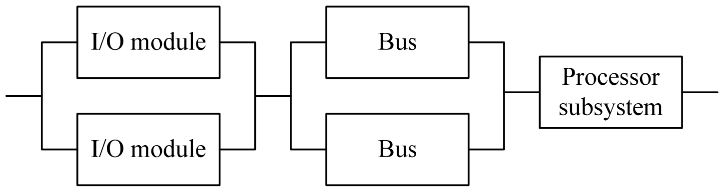
1.

a) The system consists of six error containment regions: each I/O module, each P/M module and each bus.

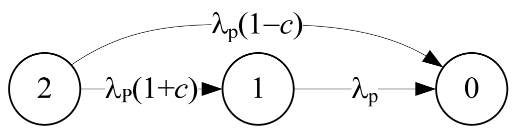
b) The system is modelled by the following reliability block diagram:



The subsystems that correspond to the I/O modules and the buses have the following reliability:

$$R_{i/o}(t) = e^{-\lambda_{i/o}t}$$
$$R_b(t) = e^{-\lambda_b t}$$
$$R_{2i/o}(t) = 2R_{i/o}(t) - R_{i/o}^2(t)$$
$$R_{2b}(t) = 2R_b(t) - R_b^2(t)$$

The processor subsystem is modelled using the following Markov model:



The transition rate matrix for the Markov model is:

$$Q = \begin{bmatrix} -2\lambda_p & \lambda_p(1+c) & \lambda_p(1-c) \\ 0 & -\lambda_p & \lambda_p \\ 0 & 0 & 0 \end{bmatrix}$$

Using the equation $P'(t) = P(t)Q$, we obtain the following system of differential equations:

$$\begin{cases} P_2'(t) = -2\lambda_p P_2(t) \\ P_1'(t) = \lambda_p(1+c)P_2(t) - \lambda_p P_1(t) \\ P_0'(t) = \lambda_p(1-c)P_2(t) + \lambda_p P_1(t) \end{cases}$$

As both modules are working from the beginning, we know that:

$$P(0) = [1 \quad 0 \quad 0].$$

We solve the equation system using the Laplace transform.

$$\begin{cases} sP_2(s) - 1 = -2\lambda_p P_2(s) \\ sP_1(s) = \lambda_p(1+c)P_2(s) - \lambda_p P_1(s) \\ P_0(s) = \lambda_p(1-c)P_2(s) + \lambda_p P_1(s) \end{cases}$$

$$P_2(s) = \frac{1}{s + 2\lambda_p}$$

$$P_1(s) = \frac{\lambda_p(1+c)}{s+\lambda_p} \cdot P_2(s) = \frac{\lambda_p(1+c)}{s+\lambda_p} \cdot \frac{1}{s+2\lambda_p} = (1+\mathrm{c})\left(\frac{1}{s+\lambda_p} - \frac{1}{s+2\lambda_p}\right)$$

Using the inverse Laplace transform, we obtain:

$$P_2(t) = e^{-2\lambda_p t}$$

$$P_1(t) = (1+\mathrm{c}) \cdot \left(e^{-\lambda_p t} - e^{-2\lambda_p t}\right)$$

$$R_{2\mathrm{PM}}(t) = P_1(t) + P_2(t) = (1+\mathrm{c}) \cdot e^{-\lambda_p t} - c \cdot e^{-2\lambda_p t}$$

The reliability of the satellite launcher is

$$R_{node}(t) = R_{2\mathrm{PM}}(t) \cdot R_{2i/o}(t) \cdot R_{2b}(t).$$

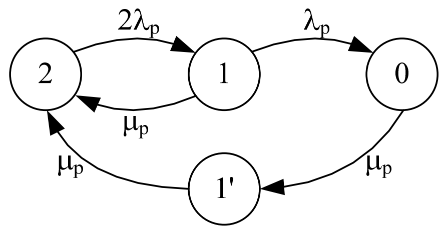
c) The MTTF of the processor subsystem is

$$MTTF_{2PM} = \int_0^\infty R_{2PM}(t)dt = \int_0^\infty (1+\mathrm{c}) \cdot e^{-\lambda_p t} - c \cdot e^{-2\lambda_p t}\, dt$$

$$= \left[-\frac{(1+c)}{\lambda_p} \cdot e^{-\lambda_p t} + \frac{c}{2\lambda_p} \cdot e^{-2\lambda_p t}\right]_0^\infty$$

$$= \frac{1+c}{\lambda_p} - \frac{c}{2\lambda_p} = \frac{2+c}{2\lambda_p}$$

2. The system is modelled by viewing the disks and the processors as two independent primary subsystems. The processor subsystem is modelled using the following Markov model:



The label of each state represents the number of working processors. We obtain the following transition rate matrix from the Markov-model:

$$Q = \begin{bmatrix} -2\lambda_p & 2\lambda_p & 0 & 0 \\ \mu_p & -(\lambda_p + \mu_p) & \lambda_p & 0 \\ 0 & 0 & -\mu_p & \mu_p \\ \mu_p & 0 & 0 & -\mu_p \end{bmatrix}$$

We also know that $P(0) = [1 \quad 0 \quad 0]$. We obtain the following system of differential equations:

$$\begin{cases} P_2'(t) = -2\lambda_p P_2(t) + \mu_p P_1(t) + \mu_p P_{1'}(t) \\ P_1'(t) = 2\lambda_p P_2(t) - (\lambda_p + \mu_p) P_1(t) \\ P_0'(t) = \lambda_p P_1(t) - \mu_p P_0(t) \\ P_{1'}'(t) = \mu_p P_0(t) - \mu_p P_{1'}(t) \end{cases}$$

Let $t \to \infty$ and use Π_i to denote $\lim_{t\to\infty} P_i(t)$. We obtain:

$$\begin{cases} 0 = -2\lambda_p \Pi_2 + \mu_p \Pi_1 + \mu_p \Pi_{1'} & (1) \\ 0 = 2\lambda_p \Pi_2 - (\lambda_p + \mu_p)\Pi_1 & (2) \\ 0 = \lambda_p \Pi_1 - \mu_p \Pi_0 & (3) \\ 0 = \mu_p \Pi_0 - \mu_p \Pi_{1'} & (4) \end{cases}$$

We also have

$$\Pi_2 + \Pi_1 + \Pi_0 + \Pi_{1'} = 1. \qquad (5)$$

We obtain the following expression for Π_1 from (3):

$$\Pi_1 = \frac{\mu_p}{\lambda_p} \Pi_0 \qquad (6)$$

Equation (4) gives:

$$\Pi_{1'} = \Pi_0 \qquad (7)$$

Equations (2) and (6) give:

$$\Pi_2 = \frac{\lambda_p + \mu_p}{2\lambda_p}\Pi_1 = \frac{\lambda_p + \mu_p}{2\lambda_p}\cdot\frac{\mu_p}{\lambda_p}\Pi_0 = \frac{\lambda_p\mu_p + \mu_p^2}{2\lambda_p^2}\Pi_0 \qquad (8)$$

An expression for Π_0 is obtained using (5):

$$\begin{aligned}
&1 = \Pi_2 + \Pi_1 + \Pi_0 + \Pi_{1'} \\
&= \frac{\lambda_p\mu_p + \mu_p^2}{2\lambda_p^2}\Pi_0 + \frac{\mu_p}{\lambda_p}\Pi_0 + \Pi_0 + \Pi_0 \\
&= \Pi_0\left(\frac{\lambda_p\mu_p + \mu_p^2}{2\lambda_p^2} + \frac{\mu_p}{\lambda_p} + 2\right) = \Pi_0 \cdot \frac{\lambda_p\mu_p + \mu_p^2 + 2\lambda_p\mu_p + 4\lambda_p^2}{2\lambda_p^2} \\
&\Rightarrow \Pi_0 = \frac{2\lambda_p^2}{3\lambda_p\mu_p + \mu_p^2 + 4\lambda_p^2}
\end{aligned}$$

The steady-state availability of the processors is:

$$\begin{aligned}
&\lim_{t\to\infty} A_{processors}(t) = \Pi_2 + \Pi_1 = \frac{2\lambda_p^2}{3\lambda_p\mu_p + \mu_p^2 + 4\lambda_p^2}\cdot\left(\frac{\lambda_p\mu_p + \mu_p^2}{2\lambda_p^2} + \frac{\mu_p}{\lambda_p}\right) \\
&= \frac{2\lambda_p^2}{3\lambda_p\mu_p + \mu_p^2 + 4\lambda_p^2}\cdot\frac{3\lambda_p\mu_p + \mu_p^2}{2\lambda_p^2} \\
&= \frac{3\lambda_p\mu_p + \mu_p^2}{3\lambda_p\mu_p + \mu_p^2 + 4\lambda_p^2}
\end{aligned}$$

The disk subsystem is modelled using the following Markov-model:

$$\begin{tikzcd}
2 \arrow[r, bend left, "2\lambda_d"] & 1 \arrow[l, bend left, "\mu_d"] \arrow[r, bend left, "\lambda_d"] & 0 \arrow[dl, bend left, "\mu_d"] \\
& 1' \arrow[ul, bend left, "\mu_d"] &
\end{tikzcd}$$

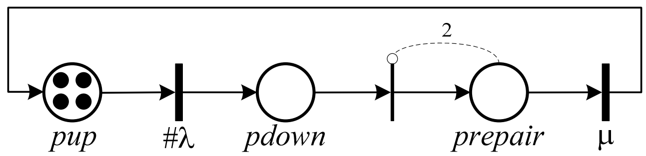
The Markov-model above is the same as for the processor subsystem, except for the failure rates. The steady-state availability of the disk subsystem is

$$\lim_{t\to\infty} A_{disks}(t) = \frac{3\lambda_d\mu_d + \mu_d^2}{3\lambda_d\mu_d + \mu_d^2 + 4\lambda_d^2}.$$

The steady-state availability of the system is calculated as

$$\lim_{t\to\infty} A_{system}(t) = A_{disks}(t)\cdot A_{processors}(t).$$

3. The system is described by the following GSPN model:



Markings of the GSPN model are represented as (*#pup #pdown #prepair*). The marking (0 2 2) corresponds to the event that the system is unavailable.