1a)
The FTU can be viewed as a parallel system with three computer nodes:

$$R_{sys} = 1 - \prod_{i=1}^{3}(1 - R_{CM}) = 1 - (1 - R_{CM})^3 = 3R_{CM} - 3R_{CM}^{\ 2} + R_{CM}^{\ 3}$$

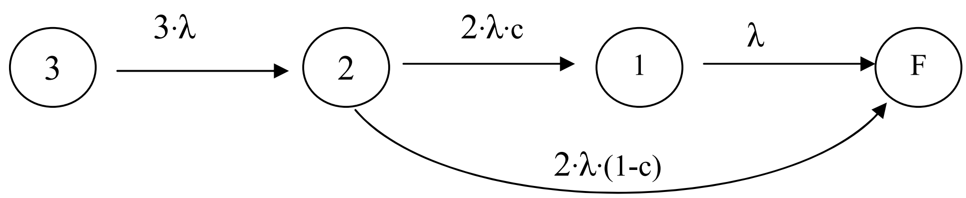
$$R_{CM}(t) = e^{-\lambda t} \Rightarrow R_{sys}(t) = 3e^{-\lambda t} - 3e^{-2\lambda t} + e^{-3\lambda t}$$

1b)

$$MTTF = \int_0^\infty R_{sys}(t)dt = \int_0^\infty 3e^{-\lambda t} - 3e^{-2\lambda t} + e^{-3\lambda t}dt = \left[-\frac{3}{\lambda}e^{-\lambda t} + \frac{3}{2\lambda}e^{-2\lambda t} - \frac{1}{3\lambda}e^{-3\lambda t}\right]_0^\infty =$$

$$= \frac{3}{\lambda} - \frac{3}{2\lambda} + \frac{1}{3\lambda} = \frac{18 - 9 + 2}{6\lambda} = \frac{11}{6\lambda}$$

1c)



States:
- 3: Three computer nodes OK
- 2: Two computer nodes OK, one computer node broken
- 1: One computer node OK, two computer nodes broken
- F: All computer nodes broken

This gives the following equation system:

$$\overline{P}'(t) = \overline{P}(t) \cdot \overline{Q}$$

$$\overline{P}(t) = \left[P_3(t)\ \ P_2(t)\ \ P_1(t)\ \ P_F(t)\right]$$

$$\overline{Q} = \begin{bmatrix} -3\lambda & 3\lambda & 0 & 0 \\ 0 & -2\lambda & 2\lambda c & 2\lambda(1-c) \\ 0 & 0 & -\lambda & \lambda \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

We obtain:

$$P_3'(t) = -3\lambda \cdot P_3(t)$$

$$P_2'(t) = 3\lambda \cdot P_3(t) - 2\lambda \cdot P_2(t)$$

$$P_1'(t) = 2\lambda c \cdot P_2(t) - \lambda \cdot P_1(t)$$

$$P_F'(t) = 2\lambda(1-c) \cdot P_2(t) + \lambda \cdot P_1(t)$$

We solve the equation system using Laplace transform:

$$s \cdot \widetilde{P}_3(s) - 1 = -3\lambda \cdot \widetilde{P}_3(s) \qquad (1)$$

$$s \cdot \widetilde{P}_2(s) = 3\lambda \cdot \widetilde{P}_3(s) - 2\lambda \cdot \widetilde{P}_2(s) \qquad (2)$$

$$s \cdot \widetilde{P}_1(s) = 2\lambda c \cdot \widetilde{P}_2(s) - \lambda \cdot \widetilde{P}_1(s) \qquad (3)$$

From (1) , (2) and (3) we obtain:

$$\tilde{P}_3(s) = \frac{1}{s+3\lambda}$$

$$\tilde{P}_2(s) = \frac{3\lambda}{(s+2\lambda)\cdot(s+3\lambda)} = 3\lambda\left[\frac{A}{s+2\lambda} + \frac{B}{s+3\lambda}\right] = \left\{\mathrm{A} = \frac{1}{\lambda}, \mathrm{B} = -\frac{1}{\lambda}\right\} = \frac{3}{s+2\lambda} - \frac{3}{s+3\lambda}$$

$$\tilde{P}_1(s) = \frac{2\lambda c}{s+\lambda}\cdot\tilde{P}_2(s) = \frac{6\lambda c}{(s+\lambda)\cdot(s+2\lambda)} - \frac{6\lambda c}{(s+\lambda)\cdot(s+3\lambda)}$$

$$= 6\lambda c\cdot\left[\frac{A}{s+\lambda} + \frac{B}{s+2\lambda} - \left(\frac{C}{s+\lambda} + \frac{D}{s+3\lambda}\right)\right] = \left\{\mathrm{A} = \frac{1}{\lambda}, \mathrm{B} = -\frac{1}{\lambda}, \mathrm{C} = \frac{1}{2\lambda}, \mathrm{D} = -\frac{1}{2\lambda}\right\}$$

$$= 6c\cdot\left[\frac{1}{s+\lambda} - \frac{1}{s+2\lambda} - \frac{1}{2(s+\lambda)} + \frac{1}{2(s+3\lambda)}\right] = \frac{3c}{s+\lambda} - \frac{6c}{s+2\lambda} + \frac{3c}{s+3\lambda}$$

The inverse Laplace transform give:

$$P_3(t) = e^{-3\lambda t}$$

$$P_2(t) = 3e^{-2\lambda t} - 3e^{-3\lambda t}$$

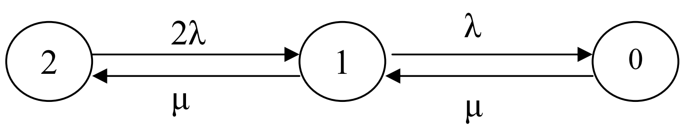
$$P_1(t) = 3c\cdot e^{-3\lambda t} - 6c\cdot e^{-2\lambda t} + 3c\cdot e^{-\lambda t}$$

The reliability is:

$$R(t) = P_3(t) + P_2(t) + P_1(t) = e^{-3\lambda t} + 3e^{-2\lambda t} - 3e^{-3\lambda t} + 3c\cdot e^{-3\lambda t} - 6c\cdot e^{-2\lambda t} + 3c\cdot e^{-\lambda t}$$

$$= 3c\cdot e^{-\lambda t} + 3(1-2c)\cdot e^{-2\lambda t} + (3c-2)\cdot e^{-3\lambda t}$$

2a)



States:

2: Two computer nodes OK
1: One computer node OK, one computer node broken
0: Two computer nodes broken

Birth-and-death process:

$$\Pi_1 = \frac{2\lambda}{\mu}\cdot\Pi_2 \quad (1)$$

$$\Pi_0 = \frac{\lambda}{\mu}\cdot\Pi_1 = \frac{2\lambda^2}{\mu^2}\cdot\Pi_2 \quad (2)$$

$$1 = \Pi_2 + \Pi_1 + \Pi_0 \quad (3)$$

From (1), (2) and (3), we obtain:

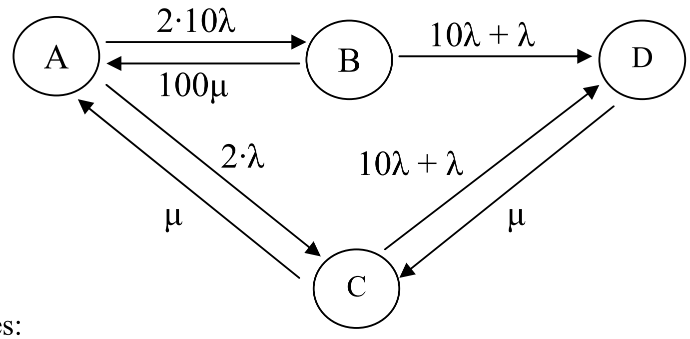
$$\Pi_2\cdot\left(1 + \frac{2\lambda}{\mu} + \frac{2\lambda^2}{\mu^2}\right) = 1 \quad \Rightarrow \quad \Pi_2 = \frac{\mu^2}{\mu^2 + 2\lambda\mu + 2\lambda^2} \quad (4)$$

From (1) and (4), we obtain:

$$\Pi_1 = \frac{2\lambda\mu}{\mu^2 + 2\lambda\mu + 2\lambda^2}$$

$$A_{(\infty)} = \Pi_2 + \Pi_1 = \frac{\mu^2 + 2\lambda\mu}{\mu^2 + 2\lambda\mu + 2\lambda^2}$$

2b)



States:

A: Two nodes OK
B: One node OK, one node broken due to transient fault
C: One node OK, one node broken
D: Two nodes broken

This gives the following equation system:

$$\overline{P}'(t) = \overline{P}(t) \cdot \overline{Q}$$
$$\overline{P}(t) = \begin{bmatrix} P_A & P_2 & P_1 & P_0 \end{bmatrix}$$

$$\overline{Q} = \begin{bmatrix} -22\lambda & 20\lambda & 2\lambda & 0 \\ 100\mu & -(11\lambda + 100\mu) & 0 & 11\lambda \\ \mu & 0 & -(11\lambda + \mu) & 11\lambda \\ 0 & 0 & \mu & -\mu \end{bmatrix}$$

Let $t \to \infty$ => $P_i'(t) = 0, \ P_i(t) = \Pi_i$:

$$0 = -22\lambda \cdot \Pi_A + 100\mu \cdot \Pi_B + \mu \cdot \Pi_C \qquad (1)$$
$$0 = 20\lambda \cdot \Pi_A - (11\lambda + 100\mu) \cdot \Pi_B \qquad (2)$$
$$0 = 2\lambda \cdot \Pi_A - (11\lambda + \mu) \cdot \Pi_1 + \mu \cdot \Pi_D \qquad (3)$$
$$0 = 11\lambda \cdot \Pi_B + 11\lambda \cdot \Pi_C - \mu \cdot \Pi_D \qquad (4)$$
$$1 = \Pi_A + \Pi_B + \Pi_C + \Pi_D \qquad (5)$$

From (2), we get:

$$\Pi_A = \frac{11\lambda + 100\mu}{20\lambda} \cdot \Pi_B \qquad (6)$$

From (1) and (6), we get:

$$\mu \cdot \Pi_c = \frac{22(11\lambda + 100\mu)}{20} \cdot \Pi_B - 100\mu \cdot \Pi_B \quad => \quad \Pi_c = \frac{121\lambda + 100\mu}{10\mu} \cdot \Pi_B \qquad (7)$$

From (4) and (7), we get:

$$\Pi_D = \frac{11\lambda}{\mu} \cdot \Pi_B + \frac{11\lambda}{\mu} \cdot \left(\frac{121\lambda + 100\mu}{10\mu} \right) \cdot \Pi_B = \frac{1331\lambda^2 + 1210\lambda\mu}{10\mu^2} \cdot \Pi_B \qquad (8)$$

From (5), (6), (7) and (8), we obtain:

$$1 = \left(\frac{11\lambda + 100\mu}{20\lambda} + 1 + \frac{121\lambda + 100\mu}{10\mu} + \frac{1331\lambda^2 + 1210\lambda\mu}{10\mu^2} \right) \cdot \Pi_B =$$

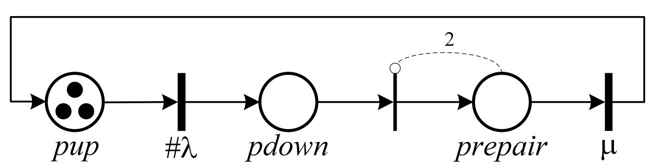
$$= \frac{11\lambda\mu^2 + 100\mu^3 + 20\lambda\mu^2 + 242\lambda^2\mu + 200\lambda\mu^2 + 2662\lambda^3 + 2420\lambda^2\mu}{20\lambda\mu^2} \cdot \Pi_B =>$$

$$\Pi_B = \frac{20\lambda\mu^2}{100\mu^3 + 231\lambda\mu^2 + 2662\lambda^2\mu + 2662\lambda^3}$$

$$A_{(\infty)} = \Pi_A + \Pi_B + \Pi_C = \frac{(11\lambda + 100\mu) \cdot \mu^2 + 20\lambda\mu^2 + (121\lambda + 100\mu) \cdot 2\lambda\mu}{100\mu^3 + 231\lambda\mu^2 + 2662\lambda^2\mu + 2662\lambda^3} =$$

$$= \frac{100\mu^3 + 231\lambda\mu^2 + 242\lambda^2\mu}{100\mu^3 + 231\lambda\mu^2 + 2662\lambda^2\mu + 2662\lambda^3}$$

3. The system is described by the following GSPN model:



Markings of the GSPN model are represented as (*#pup #pdown #prepair*). The marking (0 1 2) corresponds to the event that the system is unavailable.