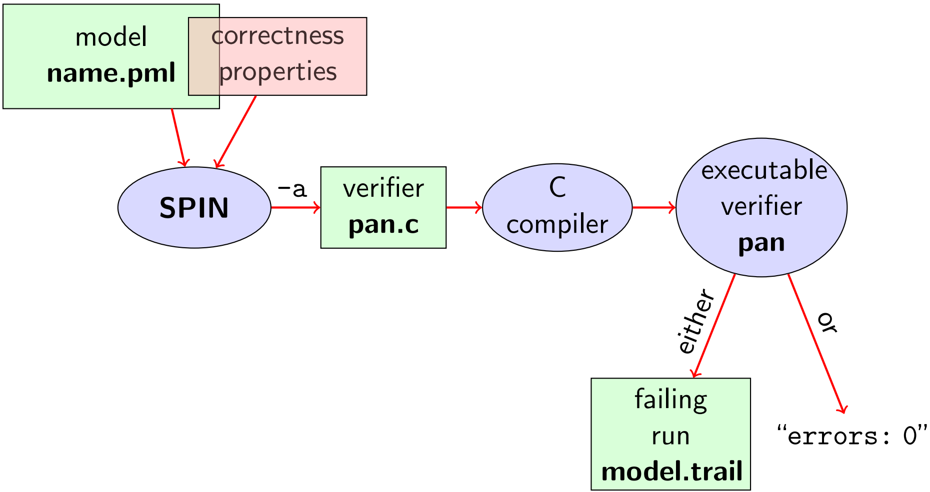
Software Engineering using Formal Methods

Model Checking with Temporal Logic

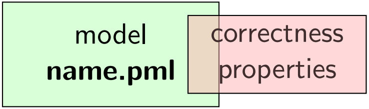
Wolfgang Ahrendt, Josef Svenningsson, Meng Wang

25th September 2012

Model Checking with SPIN

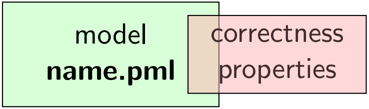


Stating Correctness Properties



Correctness properties can be stated within, or outside, the model.

Stating Correctness Properties

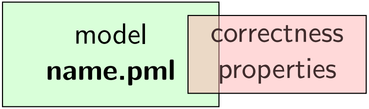


Correctness properties can be stated within, or outside, the model.

stating properties within model using

- assertion statements ✔

Stating Correctness Properties

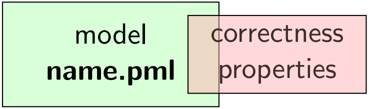


Correctness properties can be stated within, or outside, the model.

stating properties within model using

- assertion statements ✔
- meta labels
  - `end` labels ✔
  - `accept` labels
  - `progress` labels

Stating Correctness Properties



Correctness properties can be stated within, or outside, the model.

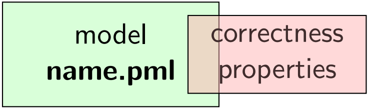
stating properties within model using

- assertion statements ✔
- meta labels
  - `end` labels ✔
  - `accept` labels
  - `progress` labels

stating properties outside model using

- never claims
- temporal logic formulas

Stating Correctness Properties



Correctness properties can be stated within, or outside, the model.

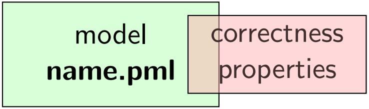
stating properties within model using

- assertion statements ✔
- meta labels
  - `end` labels ✔
  - `accept` labels
  - `progress` labels

stating properties outside model using

- never claims
- temporal logic formulas (today's main topic)

Stating Correctness Properties



Correctness properties can be stated within, or outside, the model.

stating properties within model using

- assertion statements ✔
- meta labels
  - end labels ✔
  - accept labels (briefly)
  - progress labels

stating properties outside model using

- never claims (briefly)
- temporal logic formulas (today's main topic)

Preliminaries

1. Accept labels in PROMELA ↔ Büchi automata
2. Fairness

Preliminaries 1: Acceptance Cycles

Definition (Accept Location)

A location marked with an accept label of the form "`accept`*xxx*`:`" is called an accept location.

Preliminaries 1: Acceptance Cycles

Definition (Accept Location)

A location marked with an accept label of the form "`accept`*xxx*`:`" is called an accept location.

Accept locations can be used to specify cyclic behavior

Definition (Acceptance Cycle)

A run which infinitely often passes through an accept location is called an acceptance cycle.

Acceptance cycles are mainly used in never claims (see below), to define forbidden infinite behavior

Preliminaries 2: Fairness

Does this PROMELA model terminate in each run?

Demo: `start/fair.pml`

```
byte n = 0;
bool flag = false;

active proctype P() {
  do :: flag -> break
     :: else -> n = 5 - n
  od
}
active proctype Q() {
  flag = true
}
```

Preliminaries 2: Fairness

Does this PROMELA model terminate in each run?

Demo: `start/fair.pml`

```
byte n = 0;
bool flag = false;

active proctype P() {
  do ::  flag -> break
     ::  else -> n = 5 - n
  od
}
active proctype Q() {
  flag = true
}
```

Termination guaranteed only if scheduling is (weakly) fair!

Preliminaries 2: Fairness

Does this PROMELA model terminate in each run?

Demo: `start/fair.pml`

```
byte n = 0;
bool flag = false;

active proctype P() {
  do :: flag -> break
     :: else -> n = 5 - n
  od
}
active proctype Q() {
  flag = true
}
```

Termination guaranteed only if scheduling is (weakly) fair!

Definition (Weak Fairness)

A run is called weakly fair iff the following holds:
each continuously executable statement is executed eventually.

Model Checking of Temporal Properties

Many correctness properties not expressible by assertions

- all properties that involve state changes
- temporal logic expressive enough to characterize many (but not all) properties

In this course: "temporal logic" synonymous with "linear temporal logic"

Today: model checking of properties formulated in temporal logic

Beyond Assertions

Locality of Assertions

Assertions talk only about the state at their location in the code

Beyond Assertions

Locality of Assertions

Assertions talk only about the state at their location in the code

Example

Mutual exclusion enforced by adding assertion to each critical section

```
critical++;
assert( critical <= 1 );
critical--;
```

Beyond Assertions

Locality of Assertions

Assertions talk only about the state at their location in the code

Example

Mutual exclusion enforced by adding assertion to each critical section

```
critical++;
assert( critical <= 1 );
critical--;
```

Drawbacks

- no separation of concerns (model vs. correctness property)
- changing assertions is error prone (easily out of sync)
- easy to forget assertions:
  correctness property might be violated at unexpected locations
- many interesting properties not expressible via assertions

Temporal Correctness Properties

Examples of properties more conveniently expressed as global properties than as assertions:

Temporal Correctness Properties

Examples of properties more conveniently expressed as global properties than as assertions:

Mutual Exclusion

"`critical <= 1` holds throughout any run"

Temporal Correctness Properties

Examples of properties more conveniently expressed as global properties than as assertions:

Mutual Exclusion

"`critical <= 1` holds throughout any run"

Array Index within Bounds (given array `a` of length len)

"`0 <= i <=` len`-1` holds throughout any run"

Temporal Correctness Properties

Examples of properties more conveniently expressed as global properties than as assertions:

Mutual Exclusion

"`critical <= 1` holds throughout any run"

Array Index within Bounds (given array `a` of length len)

"`0 <= i <=` len`-1` holds throughout any run"

Examples of properties impossible to express as assertions:

Temporal Correctness Properties

Examples of properties more conveniently expressed as global properties than as assertions:

Mutual Exclusion

"`critical <= 1` holds throughout any run"

Array Index within Bounds (given array `a` of length len)

"`0 <= i <=` len`-1` holds throughout any run"

Examples of properties impossible to express as assertions:

Absence of Deadlock

"If several processes try to enter their critical section, eventually one of them does so."

Temporal Correctness Properties

Examples of properties more conveniently expressed as global properties than as assertions:

Mutual Exclusion

"`critical <= 1` holds throughout any run"

Array Index within Bounds (given array `a` of length len)

"`0 <= i <=` len`-1` holds throughout any run"

Examples of properties impossible to express as assertions:

Absence of Deadlock

"If several processes try to enter their critical section, eventually one of them does so."

Absence of Starvation

"If one process tries to enter its critical section, eventually that process does so."

Temporal Correctness Properties

Examples of properties more conveniently expressed as global properties than as assertions:

Mutual Exclusion

"`critical <= 1` holds throughout any run"

Array Index within Bounds (given array `a` of length len)

"`0 <= i <=` len`-1` holds throughout any run"

Examples of properties impossible to express as assertions:

Absence of Deadlock

"If several processes try to enter their critical section, eventually one of them does so."

Absence of Starvation

"If one process tries to enter its critical section, eventually that process does so."

All of these are temporal properties $\Rightarrow$ use temporal logic

Boolean Temporal Logic

Numerical variables in expressions

- Expressions such as `0 <= i <= len-1` contain numerical variables
- Propositional LTL as introduced so far only knows propositions
- Slight generalisation of LTL required

Boolean Temporal Logic

Numerical variables in expressions

- Expressions such as `0 <= i <= len-1` contain numerical variables
- Propositional LTL as introduced so far only knows propositions
- Slight generalisation of LTL required

In Boolean Temporal Logic atomic building blocks are Boolean expressions over PROMELA variables

Boolean Temporal Logic over PROMELA

Set For_{BTL} of Boolean Temporal Formulas (simplified)

- all global PROMELA variables and constants of type **bool**/**bit** are $\in For_{BTL}$

Boolean Temporal Logic over PROMELA

Set For_{BTL} of Boolean Temporal Formulas (simplified)

- all global PROMELA variables and constants of type **bool**/**bit** are $\in For_{BTL}$
- if `e1` and `e2` are numerical PROMELA expressions, then all of `e1==e2`, `e1!=e2`, `e1<e2`, `e1<=e2`, `e1>e2`, `e1>=e2` are $\in For_{BTL}$

Boolean Temporal Logic over PROMELA

Set For_{BTL} of Boolean Temporal Formulas (simplified)

- all global PROMELA variables and constants of type **bool**/**bit** are $\in For_{BTL}$
- if `e1` and `e2` are numerical PROMELA expressions, then all of `e1==e2`, `e1!=e2`, `e1<e2`, `e1<=e2`, `e1>e2`, `e1>=e2` are $\in For_{BTL}$
- if `P` is a process and `l` is a label in `P`, then `P@l` is $\in For_{BTL}$ (`P@l` reads "`P` is at `l`")

Boolean Temporal Logic over PROMELA

Set For_{BTL} of Boolean Temporal Formulas (simplified)

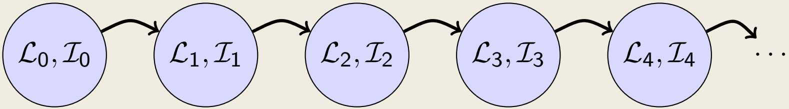
- all global PROMELA variables and constants of type **bool**/**bit** are $\in For_{BTL}$
- if `e1` and `e2` are numerical PROMELA expressions, then all of `e1==e2`, `e1!=e2`, `e1<e2`, `e1<=e2`, `e1>e2`, `e1>=e2` are $\in For_{BTL}$
- if `P` is a process and `l` is a label in `P`, then `P@l` is $\in For_{BTL}$ (`P@l` reads "`P` is at `l`")
- if ϕ and ψ are formulas $\in For_{BTL}$, then all of

$$!\phi, \quad \phi \;\&\&\; \psi, \quad \phi \;||\; \psi, \quad \phi \;\text{->}\; \psi, \quad \phi \;\text{<->}\; \psi$$
$$[\,]\phi, \quad <>\phi, \quad \phi\,\mathtt{U}\,\psi$$

are $\in For_{BTL}$

Semantics of Boolean Temporal Logic

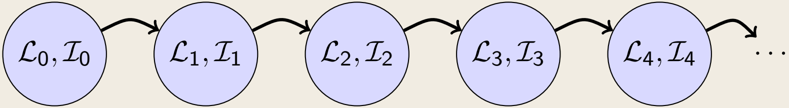
A run σ through a PROMELA model M is a chain of states



- $\mathcal{L}_j$ maps each running process to its current location counter
- From $\mathcal{L}_j$ to $\mathcal{L}_{j+1}$, only one of the location counters has advanced (exception: channel rendezvous)
- $\mathcal{I}_j$ maps each variable in M to its current value

Semantics of Boolean Temporal Logic

A run σ through a PROMELA model M is a chain of states

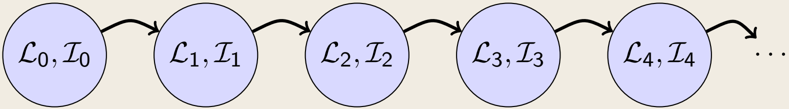


- $\mathcal{L}_j$ maps each running process to its current location counter
- From $\mathcal{L}_j$ to $\mathcal{L}_{j+1}$, only one of the location counters has advanced (exception: channel rendezvous)
- $\mathcal{I}_j$ maps each variable in M to its current value

Arithmetic and relational expressions are interpreted in states as expected; e.g. $\mathcal{L}_j, \mathcal{I}_j \models \texttt{x<y}$ iff $\mathcal{I}_j(\texttt{x}) < \mathcal{I}_j(\texttt{y})$

Semantics of Boolean Temporal Logic

A run σ through a PROMELA model M is a chain of states



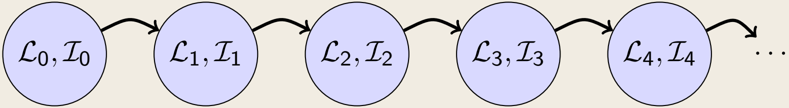
- $\mathcal{L}_j$ maps each running process to its current location counter
- From $\mathcal{L}_j$ to $\mathcal{L}_{j+1}$, only one of the location counters has advanced (exception: channel rendezvous)
- $\mathcal{I}_j$ maps each variable in M to its current value

Arithmetic and relational expressions are interpreted in states as expected; e.g. $\mathcal{L}_j, \mathcal{I}_j \models \texttt{x<y}$ iff $\mathcal{I}_j(\texttt{x}) < \mathcal{I}_j(\texttt{y})$

$\mathcal{L}_j, \mathcal{I}_j \models \texttt{P@l}$ iff $\mathcal{L}_j(\texttt{P})$ is the location labeled with `l`

Semantics of Boolean Temporal Logic

A run σ through a PROMELA model M is a chain of states



- $\mathcal{L}_j$ maps each running process to its current location counter
- From $\mathcal{L}_j$ to $\mathcal{L}_{j+1}$, only one of the location counters has advanced (exception: channel rendezvous)
- $\mathcal{I}_j$ maps each variable in M to its current value

Arithmetic and relational expressions are interpreted in states as expected; e.g. $\mathcal{L}_j, \mathcal{I}_j \models \texttt{x<y}$ iff $\mathcal{I}_j(\texttt{x}) < \mathcal{I}_j(\texttt{y})$

$\mathcal{L}_j, \mathcal{I}_j \models \texttt{P@l}$ iff $\mathcal{L}_j(\texttt{P})$ is the location labeled with `l`

Evaluating other formulas $\in For_{BTL}$ in runs σ: see previous lecture

Safety Properties

Safety Properties

...are formulas of the form $[]\phi$

- state that something good (ϕ) is guaranteed throughout each run
- equivalently: in $[]\neg\psi$ something bad (ψ) never happens

Safety Properties

Safety Properties

. . . are formulas of the form $[]\phi$

- state that something good (ϕ) is guaranteed throughout each run
- equivalently: in $[]\neg\psi$ something bad (ψ) never happens

Example

TL formula `[](critical <= 1)`

"it is guaranteed throughout each run that at most one process visits its critical section at any time"

Safety Properties

Safety Properties

. . . are formulas of the form $[]\phi$

- state that something good (ϕ) is guaranteed throughout each run
- equivalently: in $[]\neg\psi$ something bad (ψ) never happens

Example

TL formula `[](critical <= 1)`

"it is guaranteed throughout each run that at most one process visits its critical section at any time"

or, equivalently:
"it will never happen that more than one process visits its critical section"

Applying Temporal Logic to Critical Section Problem

We want to verify `[](critical<=1)` as a correctness property of:

```
active proctype P() {
  do :: /* non-critical activity */
        atomic {
          !inCriticalQ;
          inCriticalP = true
        }
        critical++;
        /* critical activity */
        critical--;
        inCriticalP = false
  od
}

/* similarly for process Q */
```

Model Checking a Safety Property with JSPIN

Alternative 1: `ltl` in model file

1. add definition of TL formula to PROMELA file

   Example `ltl m { [](critical <= 1) }`

   General `ltl <name> { <TL formula> }`

   can define more than one formula
2. load PROMELA file in JSPIN
3. ensure `Safety` is selected
4. select `Verify`
   - JSPIN always selects first formula
   - use command line `./pan -N <name>` to select arbitrary formulas
5. (if necessary) select `Stop` to terminate too long verification

Demo: `safety1.pml`

Model Checking a Safety Property with jSpin

Alternative 1: `ltl` in model file

1. add definition of TL formula to Promela file

   Example `ltl m { [](critical <= 1) }`

   General `ltl <name> { <TL formula> }`

   can define more than one formula

2. load Promela file in jSpin
3. ensure `Safety` is selected
4. select `Verify`
   - jSpin always selects first formula
   - use command line `./pan -N <name>` to select arbitrary formulas
5. (if necessary) select `Stop` to terminate too long verification

Demo: `safety1.pml`

`ltl` definitions not part of Ben Ari's book (Spin $\geq$ 6): ignore 5.3.2, etc.

Model Checking a Safety Property with jSpin

Alternative 2: edit 'LTL fomula' field of jSpin

1. load Promela file in jSpin (not necessarily containing `ltl ...`)
2. enter `[](critical <= 1)` in LTL text field of jSpin
3. select `Translate` to create a 'never claim', corresponding to the negation of the formula
4. ensure `Safety` is selected
5. select `Verify`
6. (if necessary) select `Stop` to terminate too long verification

Demo: `safety1.pml`

Never Claims: Processes trying to show user wrong

Büchi automaton, as PROMELA process, for negated property

1. Negated TL formula translated to '**never**' process
2. accepting locations in Büchi automaton represented with help of accept labels ("`accept`*xxx*`:`")
3. If one of these reached infinitely often, the orig. property is violated

Never Claims: Processes trying to show user wrong

Büchi automaton, as PROMELA process, for negated property

1. Negated TL formula translated to '**never**' process
2. accepting locations in Büchi automaton represented with help of accept labels ("accept*xxx*:")
3. If one of these reached infinitely often, the orig. property is violated

Example (Never claim for `<>p`, simplified for readability)

```
never {  /* !(<>p) */
  accept_xyz: /* passed ∞ often iff !(<>p) holds */
  do
  :: (!p)
  od
}
```

Model Checking against Temporal Logic Property

Theory behind SPIN

1. Represent the interleaving of all processes as a single automaton (only one process advances in each step), called $\mathcal{M}$

Model Checking against Temporal Logic Property

Theory behind SPIN

1. Represent the interleaving of all processes as a single automaton (only one process advances in each step), called $\mathcal{M}$
2. Construct Büchi automaton (never claim) $\mathcal{NC}_{\neg\phi}$ for negation of TL formula ϕ to be verified

Model Checking against Temporal Logic Property

Theory behind SPIN

1. Represent the interleaving of all processes as a single automaton (only one process advances in each step), called $\mathcal{M}$
2. Construct Büchi automaton (never claim) $\mathcal{NC}_{\neg\phi}$ for negation of TL formula ϕ to be verified
3. If

$$\mathcal{L}^{\omega}(\mathcal{M}) \cap \mathcal{L}^{\omega}(\mathcal{NC}_{\neg\phi}) = \emptyset$$

then ϕ holds in $\mathcal{M}$,
otherwise we have a counterexample

Model Checking against Temporal Logic Property

Theory behind SPIN

1. Represent the interleaving of all processes as a single automaton (only one process advances in each step), called $\mathcal{M}$
2. Construct Büchi automaton (never claim) $\mathcal{NC}_{\neg\phi}$ for negation of TL formula ϕ to be verified
3. If

$$\mathcal{L}^\omega(\mathcal{M}) \cap \mathcal{L}^\omega(\mathcal{NC}_{\neg\phi}) = \emptyset$$

   then ϕ holds in $\mathcal{M}$,
   otherwise we have a counterexample
4. To check $\mathcal{L}^\omega(\mathcal{M}) \cap \mathcal{L}^\omega(\mathcal{NC}_{\neg\phi})$ construct intersection automaton (both automata advance in each step) and search for accepting run

Model Checking a Safety Property with SPIN directly

Command Line Execution (Alt. 1)

Make sure `ltl <name> { <TL formula> }` *is in* `<file>.pml`

```
> spin -a <file>.pml
> gcc -DSAFETY -o pan pan.c
> ./pan -N <name>
```

Demo: `target/safety1.pml`

Model Checking a Safety Property with SPIN directly

Command Line Execution (Alt. 2)

Write negated TL formula in file <formulafile>.PRP (first line)

```
> spin -a -F <formulafile>.PRP <file>.pml
> gcc -DSAFETY -o pan pan.c
> ./pan
```

Model Checking a Safety Property with SPIN directly

Command Line Execution (Alt. 2)

Write negated TL formula in file <formulafile>.PRP (first line)

```
> spin -a -F <formulafile>.PRP <file>.pml
> gcc -DSAFETY -o pan pan.c
> ./pan
```

- some platforms have problems with `-F`
- in any case: `.PRP` file must be part of your lab submission

Liveness Properties

Liveness Properties

. . . formulas of the form $<>\phi$

- state that something good (ϕ) eventually happens in each run

Liveness Properties

Liveness Properties

. . . formulas of the form $<>\phi$

- state that something good (ϕ) eventually happens in each run

Example

`<>csp`

(with `csp` a variable only true in the critical section of `P`)

"in each run, process `P` visits its critical section eventually"

Applying Temporal Logic to Starvation Problem

We want to verify `<>csp` as a correctness property of:

```
active proctype P() {
  do :: /* non-critical activity */
        atomic {
          !inCriticalQ;
          inCriticalP = true
        }
        csp = true;
        /* critical activity */
        csp = false;
        inCriticalP = false
  od
}

/* similarly for process Q */
/* there, using csq         */
```

Model Checking a Liveness Property with jSpin

1. open Promela file
2. enter `<>csp` in 'LTL fomula' field
3. select `Translate` to create a 'never claim', corresponding to the negation of the formula
4. ensure that `Acceptance` is selected
   (Spin will search for *accepting* cycles through the never claim)
5. *for the moment* uncheck `Weak Fairness` (see discussion below)
6. select `Verify`

Model Checking a Liveness Property with jSpin

1. open Promela file
2. enter `<>csp` in ‘LTL fomula’ field
3. select `Translate` to create a ‘never claim’, corresponding to the negation of the formula
4. ensure that `Acceptance` is selected
   (Spin will search for *accepting* cycles through the never claim)
5. *for the moment* uncheck `Weak Fairness` (see discussion below)
6. select `Verify`

Alternative to 2. and 3., write

```
ltl l { <>csp }
```

in Promela file (as first `ltl` formula).

Verification Fails

Demo: `start/liveness1.pml`

Verification fails!

Why?

Verification Fails

Demo: `start/liveness1.pml`

Verification fails!

Why?

The liveness property on one process "had no chance"
Not even weak fairness was switched on!

Model Checking Liveness with Weak Fairness

Always check `enforce weak fairness constraint`
when verifying liveness

1. open PROMELA file
2. enter `<>csp` in `LTL` text field
3. select `Translate` to create a 'never claim', corresponding to the negation of the formula
4. ensure that `Acceptance` is selected
   (SPIN will search for *accepting* cycles through the never claim)
5. ensure `Weak Fairness` is checked
6. select `Verify`

Model Checking Liveness with Weak Fairness

Always check `enforce weak fairness constraint`
when verifying liveness

1. open PROMELA file
2. enter `<>csp` in LTL text field
3. select `Translate` to create a 'never claim', corresponding to the negation of the formula
4. ensure that `Acceptance` is selected
   (SPIN will search for *accepting* cycles through the never claim)
5. ensure `Weak Fairness` is checked
6. select `Verify`

Alternative to 2. and 3., write

```
ltl l { <>csp }
```

in PROMELA file (as first `ltl` formula).

Model Checking Liveness with SPIN directly

Command Line Execution (Alt. 1)

Make sure `ltl <name> { <TL formula> }` *is in* `<file>.pml`

```
> spin -a <file>.pml
> gcc -o pan pan.c
> ./pan -a -f [-N <name>]
```

-a acceptance cycles, -f weak fairness

Demo: `start/liveness1.pml`

Model Checking Liveness with SPIN directly

Command Line Execution (Alt. 2)

Write negated TL formula in file <formulafile>.PRP (first line)

```
> spin -a -F <formulafile>.PRP <file>.pml
> gcc -o pan pan.c
> ./pan -a -f [-N <name>]
```

-a acceptance cycles, -f weak fairness

Limitation of Weak Fairness

Verification fails again!

Why?

Limitation of Weak Fairness

Verification fails again!

Why?

Weak fairness is too weak ...

Definition (Weak Fairness)

A run is called weakly fair iff the following holds:
each continuously executable statement is executed eventually.

Note that `!inCriticalQ` is not continuously executable!

Limitation of Weak Fairness

Verification fails again!

Why?

Weak fairness is too weak . . .

Definition (Weak Fairness)

A run is called weakly fair iff the following holds:
each continuously executable statement is executed eventually.

Note that `!inCriticalQ` is not continuously executable!

Restriction to weak fairness is principal limitation of SPIN

The only way to show liveness of our example is to rewrite the model

Temporal Model Checking without Ghost Variables

We want to verify mutual exclusion without using ghost variables

```
bool inCriticalP = false , inCriticalQ = false ;

active proctype P() {
  do :: atomic {
          !inCriticalQ;
          inCriticalP = true
        }
cs:     /* critical activity */
        inCriticalP = false
  od
}

/* similar for process Q with same label cs: */

ltl m { []!(P@cs && Q@cs) }
```

Demo: `start/noGhost.pml`

Liveness Revisited

Label expressions often remove the need for ghost variables

Liveness Revisited

Label expressions often remove the need for ghost variables

- Specify liveness of `fair.pml` using labels
- Prove termination
- Weak fairness is sufficient

Demo: `target/fair.pml`

Literature for this Lecture

Ben-Ari Chapter 5
except Sections 5.3.2, 5.3.3, 5.4.2
(`ltl` replaces **#define** and `-f` option of SPIN)