Finite Automata and Formal Languages

TMV026/DIT321

Thursday 27th of May 2010

CTH: Total 60 points: $\geq$ 26: 3, $\geq$ 38: 4, $\geq$50: 5
GU: Total 60 points $\geq$26: G, $\geq$42: VG

No help material.

Answers can be written in English or Swedish. Write as clear as possible.

All answers should be well motivated. Points will be deduced when you give an unnecessarily complicated solution or when you do not properly justify your answer.

1. (6pts) Consider the following context-free grammar with start symbol S:

$$S \to 0S1S \mid 1S0S \mid \epsilon$$

Prove using induction that if $w \in \{0,1\}^*$ and $S \Rightarrow^* w$ then w has the same number of 0's than of 1's.

(Example: the word 01101 has 2 0's and 3 1's.)

2. (3pts) Define a deterministic finite automata accepting the language over $\{0,1\}$ not containing the strings with at least 3 consecutive 0's.

3. (6pts) Minimise the following automaton.

	a	b
$\to q_0$	q_3	q_5
q_1	q_6	q_3
q_2	q_6	q_4
q_3	q_6	q_6
$^*q_4$	q_0	q_5
$^*q_5$	q_2	q_4
q_6	q_1	q_6

Show the table that identifies the distinguishable states and justify the construction of the automaton.

4. (a) (4pts) Convert the following non-deterministic finite automata to an equivalent deterministic finite automata.

	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_2\}$
q_1	$\{q_3, q_4\}$	$\{q_1, q_2\}$
q_2	$\{q_1, q_2\}$	$\{q_3, q_4\}$
$^*q_3$	$\{q_3\}$	$\{q_4\}$
$^*q_4$	$\{q_4\}$	$\{q_3\}$

   (b) (2.5pts) Describe with words the language accepted by the deterministic finite automata you constructed.

   (c) (2.5pts) Give a regular expression which generates the language you described in 4b).

5. Do these two regular expressions represent the same language? Justify your answer.

   (a) (2.5pts) $(a + b)^*$ and $(a^*b^*)^*$?

   (b) (2.5pts) $a^*(a + b)^*$ and $(a + b)^*$?

6. (5pts) Which of the following languages are regular? Give a regular expression or use the Pumping lemma for regular languages to justify your answer.

   (a) $\mathcal{L} = \{b^n a^{3m} \mid n \geqslant 0, m \geqslant 0\}$

   (b) $\mathcal{L} = \{a^n b^{3n} \mid n \geqslant 0\}$

7. Give examples of languages $\mathcal{L}_1$ and $\mathcal{L}_2$ such that

   (a) (2.5pts) $\mathcal{L}_1$ is regular, $\mathcal{L}_2$ is not regular and $\mathcal{L}_1 - \mathcal{L}_2$ is regular.

   (b) (2.5pts) $\mathcal{L}_1$ and $\mathcal{L}_2$ are not regular but $\mathcal{L}_1 \cup \mathcal{L}_2$ is regular.

8. (a) (5pts) Give a context-free grammar that generates the language $\{a^i b^j c^k \mid i, j, k > 0,\ (i > k \text{ or } i < j)\}$. Explain your grammar!

   (b) (2pts) Is the grammar ambiguous? Justify.

9. Consider the following context-free grammar with starting symbol S:

$$S \rightarrow aSb \mid aSbb \mid ab \mid abb$$

   (a) (2pts) Describe informally the language generated by the grammar.

   (b) (2pts) Is the grammar ambiguous? Justify.

   (c) (2pts) Convert the grammar to an equivalent grammar in Chomsky Normal Form.

   (d) (4pts) Apply the CYK algorithm to see if the word *aabbb* belongs to the language generated by the grammar. Show the table and justify your answer.

10. (4pts) Define formally what a Turing machine is. Describe informally how it works.