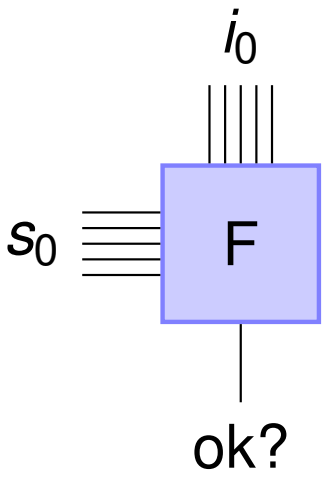
Temporal Induction and SAT-Solving

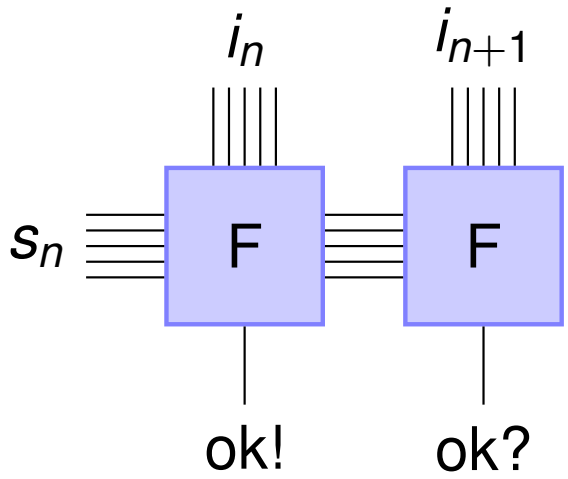
Niklas Sörensson

April 21, 2010

Simple Induction

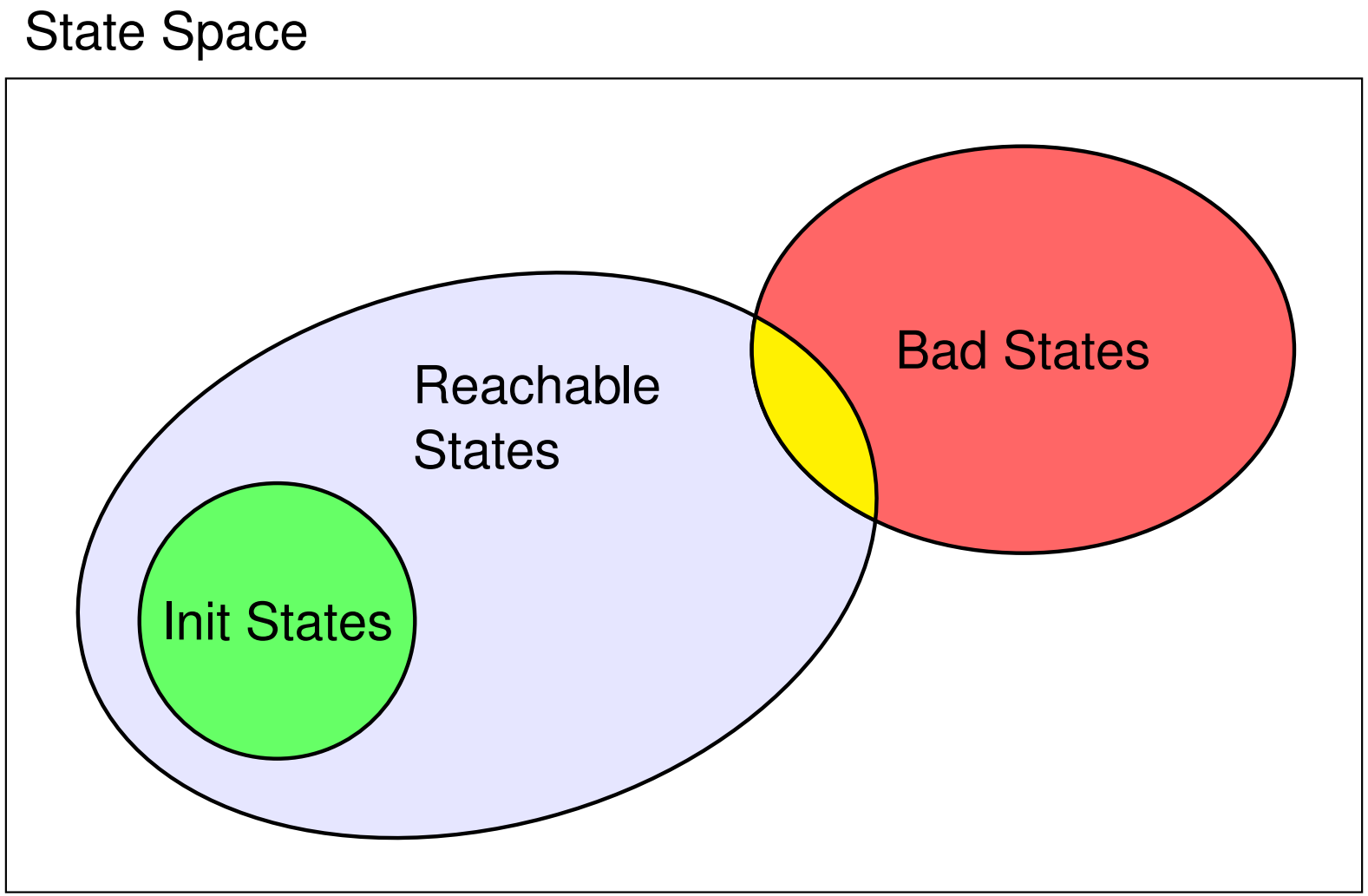


(Base-case)

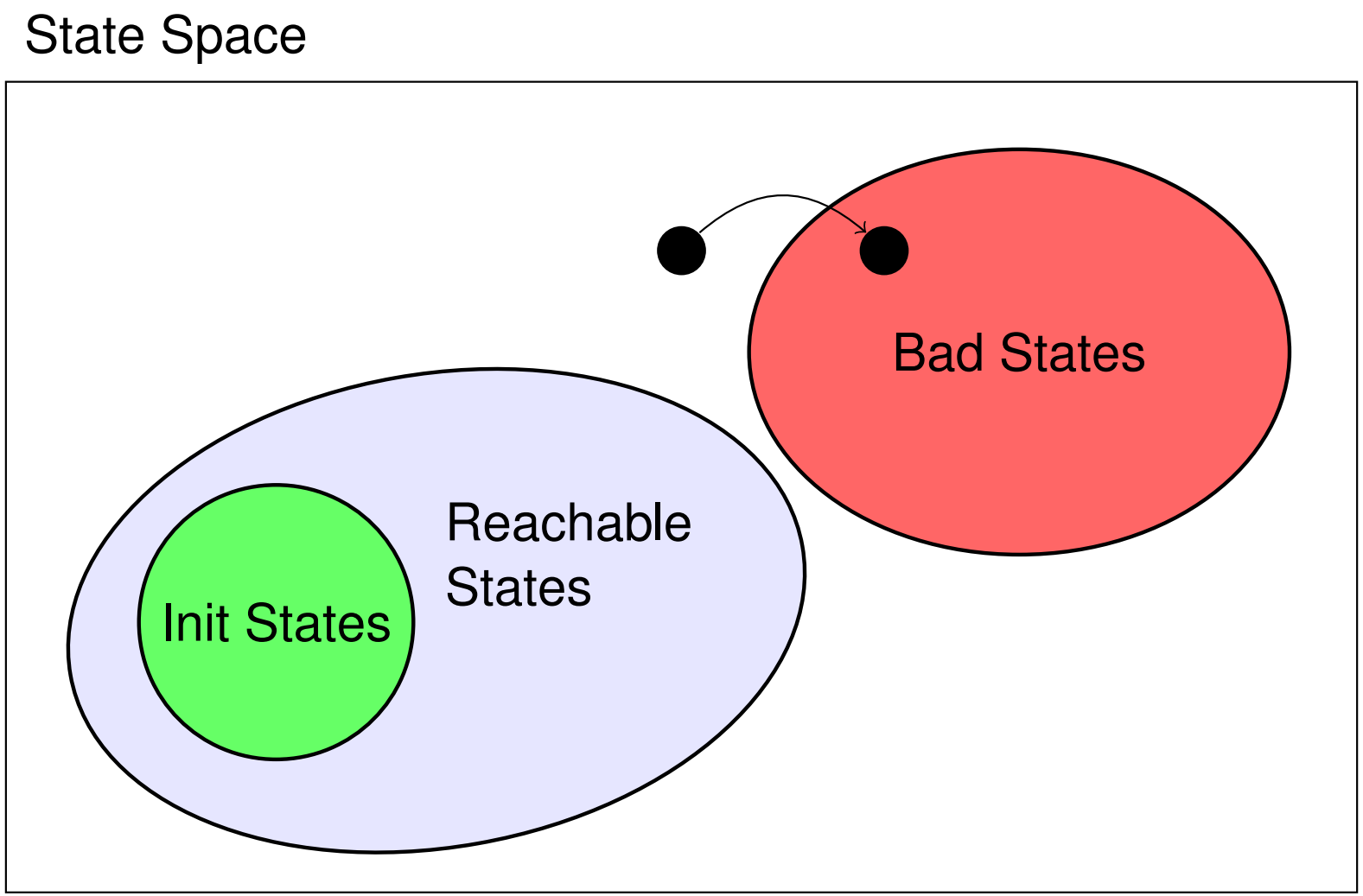


(Step-case)

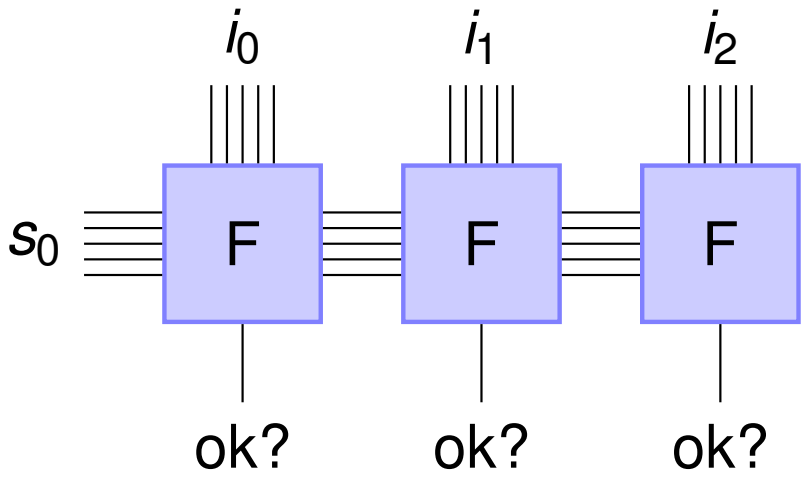
Reachability



Example: Simple Induction Fails



Induction with Depth (k=3)



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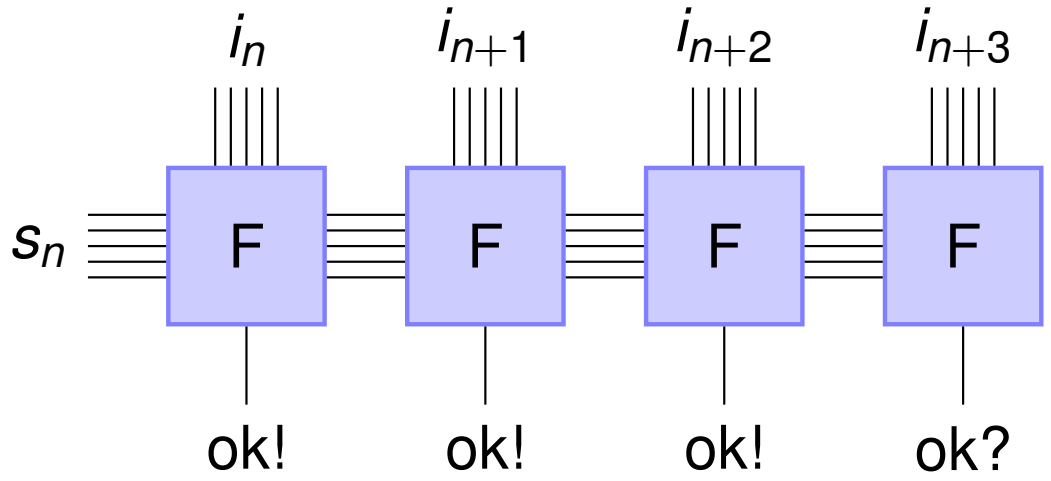

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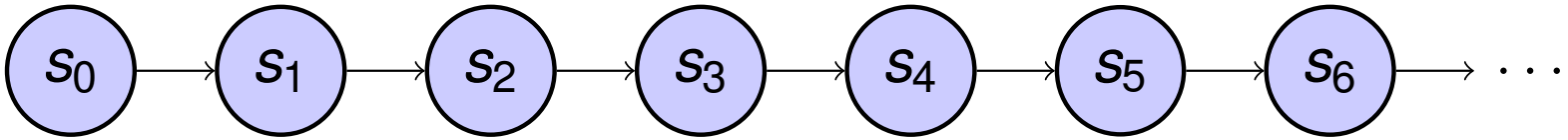

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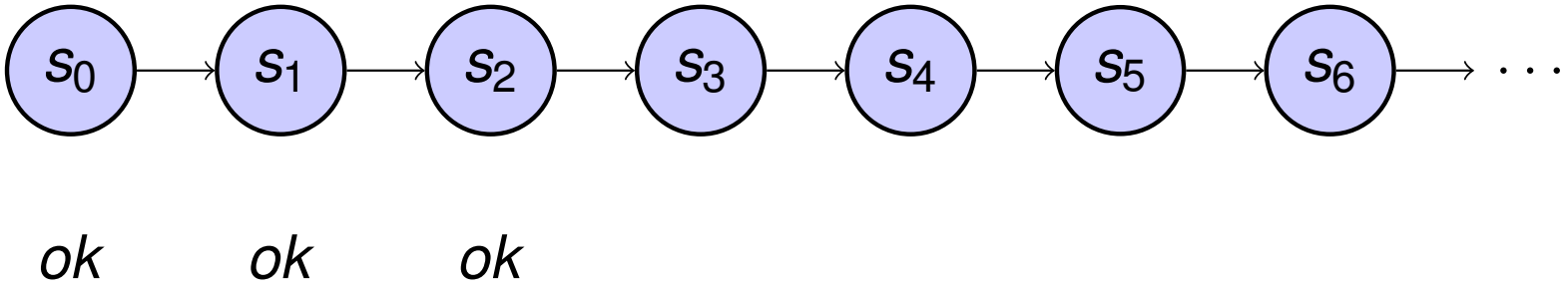

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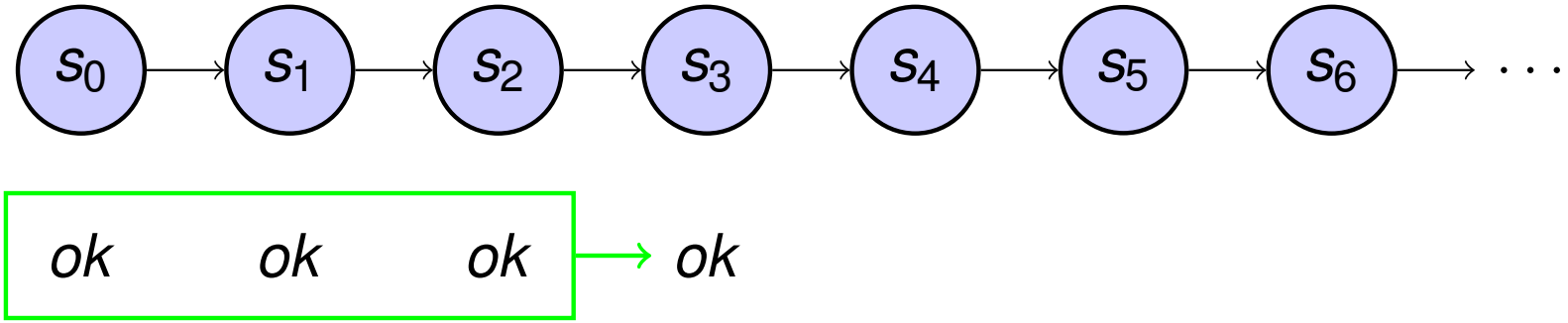

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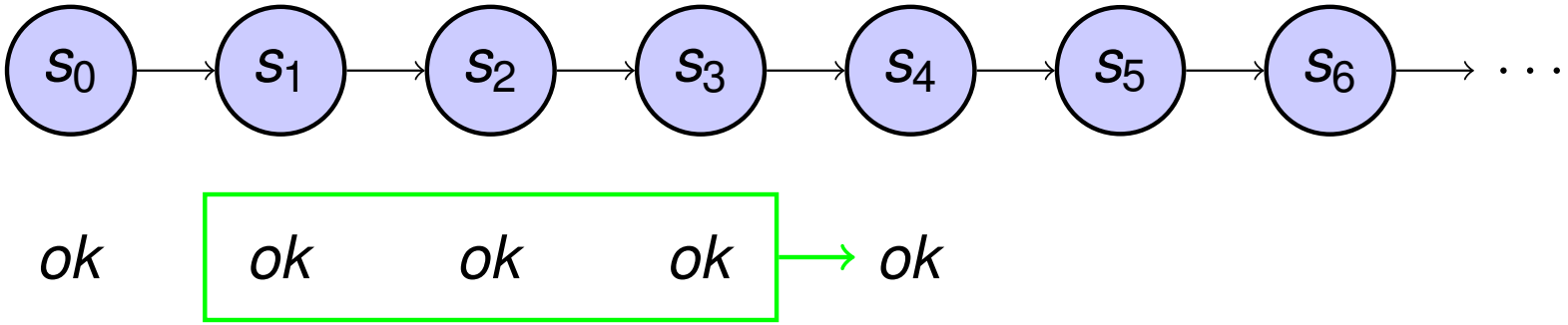

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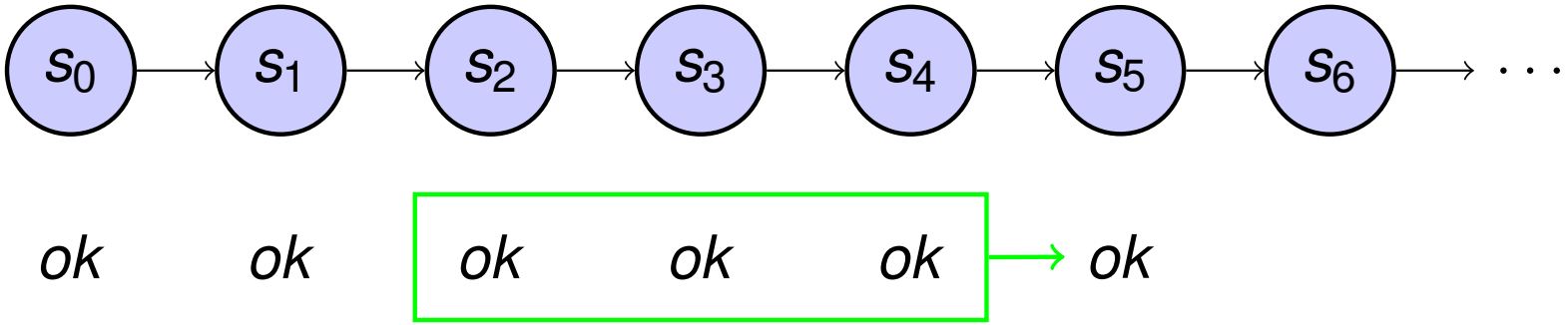

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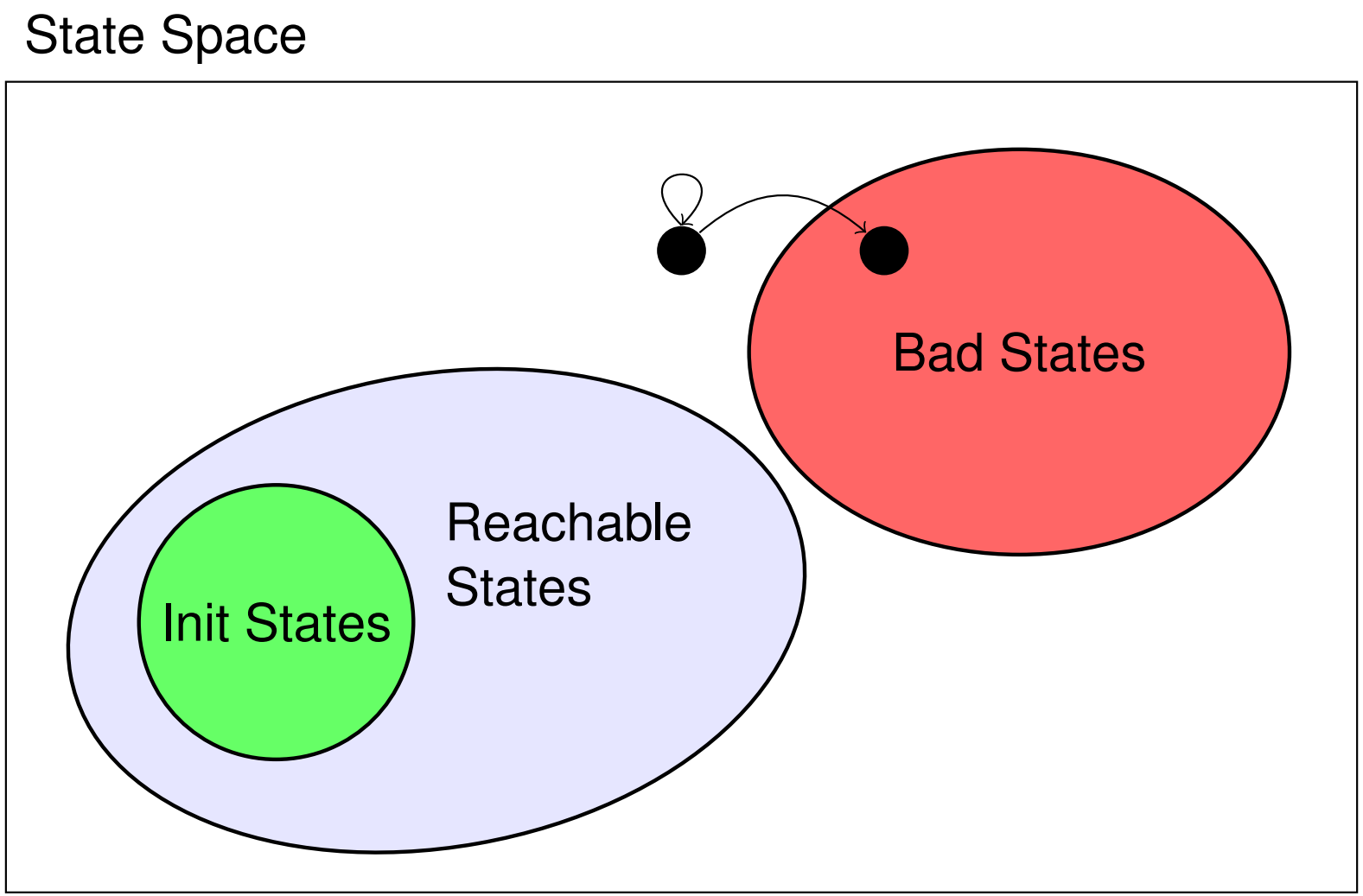

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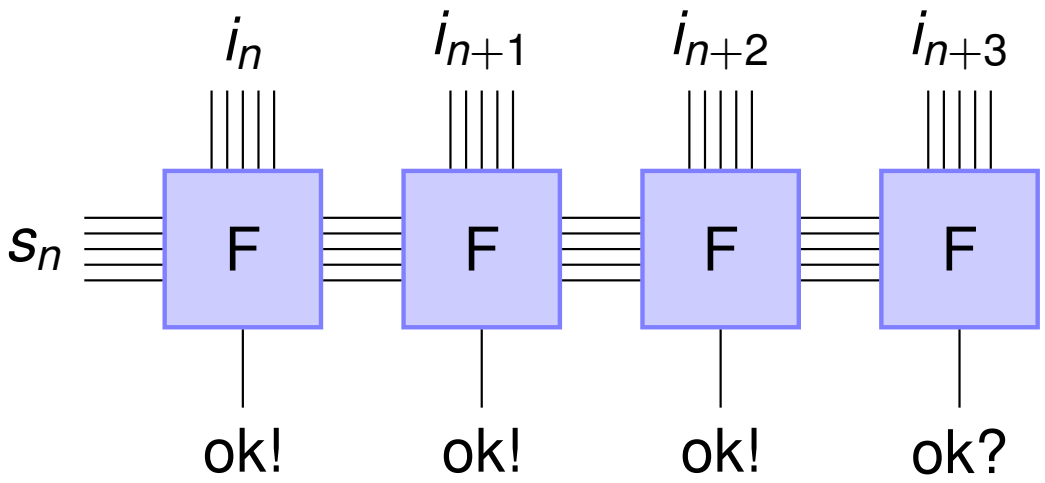

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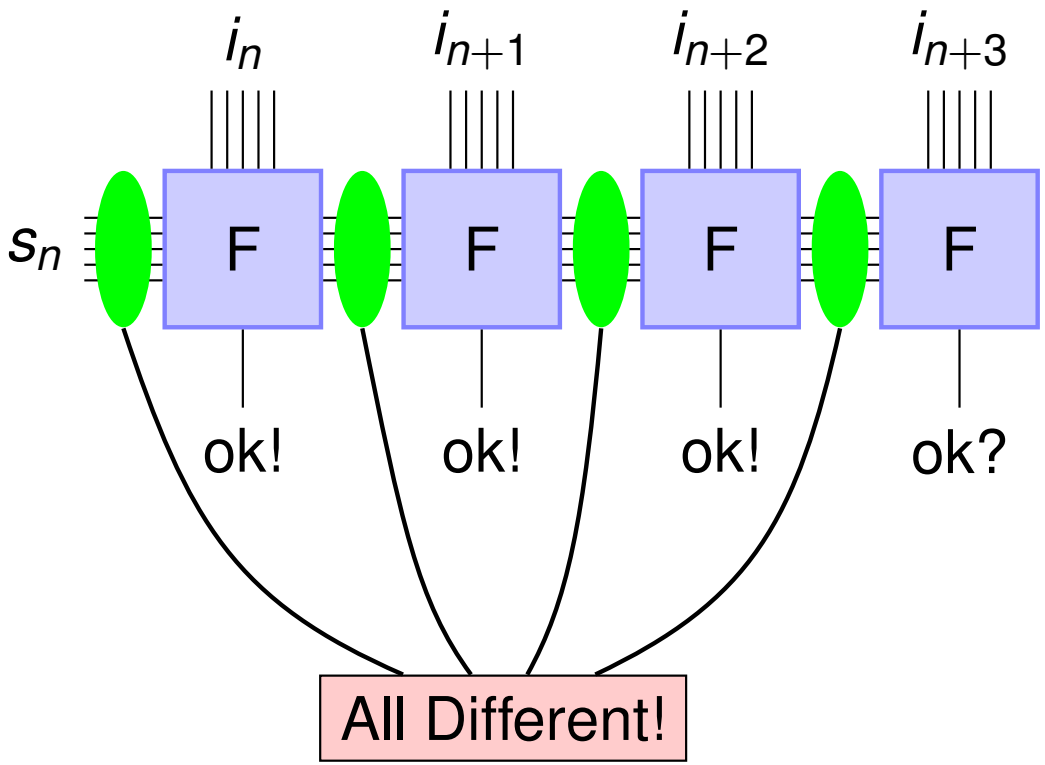

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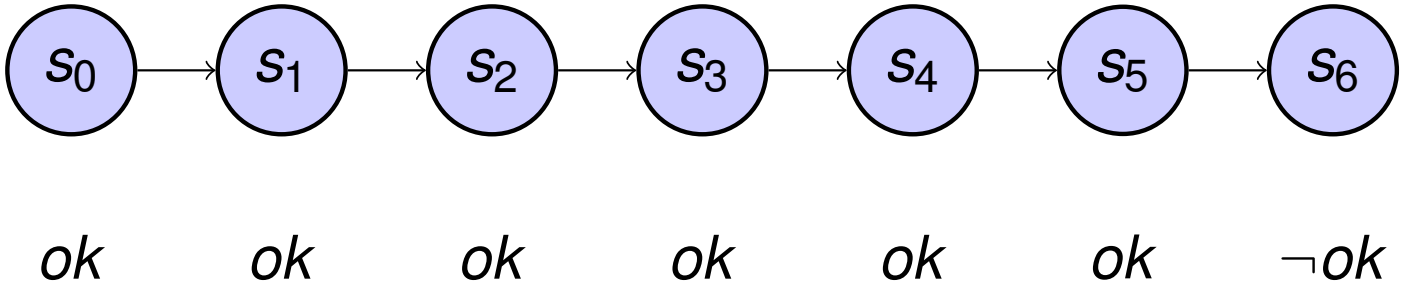

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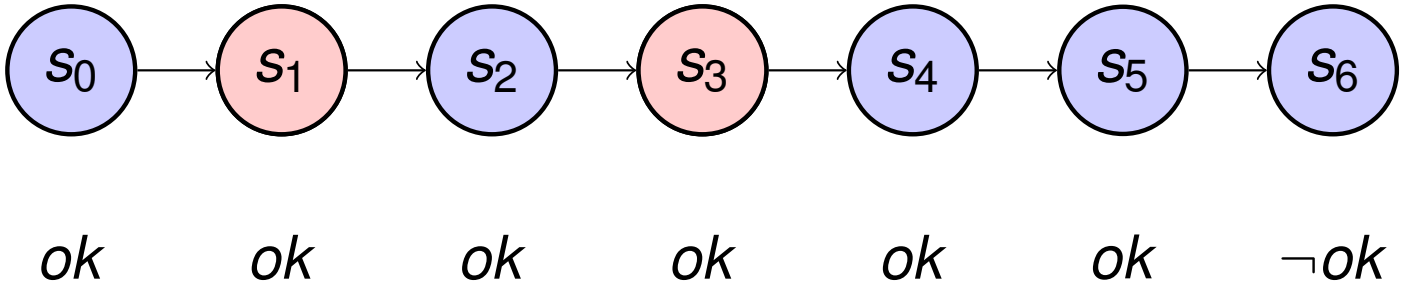

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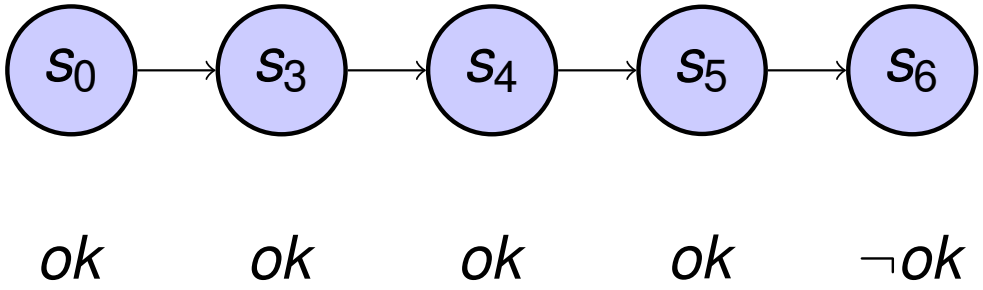

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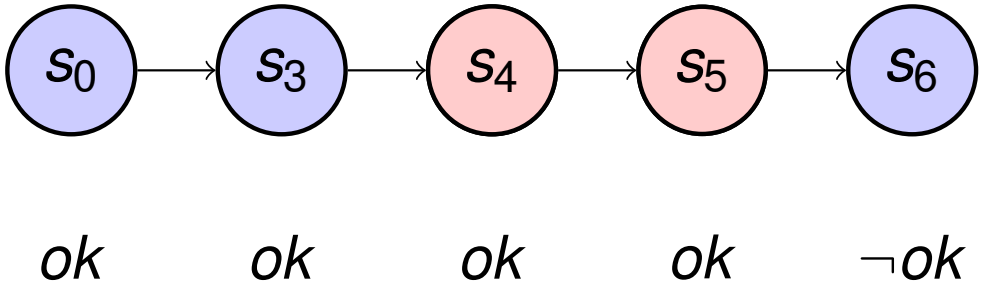

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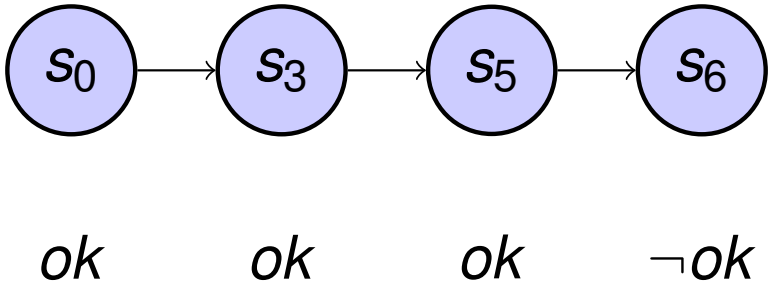

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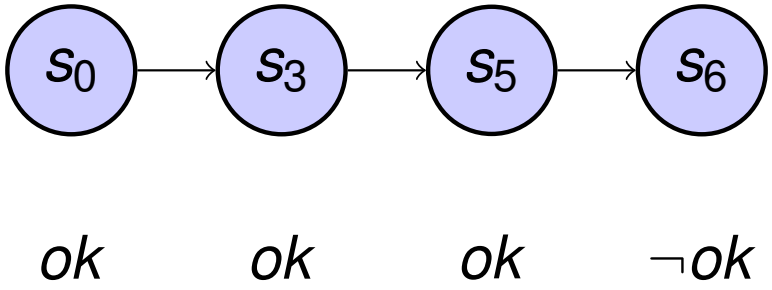
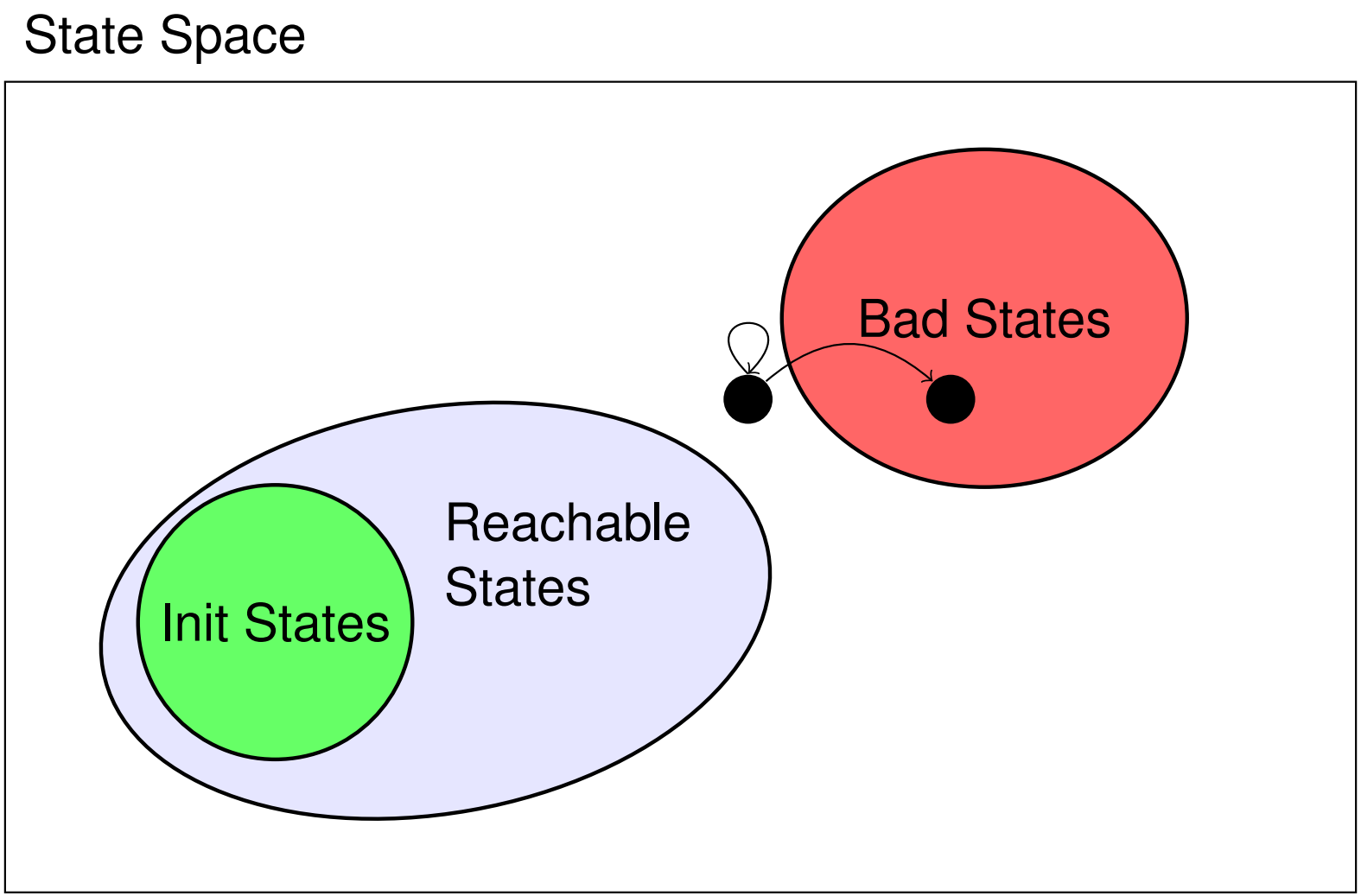


Counter-examples with loops can always be made loop-free!

Property Generalization



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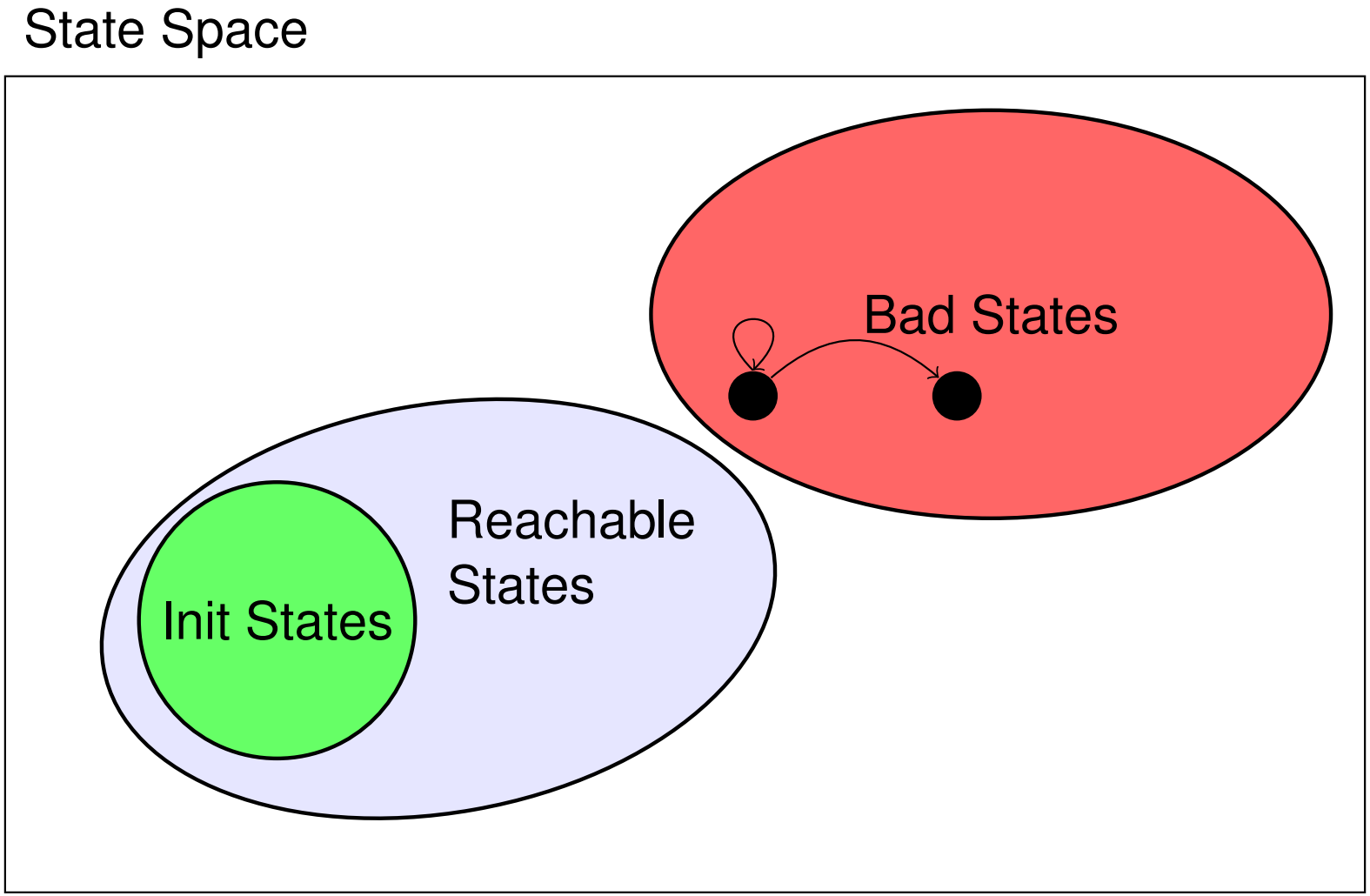

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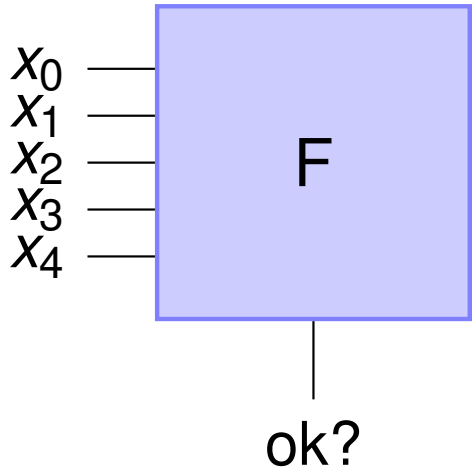
Satisfiability (SAT) in BMC and Induction



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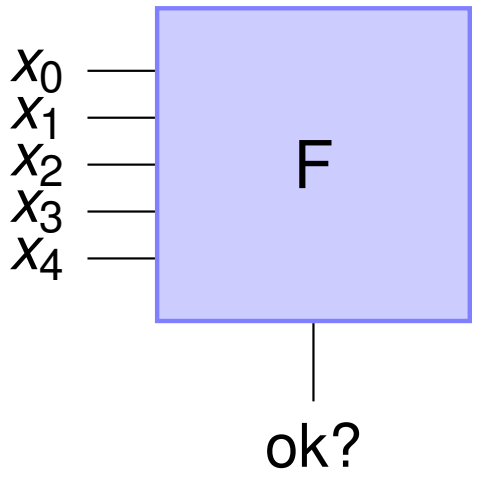

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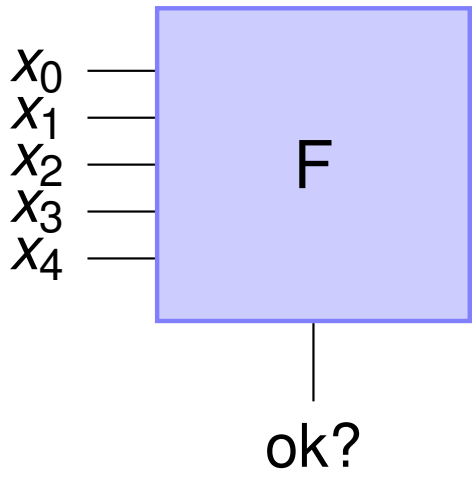

- Find values for inputs $x_0 \ldots x_n$ such that *ok* becomes false
- Or, prove that no such set of values exists

Why is it hard?

x_0	x_1	x_2	x_3	ok
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
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- Enumeration does not work
- Given k inputs there are 2^k combinations

Conjunctive Normal Form

Variables: $a, b, c, \ldots$
Literals: $a, \neg a, b, \neg b, c, \neg c, \ldots$

Clauses: *Disjunctions of literals*

Ex: $(\neg a \vee b, \neg c)$

CNF: *Conjunction of clauses*

Ex: $(\neg a \vee b) \wedge (\neg a \vee c) \wedge (\neg a \vee \neg b \vee c)$

SAT-Problem Definition

- A CNF is *satisfiable* if there is a variable assignment that evaluates all clauses to true
- (i.e. each clause contains at least one true literal)
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- $a = 1, b = 0$ is a satisfying assignment

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- Give fresh (SAT) variables for each input/gate
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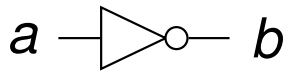

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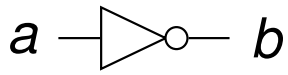

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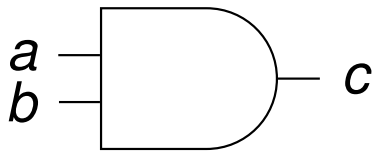

$$(\neg c \vee a)$$
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- Branch on some variable
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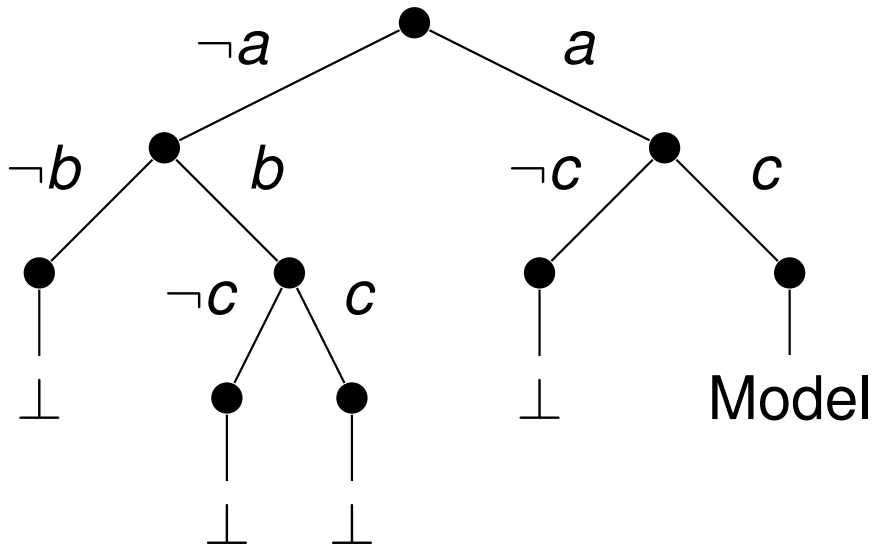

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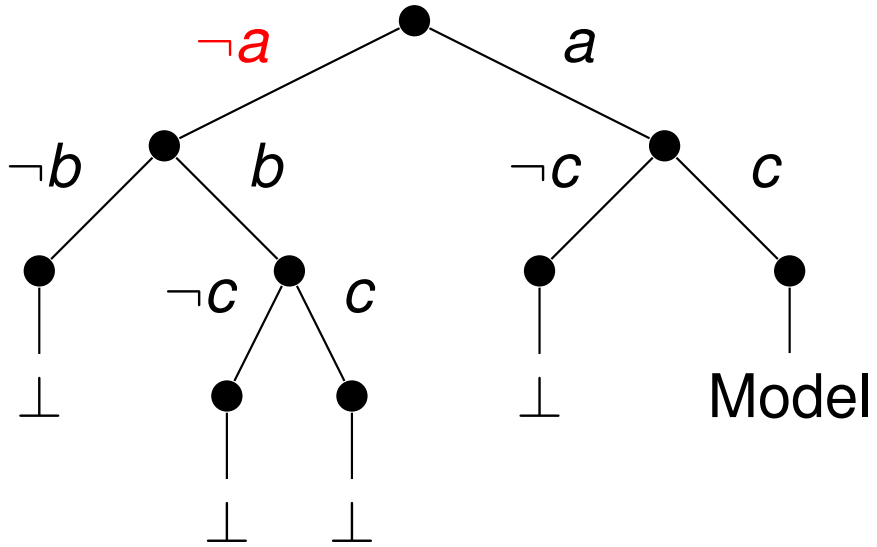

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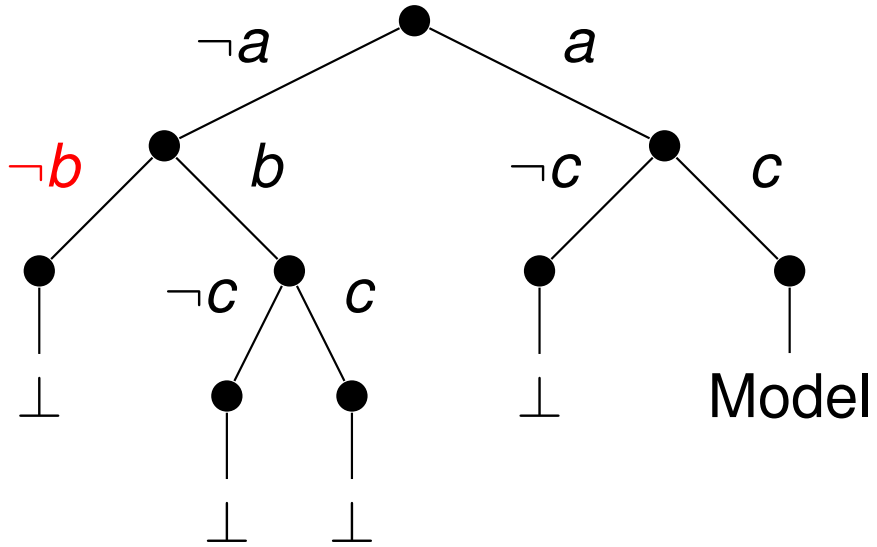

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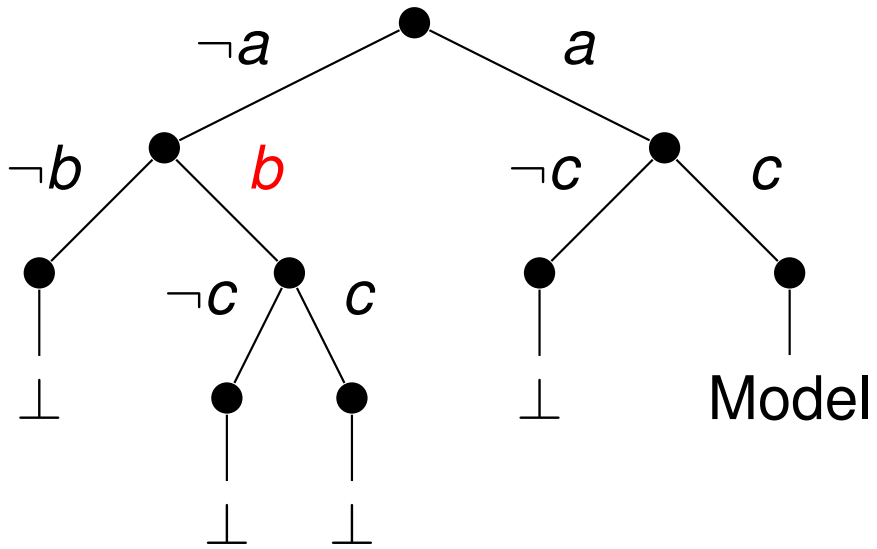

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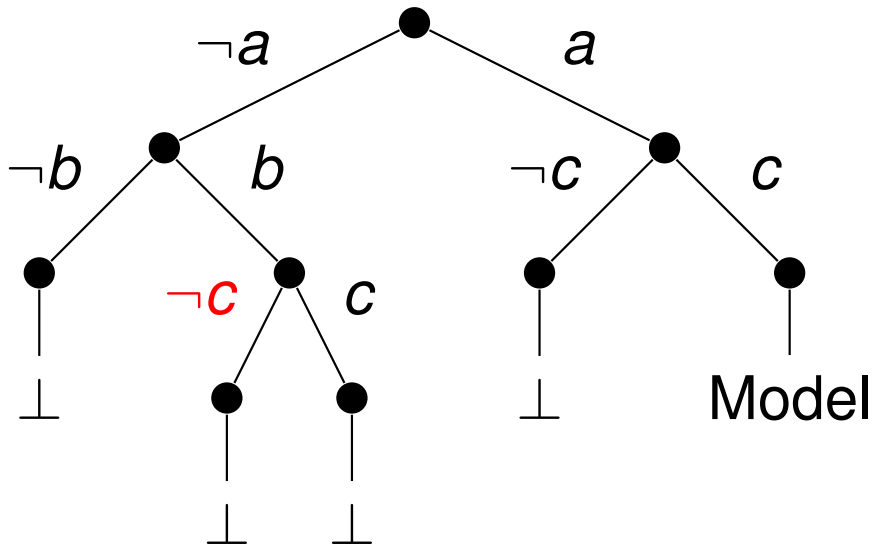

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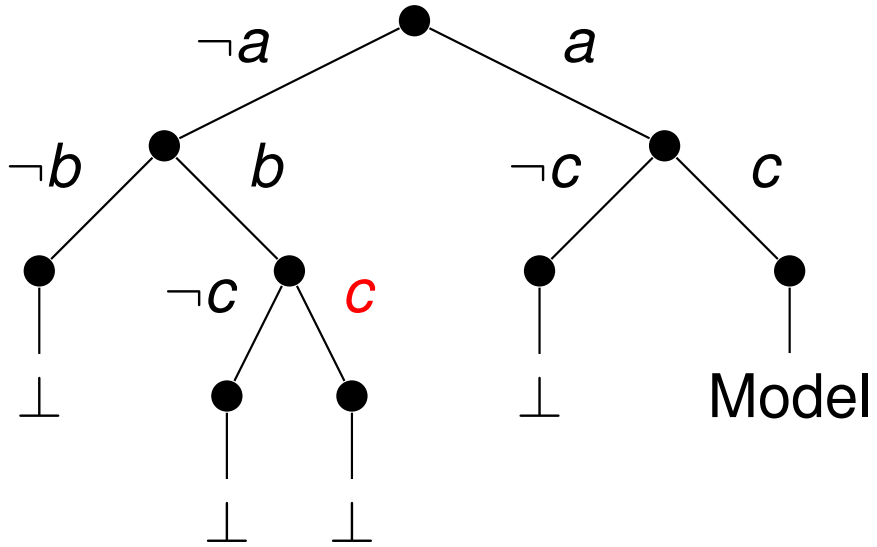

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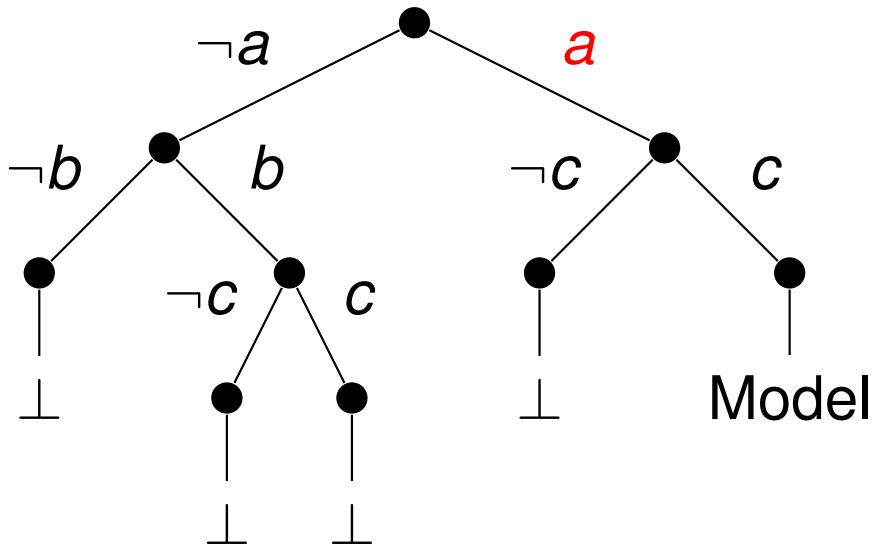

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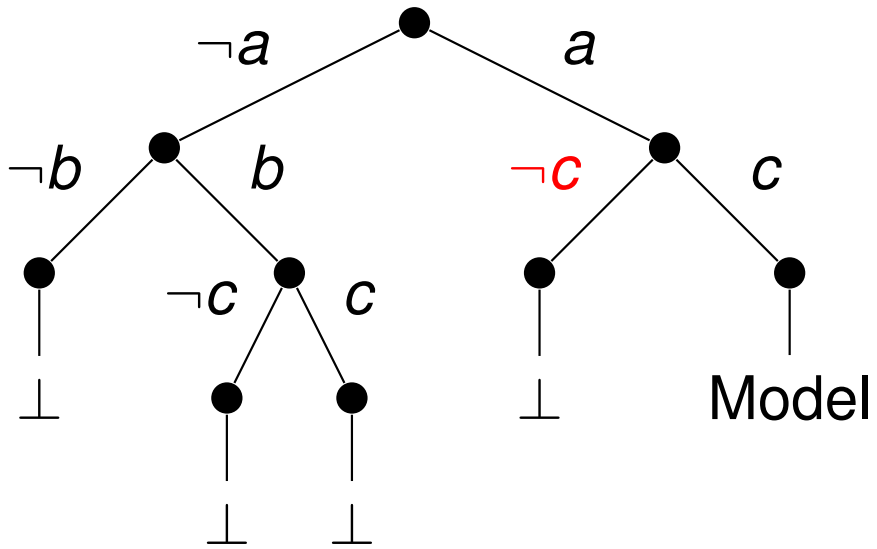

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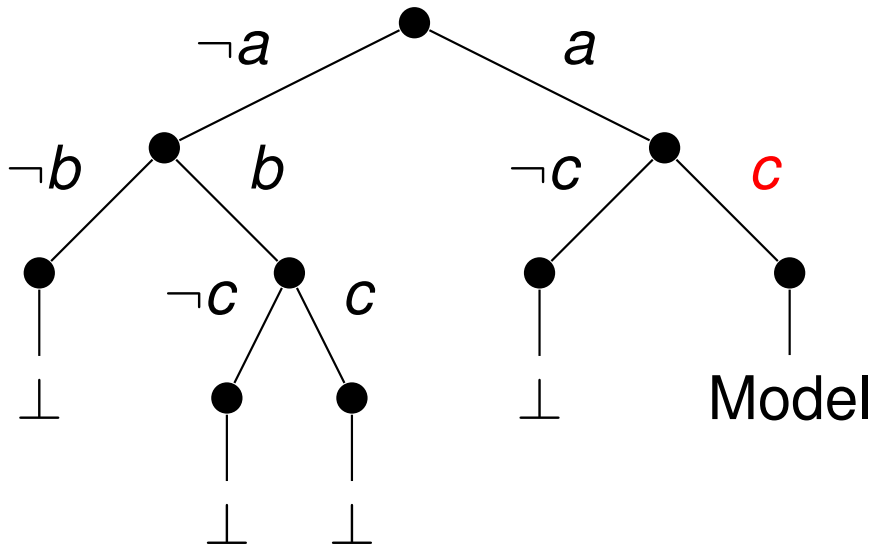

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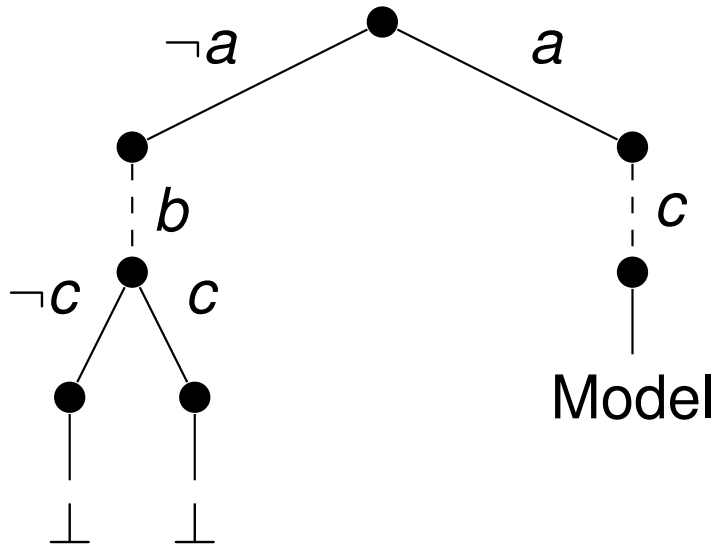

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- Restarts

SAT Research

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MiniSat

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