Finite Automata Theory and Formal Languages
TMV027/DIT321 – LP4 2013

DFA and NFA

Week 3 and partly 4

In these exercises, book sections, exercise numbers and pages refer to those in the third edition of the course book.

Those of you that are interested can implement the DFA or the NFA below.

DFA

1. Build a DFA that recognises the words over $\{0,1\}^*$ ending with the string 0100.

2. if $\mathcal{L} \subseteq \Sigma^*$ is a language, we define $\mathsf{Prefix}(\mathcal{L})$ to be the set of words that are prefix of a word in $\mathcal{L}$. Show that if $\mathcal{L}$ is regular then so is $\mathsf{Prefix}(\mathcal{L})$.

3. Let $\Sigma = \{0,1\}$ and $\mathsf{Bool} = \{\mathsf{True}, \mathsf{False}\}$. We define $f, g, h : \Sigma^* \to \mathsf{Bool}$ as follows:

$$\begin{array}{lll} f\ \epsilon = \mathsf{False} & g\ \epsilon = \mathsf{False} & h\ \epsilon = \mathsf{True} \\ f\ (0x) = g\ x & g\ (0x) = h\ x & h\ (0x) = f\ x \\ f\ (1x) = h\ x & g\ (1x) = f\ x & h\ (1x) = h\ x \end{array}$$

   Show that the language $\{x \in \Sigma^* \mid f\ x = \mathsf{True}\}$ is a *regular* language.

4. Suppose $\Sigma = \{a, b\}$. Build a DFA that accepts the words containing *bba* as a subword. Build then a DFA that accepts the words *not* containing *bba* as a subword.

5. Let $\Sigma = \{a, b, c\}$. Build a DFA D_1 that accepts the words containing *ac* as a subword. Build a DFA D_2 that accepts the words containing *ab* as a subword.

   Using the product construction, build then a DFA that accepts the words containing both *ab* and *ac* as subwords, and another DFA that accepts the words containing *ac* but *not ab* as a subword.

6. In a factory, we have the possible events a, b, c. A constraint L_1 is that if the event b occurs after the event a, then the event c should occur in between. Represent this constraint L_1 as a DFA D_1.

   Suppose that there is another constraint L_2 stating that if the event b occurs after the event c, then the event a should occur in between.

   Explain intuitively why, if we have both constraints L_1 and L_2 then the event b cannot occur after the event a.

Represent the constraint L_2 as a DFA D_2.

Do the product construction of D_1 and D_2 produce an automaton representing the conjunction of the constraint L_1 and L_2? Verify on this automaton that b cannot occur after a or c.

7. Do exercises 2.2.1, 2.2.4, 2.2.7, 2.2.8, 2.2.9, 2.2.10 and 2.2.11.

NFA and ϵ-NFA

1. Show that if $\mathcal{L} \subseteq \Sigma^*$ is regular then so is $\mathcal{L}^R = \{rev(x) \mid x \in \mathcal{L}\}$.

   (Hint: given an automaton for $\mathcal{L}$ build a NFA for $\mathcal{L}^R$)

2. A ship attempts to transmit data to shore at random intervals. The receiver must continually listen and recognise when an actual transmission starts so that it can record the data that follows. Let us assume that the start of the transmission is signaled by the string 010010 and the end of the transmission is signaled by the string 000111. Represent this behaviour with a DFA.

3. Do exercises 2.3.1, 2.3.2 and 2.3.3.

4. Do exercises 2.5.1, 2.5.2 and 2.5.3.