reverse of w;

   (e) $L_5 = \{\sharp 0^i \sharp 1^j \sharp 2^k \mid k = i * j\}$;

   (f) $L_6 = \{\sharp 0^i \sharp 1^j \mid j = i^2\}$.