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Example: scheduling using EDF

Problem: Assume a system with tasks according to the figure below. The timing properties of the tasks are given in the table. Investigate the schedulability of the tasks when EDF is used. (Note that $D_i < T_i$ for all tasks)

τ_1 τ_2 τ_3

Task	C_i	D_i	T_i
τ_1	1	1	2
τ_2	1	2	4
τ_3	1	3	8

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Example: scheduling using EDF

Simulate an execution of the tasks:

The tasks are not schedulable even though

$$U = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8} = 0.875 < 1$$

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Feasibility analysis for EDF

What analysis methods are suitable for general EDF:

- Utilization-based analysis?
Not suitable! Not general enough or exact enough
 - Does not work well for the case of $D_i < T_i$
- Response-time analysis?
Not suitable! Analysis much more complex than for DM
 - Critical instant does not necessarily occur when all tasks arrive at the same time for the first time.
 - Instead, response time of a task is maximized at some scenario where all other tasks arrive at the same time; the worst such scenario has to be identified for each task before the response time of that task can be calculated.

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Processor-demand analysis

Processor demand:

- The processor demand for a task τ_i in a given time interval $[0, L]$ is the amount of processor time that the task needs in the interval in order to meet the deadlines that fall within the interval.
- Let N_i^L represent the number of instances of τ_i that must complete execution before L .
- The total processor demand up to L is

$$C_p(0, L) = \sum_{i=1}^n N_i^L C_i$$

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Processor-demand analysis

Processor demand:

- We can calculate N_i^L by counting how many times task τ_i has arrived during the interval $[0, L - D_i]$.
- We can ignore instances of the task that arrived during the interval $[L - D_i, L]$ since $D_i > L$ for these instances.

$N_1^L = 2$
 $N_2^L = 3$

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Processor-demand analysis

Processor-demand analysis:

- We can express N_i^L as

$$N_i^L = \left\lfloor \frac{L - D_i}{T_i} \right\rfloor + 1$$

- The total processor demand is thus

$$C_p(0, L) = \sum_{i=1}^n \left(\left\lfloor \frac{L - D_i}{T_i} \right\rfloor + 1 \right) C_i$$

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Exact feasibility test for EDF (Sufficient and necessary condition)

A sufficient and necessary condition for earliest-deadline-first scheduling, for which $D_i \leq T_i$, is

$$\forall L : C_p(0, L) \leq L$$

where $C_p(0, L)$ is the total processor demand in $[0, L]$.

The processor-demand analysis and associated feasibility test was presented by S. Baruah, L. Rosier and R. Howell in 1990.

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Exact feasibility test for EDF

(Sufficient and necessary condition)

How large interval must be examined?

- Largest interval is LCM of the task's periods
- This interval can be further shortened (see course book)

How many time points must be examined?

- Only absolute deadlines need to be examined
- The set of deadlines can be further reduced (see course book)

The test can consequently be improved as follows:

$$\forall L \in K : C_p(0, L) \leq L$$

$$K = \{ D_i^k \mid D_i^k = kT_i + D_i, D_i^k \leq \text{LCM}\{T_1, \dots, T_n\}, 1 \leq i \leq n, k \geq 0 \}$$

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Example: scheduling using EDF

Problem: Assume a system with tasks according to the figure below. The timing properties of the tasks are given in the table.

- Determine, by analyzing the processor demand, whether the tasks are schedulable or not using EDF.
- Determine, by using simulation, whether the tasks are schedulable or not using EDF.



Task	C _i	D _i	T _i
τ ₁	2	3	4
τ ₂	2	7	8
τ ₃	3	12	16

We solve this on the blackboard!

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Extended processor-demand analysis

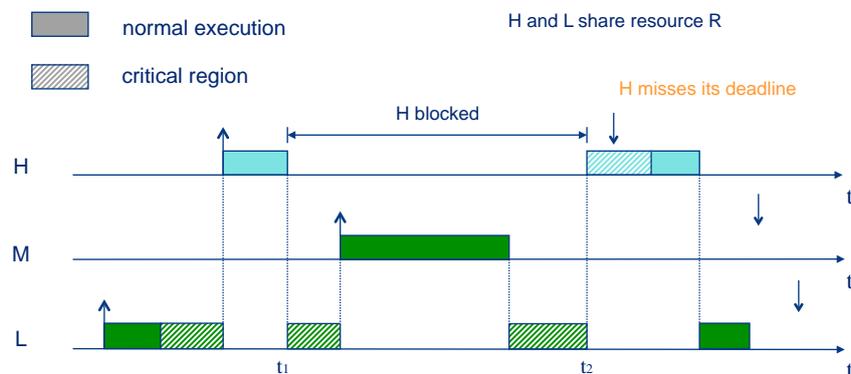
The test can be extended to handle:

- Blocking
- Start-time variations ("release jitter")
- Time offsets
- ...

In this course, we only study blocking.

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Deadline inversion



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Stack Resource Policy (SRP)

The tasks are assigned preemption levels, the properties of which are:

- The preemption level of task τ_i is denoted π_i
- Task τ_i is not allowed to preempt another task τ_j unless $\pi_i > \pi_j$
- If τ_i has higher priority than τ_j and arrives later, then τ_i must have a higher preemption level than τ_j .

Note:

- The preemption levels are static values, even though the tasks priorities may be dynamic.
- For EDF scheduling, suitable levels can be derived if tasks with shorter relative deadlines get higher preemption levels, that is:

$$\pi_i > \pi_j \Leftrightarrow D_i < D_j$$

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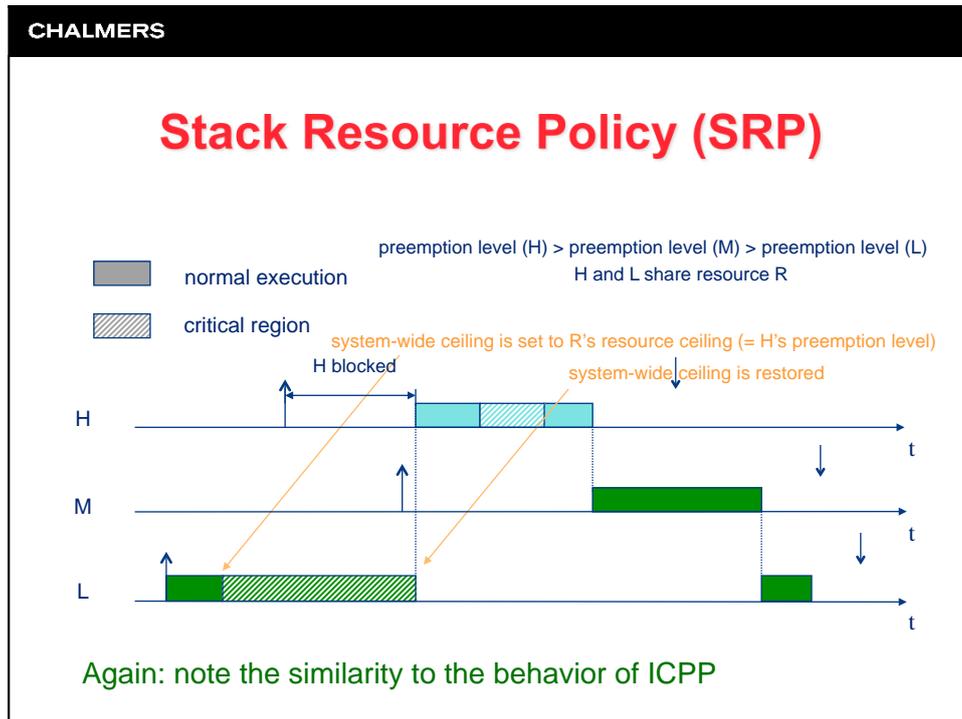
Stack Resource Policy (SRP)

Deadline inversion can be reduced with resource ceilings:

1. Each shared resource is assigned a ceiling that is always equal to the maximum preemption level among all tasks that may be blocked when requesting the resource.
2. The protocol keeps a system-wide ceiling that is equal to the maximum of the current ceilings of all resources.
3. A task with the earliest deadline is allowed to preempt only if its preemption level is higher than the system-wide ceiling.

Note:

- The original priority of the task is not changed at run-time.
- The resource ceiling is a dynamic value calculated at run-time as a function of current resource availability.
- Otherwise, the behavior of the SRP protocol is very similar to the ICPP, and SRP also exhibits identical properties regarding maximum blocking time and freedom from deadlock.



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Extended processor-demand analysis

Blocking can be accounted for in the following way:

- Blocking factor B_i represents the length of critical / non-preemptive regions that are executed by tasks with lower preemption levels than τ_i
- Tasks are indexed in the order of increasing preemption levels, that is: $\pi_i > \pi_j \Leftrightarrow i < j$

$$\forall L \in K, \forall i \in [1, n]: C_p^i(0, L) \leq L$$

$$C_p^i = \sum_{k=1}^i \left(\left\lfloor \frac{L - D_k}{T_k} \right\rfloor + 1 \right) C_k + \left(\left\lfloor \frac{L - D_i}{T_i} \right\rfloor + 1 \right) B_i$$

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Extended processor-demand analysis

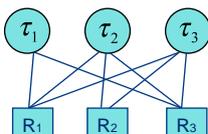
Determining the blocking factor for task τ_i :

1. Determine the worst-case resource ceiling for each critical region, that is, assume the run-time situation where the corresponding resource is unavailable.
2. Identify the tasks that have a preemption level lower than τ_i and that calls critical regions with a worst-case resource ceiling equal to or higher than the preemption level of τ_i .
3. Consider the times that these tasks lock the actual critical regions. The longest of those times constitutes the blocking factor B_i .

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Example: scheduling using EDF

Problem: Assume a system with tasks according to the figure below. The timing properties of the tasks are given in the table. Three resources R1, R2 and R3 have three, one, and three units available, respectively. The parameters H_{R1} , H_{R2} and H_{R3} represent the longest time a task may use the corresponding resource. The parameters μ_{R1} , μ_{R2} and μ_{R3} represent the number of units a task requests from the corresponding resource.



Task	C_i	D_i	T_i	H_{R1}	H_{R2}	H_{R3}	μ_{R1}	μ_{R2}	μ_{R3}
τ_1	6	10	50	2	-	2	1	-	1
τ_2	7	17	50	1	2	2	2	1	3
τ_3	10	25	50	2	3	2	3	1	1

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Example: scheduling using EDF

Problem: (cont'd)

Task τ_1 first requests R_3 and then, while using R_3 , requests R_1
 Task τ_2 first requests R_3 and then, while using R_3 , requests R_2 ;
 then, after releasing the two resources, τ_2 requests R_1
 Task τ_3 first requests R_2 and then, while using R_2 , requests R_1 ;
 then, after releasing the two resources, τ_3 requests R_3

Examine the schedulability of the tasks when the SRP (Stack Resource Policy) protocol is used.

- Derive the ceilings (dynamic and worst-case) of the resources.
- Derive the blocking factors for the tasks.
- Show whether the tasks are schedulable or not.

We solve this on the blackboard!

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Feasibility tests

Summary

	$D_i = T_i$	$D_i \leq T_i$
Static priority (RM/DM)	$U \leq n(2^{1/n} - 1)$	$\forall i : R_i = C_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_j}{T_j} \right\rceil C_j \leq D_i$
Dynamic priority (EDF)	$U \leq 1$	$\forall L : \sum_{i=1}^n \left(\left\lfloor \frac{L - D_i}{T_i} \right\rfloor + 1 \right) C_i \leq L$