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Global scheduling

General characteristics:

- All ready tasks are kept in a common (global) queue
- When selected for execution, a task can be dispatched to an arbitrary processor, even after being preempted
- Task execution is assumed to be ”greedy”:
  - If higher-priority tasks occupy all processors, a lower-priority task cannot grab a processor until the execution of a higher-priority task is complete.

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Global scheduling

Advantages:

- Supported by most multiprocessor operating systems
  - Windows NT, Solaris, Linux, ...
- Effective utilization of processing resources
  - Unused processor time can easily be reclaimed

Disadvantages:

- Weak theoretical framework
  - Few results from the uniprocessor case can be used
- Poor resource utilization for hard timing constraints
  - No more than 50% resource utilization can be guaranteed
- Suffers from several scheduling anomalies
  - Sensitive to period adjustments

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Global scheduling

Complexity of schedulability analysis for global scheduling: (Leung & Whitehead, 1982)

> The problem of deciding if a task set is schedulable on m processors with respect to global scheduling is NP-complete in the strong sense.

Consequence:

There can only exist a pseudo-polynomial time algorithm for
(i) finding an optimal static priority assignment, or
(ii) feasibility testing

But not both at the same time!

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Global scheduling

The ”root of all evil” in global scheduling: (Liu, 1969)

> Few of the results obtained for a single processor generalize directly to the multiple processor case; bringing in additional processors adds a new dimension to the scheduling problem. The simple fact that *a task can use only one processor even when several processors are free at the same time* adds a surprising amount of difficulty to the scheduling of multiple processors.

Consequence:

We're in deep trouble! (Even p-fair scheduling suffers from this.)

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Weak theoretical framework

Underlying causes:

- Dhall's effect:
  - With RM, DM and EDF, some low-utilization task sets can be unschedulable regardless of how many processors are used.
- Dependence on relative priority ordering:
  - Changing the relative priority ordering among higher-priority tasks may affect schedulability for a lower-priority task.
- Hard-to-find critical instant:
  - A critical instant does not always occur when a task arrives at the same time as all its higher-priority tasks.

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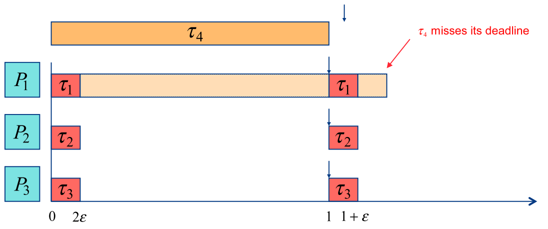
Weak theoretical framework

Dhall's effect: (Dhall & Liu, 1978)

$\tau_1 = \{C_1 = 2\varepsilon, T_1 = 1\}$
$\tau_2 = \{C_2 = 2\varepsilon, T_2 = 1\}$
$\tau_3 = \{C_3 = 2\varepsilon, T_3 = 1\}$
$\tau_4 = \{C_4 = 1, T_4 = 1+\varepsilon\}$

(RM scheduling)



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Weak theoretical framework

Dhall's effect:

- Applies for (greedy) RM, DM and EDF scheduling
- Least utilization of unschedulable task sets can be arbitrarily close to 1 no matter how many processors are used.

$$U_{global} = m\frac{2\varepsilon}{1} + \frac{1}{1+\varepsilon} \rightarrow 1$$
when $\varepsilon \rightarrow 0$

Consequence:
New multiprocessor priority-assignment schemes are needed!

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Weak theoretical framework

Impact of relative priority ordering:

- The response time of a task depends on the relative priority ordering of the higher-priority tasks
- This property does not exist for a uniprocessor system
- This means that well-known uniprocessor methods for finding optimal priority assignments (e.g., Audsley, 1991) cannot be applied

Consequence:

New methods for constructing optimal multiprocessor priority assignments are needed!

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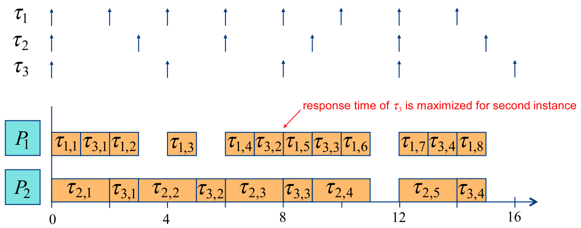
Weak theoretical framework

Hard-to-find critical instant:

$\tau_1 = \{C_1 = 1, T_1 = 2\}$
$\tau_2 = \{C_2 = 2, T_2 = 3\}$
$\tau_3 = \{C_3 = 2, T_3 = 4\}$

(RM scheduling)



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Weak theoretical framework

Hard-to-find critical instant:

- A critical instant does not always occur when a task arrives at the same time as all its higher-priority tasks.
- Finding the critical instant is a very (NP-?) hard problem
- Note: recall that knowledge about the critical instant is a fundamental property in uniprocessor feasibility tests.

Consequence:

New methods for constructing effective multiprocessor feasibility tests are needed!

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Weak theoretical framework

Underlying causes:

- Dhall's effect:
  - With RM, DM and EDF, some low-utilization task sets can be unschedulable regardless of how many processors are used.
- Dependence on relative priority ordering:
  - Changing the relative priority ordering among higher-priority tasks may affect schedulability for a lower-priority task.
- Hard-to-find critical instant:
  - A critical instant does not always occur when a task arrives at the same time as all its higher-priority tasks.

New techniques for priority assignments and schedulability tests are needed!

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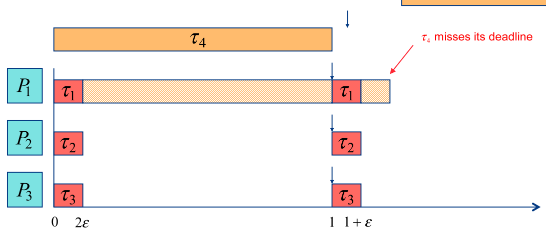
Weak theoretical framework

Dhall's effect: (Dhall & Liu, 1978)

$\tau_1 = \{C_1 = 2\varepsilon, T_1 = 1\}$
$\tau_2 = \{C_2 = 2\varepsilon, T_2 = 1\}$
$\tau_3 = \{C_3 = 2\varepsilon, T_3 = 1\}$
$\tau_4 = \{C_4 = 1, T_4 = 1+\varepsilon\}$

(RM scheduling)



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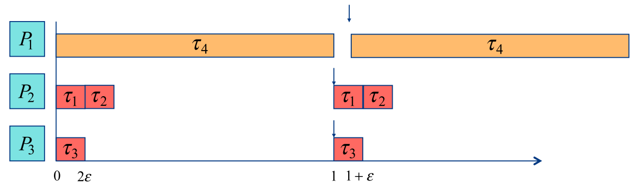
New priority-assignment scheme

How to avoid Dhall's effect:

- Problem: RM, DM & EDF only account for task deadlines! Actual computation demands are not accounted for.
- Solution: Dhall's effect can easily be avoided by letting tasks with high utilization receive higher priority:



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New priority-assignment scheme

Algorithm RM-US[m/(3m-2)]:

(Andersson, Baruah & Jonsson, 2001)

- RM-US[m/(3m-2)] assigns (static) priorities to tasks according to the following rule:

  If $U_i > m/(3m-2)$ then τ_i has the highest priority (ties broken arbitrarily)

  If $U_i \leq m/(3m-2)$ then τ_i has RM priority

- Clearly, tasks with higher utilization, $U_i = C_i / T_i$, get higher priority.

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New priority-assignment scheme

RM-US[m/(3m-2)] example:

- As an example of the priorities assigned by RM-US[m/(3m-2)], consider the following task set to be scheduled on a system with 3 identical processors:

  $\tau_1 = \{C_1 = 1, T_1 = 7\}$ $\tau_2 = \{C_2 = 2, T_2 = 10\}$
  $\tau_3 = \{C_3 = 9, T_3 = 20\}$ $\tau_4 = \{C_4 = 11, T_4 = 22\}$
  $\tau_5 = \{C_5 = 2, T_5 = 25\}$

- The utilizations of these tasks are: 0.143, 0.2, 0.45, 0.5 and 0.08, respectively.

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New priority-assignment scheme

RM-US[m/(3m-2)] example:

- For $m = 3$:

  $m/(3m-2) = 3/7 \approx 0.4286$

- Hence, tasks τ_3 and τ_4 will be assigned higher priorities, and the remaining tasks will be assigned RM priorities.
- The possible priority assignments are therefore as follows (highest-priority task listed first):

  $\tau_3, \tau_4, \tau_1, \tau_2, \tau_5,$

  or

  $\tau_4, \tau_3, \tau_1, \tau_2, \tau_5,$

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New feasibility tests

Processor utilization analysis for RM-US[m/(3m-2)]:

(Andersson, Baruah & Jonsson, 2001)

- A sufficient condition for RM-US[m/(3m-2)] scheduling on m identical processors is

$$U = \sum_{i=1}^{n} \frac{C_i}{T_i} \leq \frac{m^2}{3m-2}$$

- Question: does RM-US[m/(3m-2)] avoid Dhall's effect?

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New feasibility tests

Processor utilization analysis for RM-US[m/(3m-2)]:

- We observe that, regardless of the number of processors, the task set will always meet its deadlines as long as no more than one third of the processing capacity is used:

$$U_{RM-US[m/(3m-2)]} = \lim_{m \to \infty} \frac{m^2}{3m-2} = \frac{m}{3}$$

- RM-US[m/(3m-2)] thus avoids Dhall's effect since we can always add more processors if deadlines were missed.
- Note that this remedy was not possible with traditional RM.

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New feasibility tests

Response-time analysis for multiprocessors:

- Uses the same principle as the uniprocessor case, where the response time for a task τ_i consists of:

  C_i The task's uninterrupted execution time (WCET)

  I_i Interference from higher-priority tasks

$$R_i = C_i + I_i$$

- The difference is that the calculation of interference now has to account for the fact that higher-priority tasks can execute in parallel on the processors.

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New feasibility tests

Response-time analysis for multiprocessors:

- For the multiprocessor case, with n tasks and m processors, we observe two things:
  1. Interference can only occur when $n > m$.
  2. Interference can only affect tasks $\{\tau_k : k > m\}$ since the m highest-priority tasks will always execute in parallel without contention on the m processors.
- Consequently, interference of a task is a function of the execution overlap of its higher-priority tasks.

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New feasibility tests

Response-time analysis for multiprocessors:

- The following two observations give us the secret to analyzing the interference of a task:

  With respect to the execution overlap it can be shown that the interference is maximized when the higher-priority tasks completely overlap their execution.

  Compared to the uniprocessor case, one extra instance of each higher-priority task must be accounted for in the interference analysis.

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New feasibility tests

Response-time analysis for multiprocessors:

- The worst-case interference term is

$$I_i = \frac{1}{m} \sum_{\forall j \in hp(i)} \left(\left\lceil \frac{R_i}{T_j} \right\rceil \cdot C_j + C_j \right)$$

  where $hp(i)$ is the set of tasks with higher priority than τ_i.
- The worst-case response time for a task τ_i is thus:

$$R_i = C_i + \frac{1}{m} \sum_{\forall j \in hp(i)} \left(\left\lceil \frac{R_i}{T_j} \right\rceil \cdot C_j + C_j \right)$$

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New feasibility tests

Response-time analysis for multiprocessors:

- As before, an iterative approach can be used for finding the worst-case response time:

$$R_i^{n+1} = C_i + \frac{1}{m} \sum_{\forall j \in hp(i)} \left(\left\lceil \frac{R_i^n}{T_j} \right\rceil \cdot C_j + C_j \right)$$

- We now have a sufficient condition for static-priority scheduling on multiprocessors:

$$\forall i : R_i \le D_i$$

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Poor resource utilization

A fundamental limit: (Andersson, Baruah & Jonsson, 2001)

> The utilization guarantee bound for any static-priority multiprocessor scheduling algorithm cannot be higher than 1/2 of the capacity of the processors.

- This applies for all types of static-priority scheduling. That is, partitioned and global, greedy and p-fair scheduling.
- Hence, we can never expect to utilize more than half the processing capacity if hard timing constraints exist.
- The most resource-efficient multiprocessor real-time system is therefore one with a mix of soft and hard constraints.

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Scheduling anomalies

> **Scheduling anomaly**: A seemingly positive change in the system (reducing load or adding resources) causes a non-intuitive decrease in performance.

State-of-the-art :

- Uniprocessor systems:
  - Anomalies only found for non-preemptive scheduling (Mok, 2000)
- Multiprocessor systems:
  - Richard's anomalies for non-preemptive scheduling
  - Execution-time-based anomalies for preemptive scheduling
  - Period-based anomalies for preemptive scheduling

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Scheduling anomalies

Richard's anomalies: (Graham, 1969)

Assumptions:

- Non-preemptive scheduling
- Precedence constraints
- Restricted migration (individual task instances cannot migrate)
- Fixed execution times

Task completion times may increase as a result of:

- Changing the task priorities
- Increasing the number of processors
- Reducing task execution times
- Weakening the precedence constraints

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Scheduling anomalies

Execution-time-based anomalies: (Ha & Liu, 1994)

Assumptions:

- Preemptive scheduling
- Independent tasks
- Restricted migration (individual task instances cannot migrate)
- Fixed execution times

Task completion times may increase as a result of:

- Reducing task execution times

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Scheduling anomalies

Period-based anomalies: (Andersson & Jonsson, 2000)

Assumptions:
- Preemptive scheduling
- Independent tasks
- Full migration
- Fixed execution times

A task's completion time may increase as a result of:
- Increasing the period of a higher-priority task
- Increasing the period of the task itself

Note: increasing the periods is commonly used to reduce the load in feedback-control systems!

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Global scheduling

State-of-the-art in global scheduling:

- Static priorities:
  - The RM-US[m/(3m-2)] priority assignment scheme offers a way to circumvent Dhall's effect and a non-zero resource utilization guarantee bound of m/(3m-2) ≥ 33.3%.
  - In 2003, Baker generalized the RM-US results to DM.
- Dynamic priorities:
  - In 2002, Srinivasan & Baruah proposed the EDF-US[m/(2m-1)] scheme with a corresponding non-zero resource utilization guarantee bound of m/(2m-1) ≥ 50%.
- Optimal multiprocessor scheduling:
  - Using p-fair scheduling and dynamic priorities it is possible to achieve 100% resource utilization on a multiprocessor.