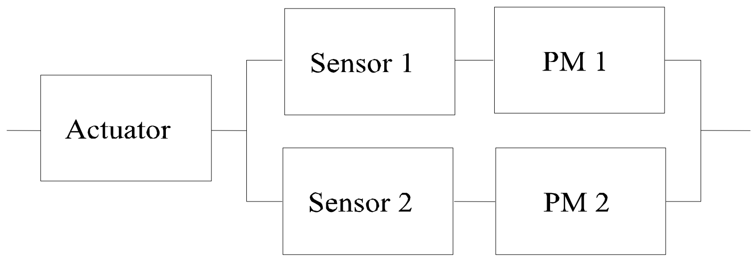
1.

a) The system consists of the following fault containment regions: S1+PM1, S2+PM2, A, and each communication bus. Thus, in total we have 5 fault containment regions.

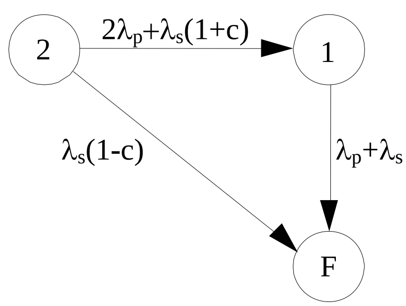
b) We obtain the following reliability block diagram:



Let R_{ps} denote the reliability of a processor/sensor pair and R_{2ps} the reliability of the processor/sensor-subsystem. Let R_a denote the reliability of the actuator and R_{sys} the reliability of the node. We then obtain:

$$R_{ps} = e^{-(\lambda_p+\lambda_s)\cdot t}$$
$$R_{2ps} = 2R_{ps} - R_{ps}^2 = 2e^{-(\lambda_p+\lambda_s)\cdot t} - e^{-2(\lambda_p+\lambda_s)\cdot t}$$
$$R_a = e_a^{-\lambda} \cdot t$$
$$R_{sys} = R_{2ps} \cdot R_a = e^{-\lambda_a \cdot t}\left(2e^{-(\lambda_p+\lambda_s)\cdot t} - e^{-2(\lambda_p+\lambda_s)\cdot t}\right)$$

c) The processor/sensor subsystem is modelled using the following Markov model:



From the Markov model, we obtain the following transition rate matrix

$$Q = \begin{bmatrix} -2(\lambda_p + \lambda_s) & 2\lambda_p + \lambda_s(1+c) & \lambda_s(1-c) \\ 0 & -(\lambda_p + \lambda_s) & \lambda_p + \lambda_s \\ 0 & 0 & 0 \end{bmatrix}$$

and the following system of differential equations

$$\begin{cases} P_2'(t) = -2(\lambda_p + \lambda_s) \cdot P_2'(t) \\ P_1'(t) = \left(2\lambda_p + \lambda_s(1+c)\right) \cdot P_2'(t) - (\lambda_p + \lambda_s) \cdot P_1'(t) \\ P_F'(t) = \lambda_s(1-c) \cdot P_2'(t) + (\lambda_p + \lambda_s) \cdot P_1'(t) \end{cases}$$

We also know that

$$P(0) = [1 \quad 0 \quad 0]$$

The equation system is solved using the Laplace transform.

$$\begin{cases} sP_2(s) - 1 = -2(\lambda_p + \lambda_s) \cdot P_2(s) \\ sP_1(s) = \left(2\lambda_p + \lambda_s(1+c)\right) \cdot P_2(s) - (\lambda_p + \lambda_s) \cdot P_1(s) \\ sP_F(s) = \lambda_s(1-c) \cdot P_2(s) + (\lambda_p + \lambda_s) \cdot P_1(s) \end{cases}$$

$$P_2(s) = \frac{1}{s + 2(\lambda_p + \lambda_s)}$$

$$P_1(s) = \frac{2\lambda_p + \lambda_s(1+c)}{s + (\lambda_p + \lambda_s)} \cdot \mathrm{P}_2(\mathrm{s}) = \frac{2\lambda_p + \lambda_s(1+c)}{s + (\lambda_p + \lambda_s)} \cdot \frac{1}{s + 2(\lambda_p + \lambda_s)}$$

$$= \frac{2\lambda_p + \lambda_s(1+c)}{\lambda_p + \lambda_s} \cdot \left(\frac{1}{s + (\lambda_p + \lambda_s)} - \frac{1}{s + 2(\lambda_p + \lambda_s)}\right)$$

$$P_2(t) = e^{-2(\lambda_p + \lambda_s) \cdot t}$$

$$P_1(t) = \frac{2\lambda_p + \lambda_s(1+c)}{\lambda_p + \lambda_s}\left(e^{-(\lambda_p + \lambda_s) \cdot t} - e^{-2(\lambda_p + \lambda_s) \cdot t}\right)$$

We denote the reliability of the processor/sensor subsystem R_{2ps}.

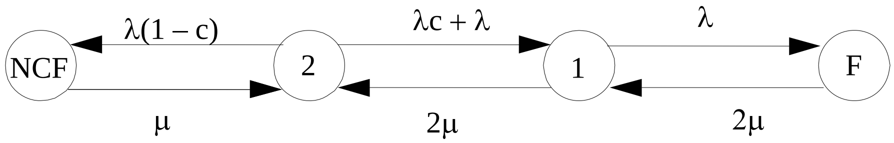
$$R_{2ps}(t) = \mathrm{P}_2(\mathrm{t}) + \mathrm{P}_1(\mathrm{t})$$

$$= e^{-(2\lambda_p + \lambda_s) \cdot t} + \frac{2\lambda_p + \lambda_s(1+c)}{\lambda_p + \lambda_s}\left(e^{-(\lambda_p + \lambda_s) \cdot t} - e^{-2(\lambda_p + \lambda_s) \cdot t}\right)$$

The reliability of the FTU is

$$R_{sys}(t) = R_{2ps}(t) \cdot R_a(t).$$

2. We use the following Markov chain model for the FTU:



$$\Pi_2 = \frac{\mu}{\lambda(1-c)}\Pi_{NCF}$$

$$\Pi_1 = \frac{\lambda(1+c)}{2\mu}\Pi_2 = \frac{1+c}{2(1-c)}\Pi_{NCF}$$

$$\Pi_F = \frac{\lambda}{2\mu}\Pi_1 = \frac{\lambda(1+c)}{4\mu(1-c)}\Pi_{NCF}$$

$$1 = \Pi_{NCF} + \Pi_2 + \Pi_1 + \Pi_F$$

$$1 = \Pi_{NCF}\left(1 + \frac{\mu}{\lambda(1-c)} + \frac{1+c}{2(1-c)} + \frac{\lambda(1+c)}{4\mu(1-c)}\right)$$

$$= (4\lambda\mu(1-c) + 4\mu\text{^}2 + 2\lambda\mu(1+c) + \lambda^2(1+c))/4\lambda\mu(1-c)$$

$$\Pi_{NCF} = \frac{4\lambda\mu(1-c)}{4\lambda\mu(1-c) + 4\mu^2 + 2\lambda\mu(1+c) + \lambda^2(1+c)}$$

a) Steady-state probability for a non-covered fault:

$$\Pi_{NCF} = \frac{4\lambda\mu(1-c)}{4\lambda\mu(1-c) + 4\mu^2 + 2\lambda\mu(1+c) + \lambda^2(1+c)}$$

b) Steady-state probability that both nodes function correctly:

$$\Pi_2 = \frac{\mu}{\lambda(1-c)}\frac{4\lambda\mu(1-c)}{4\lambda\mu(1-c) + 4\mu^2 + 2\lambda\mu(1+c) + \lambda^2(1+c)}$$

$$= \frac{4\mu^2}{4\lambda\mu(1-c) + 4\mu^2 + 2\lambda\mu(1+c) + \lambda^2(1+c)}$$

c) The steady-state availability of the FTU is

$$\Pi_2 + \Pi_1 = \left(\frac{\mu}{\lambda(1-c)} + \frac{1+c}{2(1-c)}\right)\frac{4\lambda\mu(1-c)}{4\lambda\mu(1-c) + 4\mu^2 + 2\lambda\mu(1+c) + \lambda^2(1+c)}$$

$$= \frac{2\mu + \lambda(1+c)}{2\lambda(1-c)}\ \frac{4\lambda\mu(1-c)}{4\lambda\mu(1-c) + 4\mu^2 + 2\lambda\mu(1+c) + \lambda^2(1+c)}$$

$$= \frac{4\mu^2 + 2\lambda\mu(1+c)}{4\lambda\mu(1-c) + 4\mu^2 + 2\lambda\mu(1+c) + \lambda^2(1+c)}$$

3. The cold standby system can be described using the following GSPN model.