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Multiprocessor scheduling

How are tasks assigned to processors?

- Static assignment
 - The processor(s) used for executing a task are determined before system is put in mission ("off-line")
 - Algorithms: partitioned scheduling, guided search, non-guided search, ...
- Dynamic assignment
 - The processor(s) used for executing a task are determined during system operation "on-line"
 - Algorithms: global scheduling

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Multiprocessor scheduling

How are tasks allowed to migrate?

- Partitioned scheduling
 - Each instance of a task must execute on the same processor
 - Equivalent to multiple uniprocessor systems!
- Guided search & non-guided techniques
 - Depending on migration constraints, a task may or may not execute on more than one processor
- Global scheduling
 - A task is allowed to execute on an arbitrary processor (sometimes even after being preempted)

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Partitioned scheduling

General characteristics:

- Each processor has its own queue for ready tasks
- Tasks are organized in groups, and each task group is assigned to a specific processor
- When selected for execution, a task can only be dispatched to its assigned processor

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Partitioned scheduling

Advantages:

- Mature scheduling framework
 - Most scheduling theory pertaining to uniprocessor scheduling are also applicable here
 - Uniprocessor resource-management protocols can be used
- Partitioning of tasks can be automated
 - For example, using a bin-packing algorithm

Disadvantages:

- Cannot exploit all unused execution time
 - Surplus capacity cannot be shared among processors
 - Will suffer from overly-pessimistic WCET derivation

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Partitioned scheduling

Complexity of schedulability analysis for partitioned scheduling: (Leung & Whitehead, 1982)

The problem of deciding whether a task set is schedulable on m processors with respect to partitioned scheduling is NP-complete in the strong sense.

Consequence:

There cannot be any pseudo-polynomial time algorithm for finding an optimal partition of a set of tasks unless $P = NP$.

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Partitioned scheduling

Bin-packing algorithms:

- Basic idea:
 - The problem concerns packing objects of varying sizes in boxes ("bins") with the objective of minimizing number of used boxes.
- Application to multiprocessor systems:
 - Bins are represented by processors and objects by tasks.
 - The decision whether a processor is "full" or not is derived from a utilization-based feasibility test.
- Assumptions:
 - Independent, periodic tasks
 - Preemptive, uniprocessor scheduling (RM)



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Partitioned scheduling

Bin-packing algorithms:

Rate-Monotonic-First-Fit (RMFF): (Dhall and Liu, 1978)

- Let the processors be indexed as $\mu_1, \mu_2, \dots$
- Assign the tasks in the order of increasing periods (that is, RM order).
- For each task τ_i , choose the lowest previously-used j such that τ_i , together with all tasks that have already been assigned to processor μ_j , can be feasibly scheduled according to the utilization-based RM-feasibility test.
- Processors are added if needed for RM-schedulability.

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Partitioned scheduling

Bin-packing algorithms:

The utilization guarantee bound U_{RMFF} for a system with m processors using the RMFF scheduling policy (with arbitrary task-assignment order) is

$$m(2^{1/2} - 1) \leq U_{RMFF} \leq (m+1) / \left(1 + 2^{1/(m+1)}\right) \quad (\text{Oh \& Baker, 1998})$$

Note: $(2^{1/2} - 1) \approx 0.41$

Thus: task sets whose utilization do not exceed $\approx 41\%$ of the total processor capacity is always RMFF-schedulable.

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Guided search

Branch-and-bound algorithms:

- Basic idea:
 - A set of solutions to a given problem is organized in a search tree.
 - A vertex in the search tree corresponds to a specific solution structure.
 - A goal vertex corresponds to a complete solution to the problem and is located at the highest level of the search tree.
 - The root vertex corresponds to an initial solution at the lowest level of the search tree.
 - The search for a solution starts with only the root vertex.
 - Search objective is to find a goal vertex that optimizes a given cost (performance measure).

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Guided search

Branch-and-bound algorithms:

- Basic idea (cont'd):
 - For each vertex, a set of child vertices is generated by modifying the structure of the current vertex ("branching").
 - To check if a tree branch may lead to an acceptable solution, a lower-bound function is applied to each of the child vertices.
 - If a child vertex looks promising, it will be further investigated.
 - If a child vertex will only lead to inferior solutions, that entire branch is pruned ("bounding").

Note: An initial solution could be used for making good bounding operations early in the search. When an acceptable goal vertex is reached the bounding operation can be made more accurate.

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Guided search

Branch-and-bound algorithms:

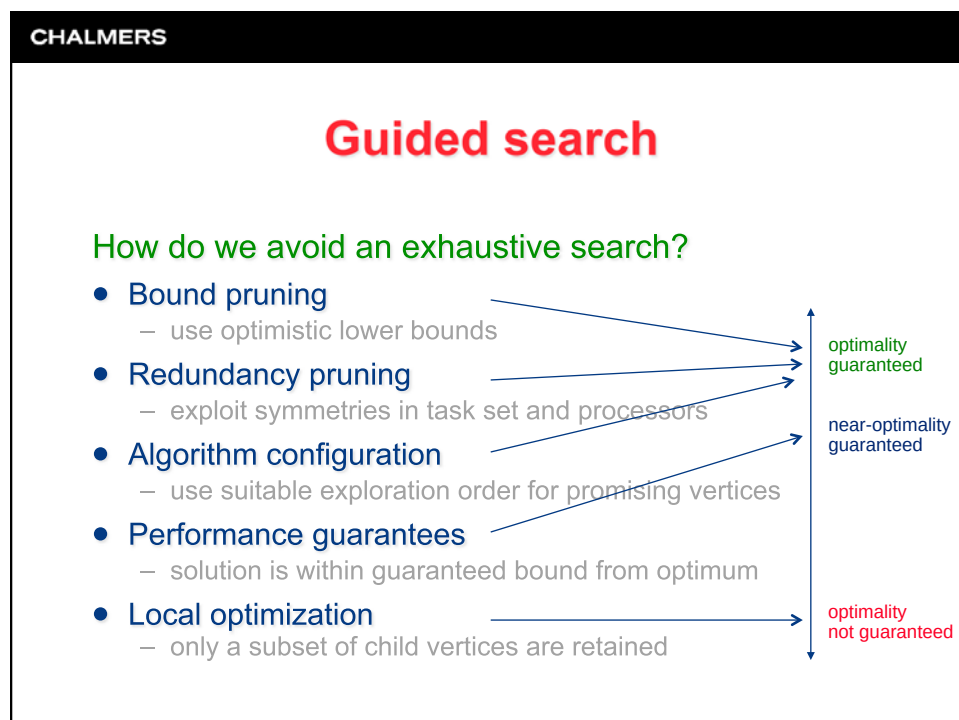
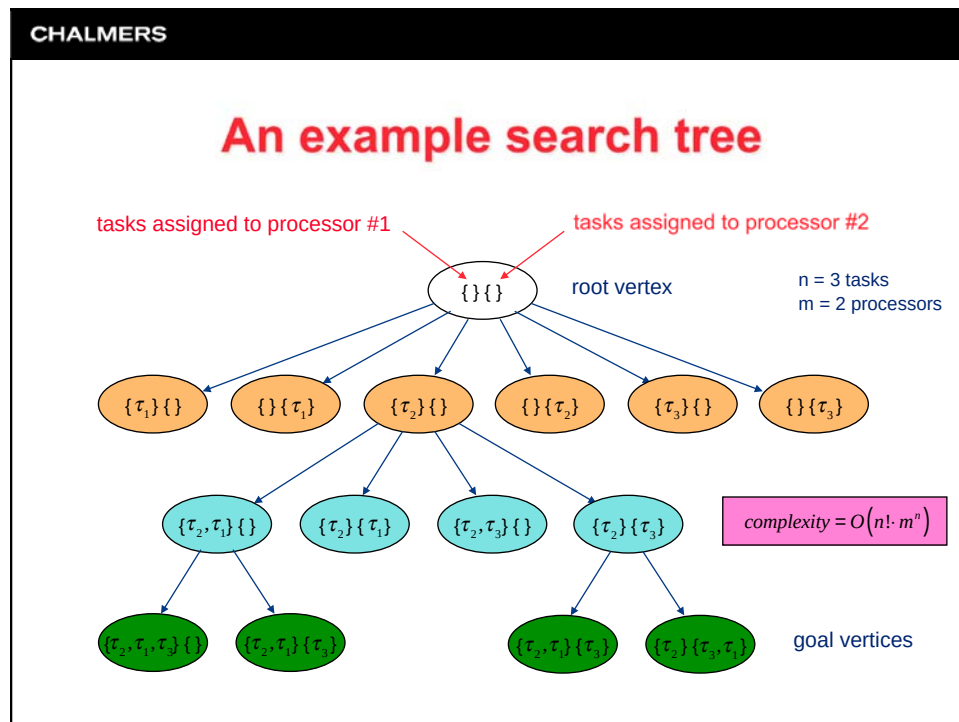
- Application to multiprocessor scheduling:
 - The search tree represents the set of all task-to-processor assignments for a given set of tasks and processors.
 - A vertex in the search tree is a partial or complete assignment of tasks to processors.
 - The root vertex corresponds to an initial (empty or complete) schedule.
 - A goal vertex corresponds to a complete schedule.
 - The purpose of the lower-bound function is to assess whether a child vertex is feasible, that is, whether the corresponding branch in the search tree contains a feasible schedule.

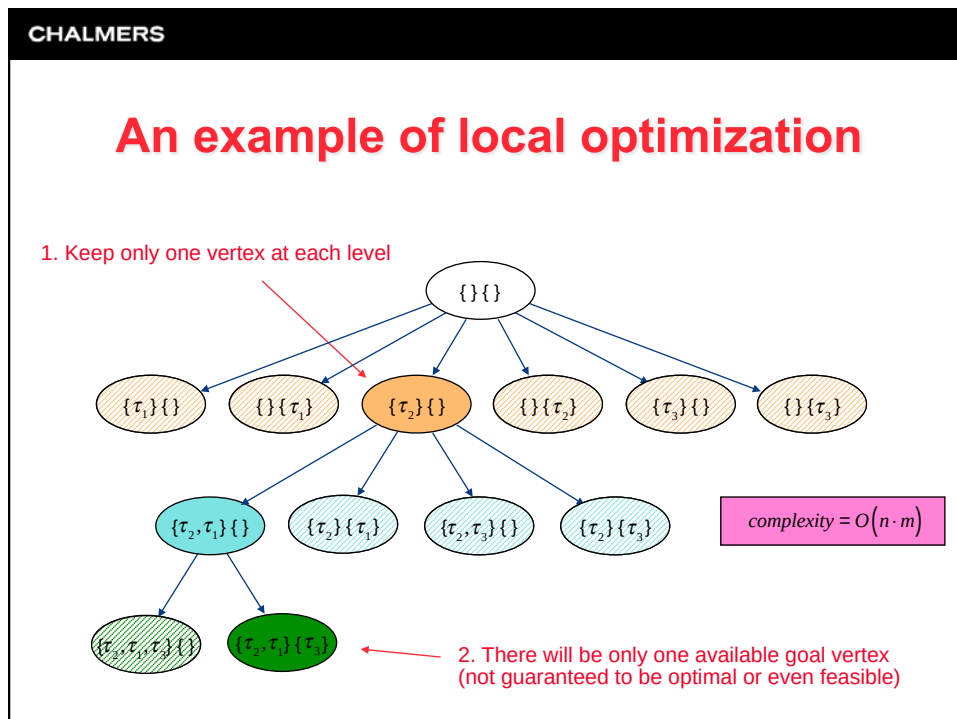
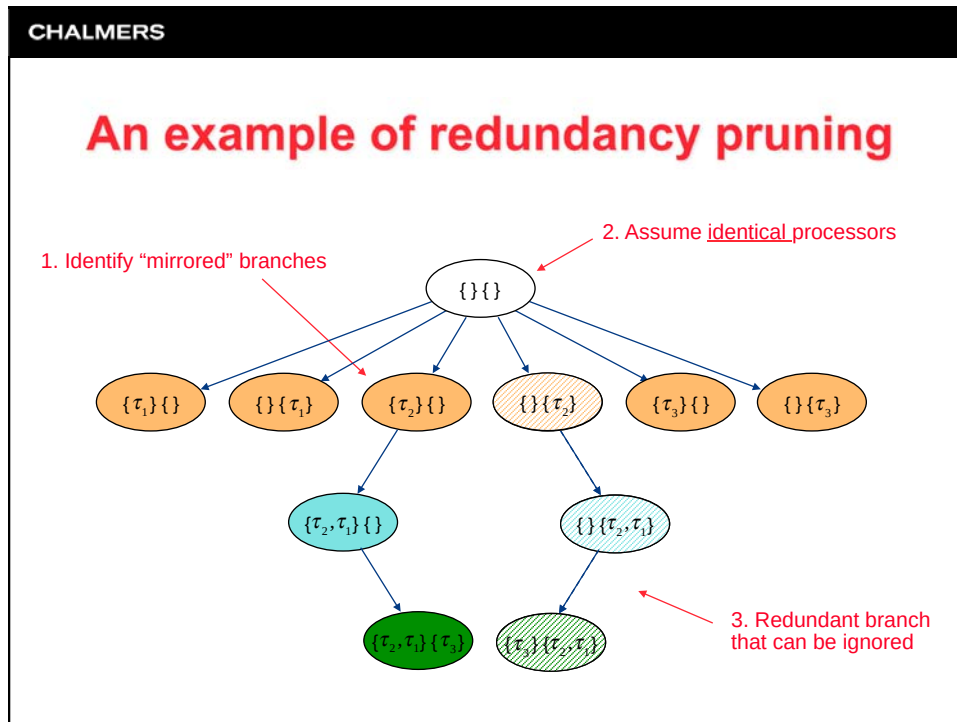
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Guided search

Branch-and-bound for multiprocessor scheduling:

- Initial schedule is empty:
 - At each vertex in the search tree, a set of ready tasks (candidates for execution) are available for scheduling.
 - Generation of a child vertex corresponds to adding one of the ready tasks to the schedule in the current vertex.
- Initial schedule is complete (but possibly suboptimal):
 - At each level of the search tree, a set of scheduling changes (e.g., modified constraints or assignments) are available.
 - Generation of a child vertex corresponds to applying one or more of the changes to the schedule in the current vertex.





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Guided search

Some (optimal) branch-and-bound algorithms:

- Distributed real-time systems: (Peng and Shin, 1989)
 - Minimizes system hazard (maximum normalized task response time)
 - Starts with an empty schedule
- Fault-tolerant real-time systems: (Hou and Shin, 1994)
 - Maximizes probability of no dynamic failure (probability that all deadlines are met in the presence of component failures)
 - Starts with an empty schedule
 - May change degree of replication and restart the algorithm

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Guided search

Some (optimal) branch-and-bound algorithms:

- Uniprocessor real-time systems: (Xu and Parnas, 1990)
 - Minimizes maximum task lateness
 - Starts with an initial (complete) schedule
 - Modifies preemption, precedence and exclusion constraints
- Multiprocessor real-time systems: (Xu, 1993)
 - Minimizes maximum task lateness
 - Starts with an initial (complete) schedule
 - Modifies preemption, precedence and exclusion constraints

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Guided search

Some good local-optimization algorithms:

- **Myopic scheduling:** (Ramamritham, Stankovic and Shiah, 1990)
 - Promising vertices are explored in the order of decreasing search-tree level; within each level, exploration order is given by a heuristic function that calculates a weighted sum of task execution time, deadline, earliest start time and laxity.
 - Lower-bound function determines for the current vertex whether it is strongly feasible, that is, whether a feasible schedule can be obtained by expanding any of its child vertices.
 - Reduces search complexity by only investigating the k child vertices with closest deadline in the check for strong feasibility.
 - Reduces search complexity by limiting the number of allowed backtracks (to vertices at lower search-tree levels)

$$\text{complexity} = O(k \cdot n \cdot m)$$

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Guided search

Some good local-optimization algorithms:

- **Pair-wise clustering:** (Ramamritham, 1995)
 - Promising vertices are explored in the order of decreasing search-tree level; within each level, exploration is made in the order of increasing task LFT (latest finishing time).
 - Lower-bound function determines for the current vertex whether it is feasible using simple heuristics that keep track of latest start time and available time resources.
 - LFT is derived from task set end-to-end deadlines.
 - Pairs of communicating tasks are clustered based on the communication volume ratio. If the ratio between the task pair's execution times and communication volume is below a certain bound, the two tasks are assigned to the same processor.

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Non-guided search

General characteristics:

- Each non-guided search is given an initial task-to-processor assignment from which the search starts.
- Within each iteration step during search, different derivable alternatives of changing the current assignment are examined.
- To check whether an alternative is feasible or not, a run-time efficient feasibility test has to be used.
- In order to help the search find better assignments, the number of deadline misses is included as a penalty into the function calculating the goodness of the assignment.

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Non-guided search

Examples:

- Simulated annealing
- Genetic optimization
- Tabu search
- Neighbourhood search
- ...

These techniques all have in common that it is sufficient to state what makes a good solution, not how to get one!

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Non-guided search

Simulated annealing: (Kirkpatrick, Gelatt and Vecchi, 1983)

- Basic idea:
 - Simulated annealing is a global optimization technique which borrows ideas from statistical physics. The technique is derived from observations of how slowly-cooled molten metal can result in a regular crystalline structure.
 - The salient property of the technique is the incorporation of random jumps from local minima to potential new solutions. As the algorithm progresses, this ability is lessened, by reducing a temperature factor, which makes larger jumps less likely.
 - The main objective of the technique is to find the lowest point in an energy landscape.

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Non-guided search

Simulated annealing:

- Application to multiprocessor scheduling:
 - The set of all task-to-processor assignments for a given set of task and processors is called the problem space. A point in the problem space is an assignment of tasks to processors.
 - The neighbor space of a point is the set of points that are reachable by moving any single task to any other processor.
 - The energy of a point in problem space is a measure of the goodness of the task assignment represented by that point.
 - The energy function determines the shape of the problem space. It can be visualized as a rugged landscape, with deep valleys representing good solutions, and high peaks representing poor or infeasible ones.

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Non-guided search

Simulated annealing:

- Algorithm:

A random starting point is chosen, and its energy E_s is evaluated.

A random point in the neighbor space is then chosen, and its energy E_n is evaluated. This point becomes the new starting point if either $E_n \leq E_s$, or if $E_n > E_s$ and

$$e^x \geq \text{random}(0,1) \text{ where } x = -(E_n - E_s) / C$$

The control variable C is analogous to the temperature factor in a thermodynamic system. During the annealing process, C is slowly reduced (cooling the system), making higher energy jumps less likely. Eventually, the system freezes into a low energy state.

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Non-guided search

Simulated annealing:

- Implementation: (Tindell, Burns & Wellings, 1992)

Neighbor function: Choose a random task and move it to a randomly-chosen processor.

Energy function: The weighted sum of the following characteristics of the assignment:

- Number of tasks assigned to the wrong processor
- Number of replicas assigned to the same processor
- Number of processors with too high a memory utilization
- Number of tasks which do not meet their deadlines
- Total communication bus utilization

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Non-guided search

Genetic optimization: (Goldberg, 1989)

- Basic idea:
 - Based on Darwin's evolution theory: "Survival of the Fittest"
 - Solutions to a problem is viewed as individuals forming a population.
 - Pair of individuals can create children (new individuals)
 - New individuals are created by applying a crossover operator to the genes of the parents
 - Genes of a new individual may mutate

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Non-guided search

Genetic optimization:

- Application to multiprocessor scheduling:
 - Tasks assignments and orderings are viewed as "chromosomes"
 - Tasks represent "genes"
 - Mutation means that a task is moved to another processor