Finite Automata and Formal Languages

TMV026/DIT321

Friday 27th of August 2010

CTH: Total 60 points: $\geq$ 26: 3, $\geq$ 38: 4, $\geq$50: 5
GU: Total 60 points $\geq$26: G, $\geq$42: VG

No help material.

Answers can be written in English or Swedish. Write as clear as possible.

All answers should be well motivated. Points will be deduced when you give an unnecessarily complicated solution or when you do not properly justify your answer.

1. (5pts) Prove the following statement using mathematical induction:

$$\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$$

   Do not forget to clearly state the base case and the inductive hypothesis!

2. (3.5pts) Define a deterministic finite automata accepting the language over $\{0,1\}$ not containing the sub-word 0101.

3. (4.5pts) Convert the following non-deterministic finite automata with ϵ-transitions to an equivalent deterministic finite automata.

	a	b	ϵ
$\rightarrow q_0$	$\{q_1\}$	$\emptyset$	$\{q_2\}$
q_1	$\{q_1\}$	$\{q_2, q_3\}$	$\emptyset$
q_2	$\{q_3\}$	$\{q_2\}$	$\emptyset$
$^*q_3$	$\emptyset$	$\emptyset$	$\emptyset$

4. (5pts) Compute, by eliminating the states in the automaton, a regular expression that generates exactly the language accepted by the following deterministic finite automata.

	a	b
$\rightarrow q_0$	q_1	q_2
q_1	q_3	q_2
q_2	q_1	q_3
$^*q_3$	$-$	$-$

5. Give regular expressions that generate exactly the following languages. Justify your answer.
   (a) (2pts) $\mathcal{L}((a+b)a^*) \cap \mathcal{L}(baa^*)$.
   (b) (2pts) $\{0,1\}^* - \mathcal{L}((0+1)^*10)$.
   (c) (2pts) The words over $\{0,1\}$ that contain exactly one pair of consecutive 0's.

6. (5pts) Do these two regular expressions represent the same language? Justify your answer.
   (a) $b + ab^* + aa^*b + aa^*ab^*$ and $a^*(b + ab^*)$?
   (b) $(ab + a)^*ab$ and $(aa^*b)^*$?

7. (a) (1.5pts) What is formally a regular language?
   (b) (1.5pts) Explain informally why the language $\{0^n1^n2^n \mid n > 0\}$ is not regular.
   (c) (3pts) State the Pumping lemma for regular languages and use it to formally prove that the language $\{0^n1^n2^n \mid n > 0\}$ is not regular.

8. (a) (4.5pts) Write a context-free grammar that generates the language

$$\{a^m b^n c^k \mid (k = m + n, k > 0) \text{ or } (n = k + m, n > 0)\}$$

   Explain the grammar
   (b) (1.5pts) Is this grammar ambiguous? Justify.

9. Consider the following context-free grammar with starting symbol S:

$$\begin{array}{ll} S \to a \mid aA \mid B \mid C & \\ A \to aB \mid \epsilon & B \to Aa \\ C \to cCD & D \to ddd \end{array}$$

   (a) (7.5pts) Simplify the grammar by successively applying the following steps (that is, steps ii. to iv. should be performed to the grammar obtained in steps i. to iii. respectively):
      i. Identify and eliminate the ϵ-productions.
      ii. Identify and eliminate the unit productions.
      iii. Identify and eliminate the non-generating symbols.
      iv. Identify and eliminate the non-reachable symbols.
   (b) (2pts) Which is the language generated by this grammar?
   (c) (1.5pts) Is this grammar ambiguous? Justify.

10. (4pts) Consider the following context-free grammar G with starting symbol S:

$$\begin{array}{ll} S \to AB \mid BC & A \to BA \mid a \\ B \to CC \mid b & C \to AB \mid a \end{array}$$

   Apply the CYK algorithm to determine whether $baaab \in \mathcal{L}(G)$ or not. Show the resulting table and justify your answer.

11. (4pts) Consider the following Turing machine defined by the tuple $(\{q_0, q_1\}, \{0, 1, a, b\}, \delta, q_0, \square, \{q_1\})$, where the transition function δ is as following:

$$\begin{array}{l} \delta(q_0, 0) = (q_0, a, \mathsf{R}) \\ \delta(q_0, 1) = (q_0, b, \mathsf{R}) \\ \delta(q_0, \square) = (q_1, \square, \mathsf{R}) \end{array}$$

   What does this Turing machine do? When does it stop? Explain as much as you can.