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Multiprocessor scheduling

How are tasks assigned to processors?

- Static assignment
  - The processor(s) used for executing a task are determined before system is put in mission ("off-line")
  - Algorithms: partitioned scheduling, guided search, non-guided search, ...
- Dynamic assignment
  - The processor(s) used for executing a task are determined during system operation "on-line"
  - Algorithms: global scheduling

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Multiprocessor scheduling

How are tasks allowed to migrate?

- Partitioned scheduling
  - Each instance of a task must execute on the same processor
  - Equivalent to multiple uniprocessor systems!
- Guided search & non-guided techniques
  - Depending on migration constraints, a task may or may not execute on more than one processor
- Global scheduling
  - A task is allowed to execute on an arbitrary processor (sometimes even after being preempted)

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Partitioned scheduling

General characteristics:

- Each processor has its own queue for ready tasks
- Tasks are organized in groups, and each task group is assigned to a specific processor
- When selected for execution, a task can only be dispatched to its assigned processor

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Partitioned scheduling

Advantages:

- Mature scheduling framework
  - Most scheduling theory pertaining to uniprocessor scheduling are also applicable here
  - Uniprocessor resource-management protocols can be used
- Partitioning of tasks can be automated
  - For example, using a bin-packing algorithm

Disadvantages:

- Cannot exploit all unused execution time
  - Surplus capacity cannot be shared among processors
  - Will suffer from overly-pessimistic WCET derivation

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Partitioned scheduling

Complexity of schedulability analysis for partitioned scheduling: (Leung & Whitehead, 1982)

The problem of deciding whether a task set is schedulable on m processors with respect to partitioned scheduling is NP-complete in the strong sense.

Consequence:

There cannot be any pseudo-polynomial time algorithm for finding an optimal partition of a set of tasks unless P = NP.

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Partitioned scheduling

Bin-packing algorithms:

- Basic idea:
  - The problem concerns packing objects of varying sizes in boxes ("bins") with the objective of minimizing number of used boxes.
- Application to multiprocessor systems:
  - Bins are represented by processors and objects by tasks.
  - The decision whether a processor is "full" or not is derived from a utilization-based feasibility test.
- Assumptions:
  - Independent, periodic tasks
  - Preemptive, uniprocessor scheduling (RM)

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Partitioned scheduling

Bin-packing algorithms:

Rate-Monotonic-First-Fit (RMFF): (Dhall and Liu, 1978)

- Let the processors be indexed as $\mu_1, \mu_2, \ldots$
- Assign the tasks in the order of increasing periods (that is, RM order).
- For each task τ_i, choose the lowest previously-used j such that τ_i, together with all tasks that have already been assigned to processor μ_j, can be feasibly scheduled according to the utilization-based RM-feasibility test.
- Processors are added if needed for RM-schedulability.

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Partitioned scheduling

Bin-packing algorithms:

The utilization guarantee bound U_{RMFF} for a system with m processors using the RMFF scheduling policy (with arbitrary task-assignment order) is

$$m\left(2^{1/2}-1\right) \le U_{RMFF} \le (m+1)/\left(1+2^{1/(m+1)}\right)$$ (Oh & Baker, 1998)

Note: $\left(2^{1/2}-1\right) \approx 0.41$

Thus: task sets whose utilization do not exceed ≈ 41% of the total processor capacity is always RMFF-schedulable.

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Guided search

Branch-and-bound algorithms:

- Basic idea:
  - A set of solutions to a given problem is organized in a search tree.
  - A vertex in the search tree corresponds to a specific solution structure.
  - A goal vertex corresponds to a complete solution to the problem and is located at the highest level of the search tree.
  - The root vertex corresponds to an initial solution at the lowest level of the search tree.
  - The search for a solution starts with only the root vertex.
  - Search objective is to find a goal vertex that optimizes a given cost (performance measure).

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Guided search

Branch-and-bound algorithms:

- Basic idea (cont'd):
  - For each vertex, a set of child vertices is generated by modifying the structure of the current vertex ("branching").
  - To check if a tree branch may lead to an acceptable solution, a lower-bound function is applied to each of the child vertices.
  - If a child vertex looks promising, it will be further investigated.
  - If a child vertex will only lead to inferior solutions, that entire branch is pruned ("bounding").

Note: An initial solution could be used for making good bounding operations early in the search. When an acceptable goal vertex is reached the bounding operation can be made more accurate.

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Guided search

Branch-and-bound algorithms:

- Application to multiprocessor scheduling:
  - The search tree represents the set of all task-to-processor assignments for a given set of tasks and processors.
  - A vertex in the search tree is a partial or complete assignment of tasks to processors.
  - The root vertex corresponds to an initial (empty or complete) schedule.
  - A goal vertex corresponds to a complete schedule.
  - The purpose of the lower-bound function is to assess whether a child vertex is feasible, that is, whether the corresponding branch in the search tree contains a feasible schedule.

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Guided search

Branch-and-bound for multiprocessor scheduling:

- Initial schedule is empty:
  - At each vertex in the search tree, a set of ready tasks (candidates for execution) are available for scheduling.
  - Generation of a child vertex corresponds to adding one of the ready tasks to the schedule in the current vertex.
- Initial schedule is complete (but possibly suboptimal):
  - At each level of the search tree, a set of scheduling changes (e.g., modified constraints or assignments) are available.
  - Generation of a child vertex corresponds to applying one or more of the changes to the schedule in the current vertex.

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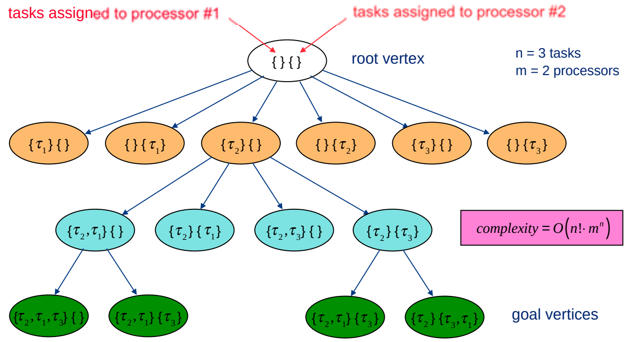
An example search tree



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Guided search

How do we avoid an exhaustive search?

- Bound pruning
  - use optimistic lower bounds
- Redundancy pruning
  - exploit symmetries in task set and processors
- Algorithm configuration
  - use suitable exploration order for promising vertices
- Performance guarantees
  - solution is within guaranteed bound from optimum
- Local optimization
  - only a subset of child vertices are retained

optimality guaranteed

near-optimality guaranteed

optimality not guaranteed

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Guided search

How do we avoid an exhaustive search?

- Bound pruning
  - use optimistic lower bounds

Additional reading:
Read the paper by Jonsson and Shin (ICPP'97)
Study how different vertex selection rules and estimated bounds affect the performance of the search algorithm

- Performance guarantees
  - solution is within guaranteed bound from optimum
- Local optimization
  - only a subset of child vertices are retained

optimality not guaranteed

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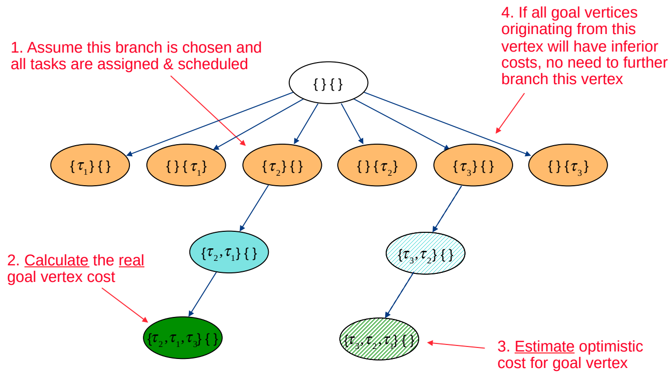
An example of bound pruning



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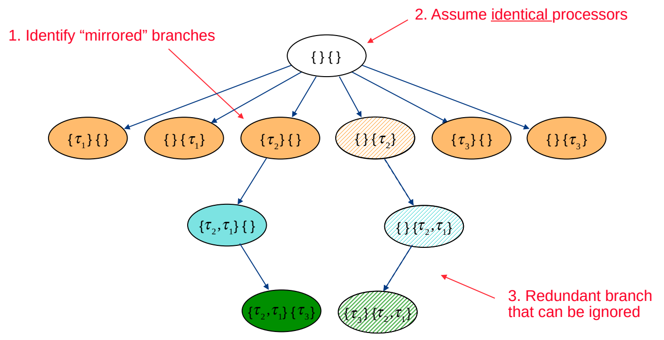
An example of redundancy pruning



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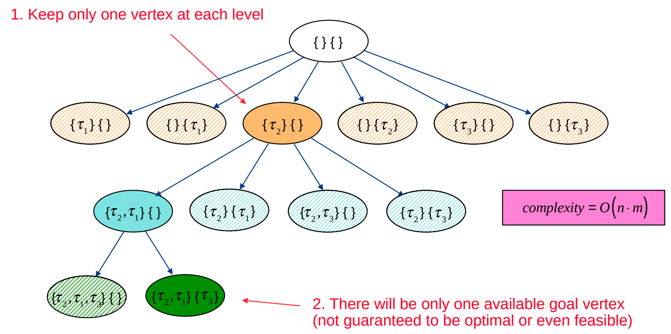
An example of local optimization



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Guided search

Some (optimal) branch-and-bound algorithms:

- Distributed real-time systems: (Peng and Shin, 1989)
  - Minimizes system hazard (maximum normalized task response time)
  - Starts with an empty schedule
- Fault-tolerant real-time systems: (Hou and Shin, 1994)
  - Maximizes probability of no dynamic failure (probability that all deadlines are met in the presence of component failures)
  - Starts with an empty schedule
  - May change degree of replication and restart the algorithm

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Guided search

Some (optimal) branch-and-bound algorithms:

- Uniprocessor real-time systems: (Xu and Parnas, 1990)
  - Minimizes maximum task lateness
  - Starts with an initial (complete) schedule
  - Modifies preemption, precedence and exclusion constraints
- Multiprocessor real-time systems: (Xu, 1993)
  - Minimizes maximum task lateness
  - Starts with an initial (complete) schedule
  - Modifies preemption, precedence and exclusion constraints

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Guided search

Some good local-optimization algorithms:

- Myopic scheduling: (Ramamritham, Stankovic and Shiah, 1990)
  - Promising vertices are explored in the order of decreasing search-tree level; within each level, exploration order is given by a heuristic function that calculates a weighted sum of task execution time, deadline, earliest start time and laxity.
  - Lower-bound function determines for the current vertex whether it is strongly feasible, that is, whether a feasible schedule can be obtained by expanding any of its child vertices.
  - Reduces search complexity by only investigating the *k* child vertices with closest deadline in the check for strong feasibility.
  - Reduces search complexity by limiting the number of allowed backtracks (to vertices at lower search-tree levels)

$$complexity = O(k \cdot n \cdot m)$$

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Guided search

Some good local-optimization algorithms:

- Pair-wise clustering: (Ramamritham, 1995)
  - Promising vertices are explored in the order of decreasing search-tree level; within each level, exploration is made in the order of increasing task LFT (latest finishing time).
  - Lower-bound function determines for the current vertex whether it is feasible using simple heuristics that keep track of latest start time and available time resources.
  - LFT is derived from task set end-to-end deadlines.
  - Pairs of communicating tasks are clustered based on the communication volume ratio. If the ratio between the task pair's execution times and communication volume is below a certain bound, the two tasks are assigned to the same processor.

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Non-guided search

General characteristics:

- Each non-guided search is given an initial task-to-processor assignment from which the search starts.
- Within each iteration step during search, different derivable alternatives of changing the current assignment are examined.
- To check whether an alternative is feasible or not, a run-time efficient feasibility test has to be used.
- In order to help the search find better assignments, the number of deadline misses is included as a penalty into the function calculating the goodness of the assignment.

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Non-guided search

Examples:

- Simulated annealing
- Genetic optimization

- Tabu search
- Neighbourhood search
- ...

These techniques all have in common that it is sufficient to state what makes a good solution, not how to get one!

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Non-guided search

Simulated annealing: (Kirkpatrick, Gelatt and Vecchi, 1983)

- Basic idea:
  - Simulated annealing is a global optimization technique which borrows ideas from statistical physics. The technique is derived from observations of how slowly-cooled molten metal can result in a regular crystalline structure.
  - The salient property of the technique is the incorporation of random jumps from local minima to potential new solutions. As the algorithm progresses, this ability is lessened, by reducing a temperature factor, which makes larger jumps less likely.
  - The main objective of the technique is to find the lowest point in an energy landscape.

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Non-guided search

Simulated annealing:

- Application to multiprocessor scheduling:
  - The set of all task-to-processor assignments for a given set of task and processors is called the problem space. A point in the problem space is an assignment of tasks to processors.
  - The neighbor space of a point is the set of points that are reachable by moving any single task to any other processor.
  - The energy of a point in problem space is a measure of the goodness of the task assignment represented by that point.
  - The energy function determines the shape of the problem space. It can be visualized as a rugged landscape, with deep valleys representing good solutions, and high peaks representing poor or infeasible ones.

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Non-guided search

Simulated annealing:

- Algorithm:

  A random starting point is chosen, and its energy E_s is evaluated. A random point in the neighbor space is then chosen, and its energy E_n is evaluated. This point becomes the new starting point if either $E_n \leq E_s$, or if $E_n > E_s$ and

  $e^x \geq \text{random}(0,1)$ where $x = -(E_n - E_s)/C$

  The control variable C is analogous to the temperature factor in a thermodynamic system. During the annealing process, C is slowly reduced (cooling the system), making higher energy jumps less likely. Eventually, the system freezes into a low energy state.

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Non-guided search

Simulated annealing:

- Implementation: (Tindell, Burns & Wellings, 1992)

  Neighbor function: Choose a random task and move it to a randomly-chosen processor.

  Energy function: The weighted sum of the following characteristics of the assignment:

  - Number of tasks assigned to the wrong processor
  - Number of replicas assigned to the same processor
  - Number of processors with too high a memory utilization
  - Number of tasks which do not meet their deadlines
  - Total communication bus utilization

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Non-guided search

Genetic optimization: (Goldberg, 1989)

- Basic idea:
  - Based on Darwin's evolution theory: "Survival of the Fittest"
  - Solutions to a problem is viewed as individuals forming a population.
  - Pair of individuals can create children (new individuals)
  - New individuals are created by applying a crossover operator to the genes of the parents
  - Genes of a new individual may mutate

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Non-guided search

Genetic optimization:

- Application to multiprocessor scheduling:
  - Tasks assignments and orderings are viewed as "chromosomes"
  - Tasks represent "genes"
  - Mutation means that a task is moved to another processor