

## Multiprocessor scheduling

### How are tasks assigned to processors?

- Static assignment
  - The processor(s) used for executing a task are determined before system is put in mission ("off-line")
  - Approaches: partitioned scheduling, guided search, non-guided search, ...
- Dynamic assignment
  - The processor(s) used for executing a task are determined during system operation "on-line"
  - Approach: global scheduling

## Multiprocessor scheduling

### How are tasks allowed to migrate?

- Partitioned scheduling (no migration!)
  - Each instance of a task must execute on the same processor
  - Equivalent to multiple uniprocessor systems!
- Guided search & non-guided techniques
  - Depending on migration constraints, a task may or may not execute on more than one processor
- Global scheduling (full migration!)
  - A task is allowed to execute on an arbitrary processor (sometimes even after being preempted)

## Partitioned scheduling

### General characteristics:

- Each processor has its own queue for ready tasks
- Tasks are organized in groups, and each task group is assigned to a specific processor
- When selected for execution, a task can only be dispatched to its assigned processor

## Partitioned scheduling

### Advantages:

- Mature scheduling framework
  - Most uniprocessor scheduling theory also applicable here
  - Uniprocessor resource-management protocols can be used
- Supported by automotive industry
  - AUTOSAR prescribes partitioned scheduling

### Disadvantages:

- Cannot exploit all unused execution time
  - Surplus capacity cannot be shared among processors
  - Will suffer from overly-pessimistic WCET derivation

## Partitioned scheduling

Complexity of schedulability analysis for partitioned scheduling: (Leung & Whitehead, 1982)

The problem of deciding whether a task set (synchronous or asynchronous) is schedulable on  $m$  processors with respect to partitioned scheduling is NP-complete in the strong sense.

**Consequence:**

There cannot be any pseudo-polynomial time algorithm for finding an optimal partition of a set of tasks unless  $P = NP$ .

## Partitioned scheduling

**Bin-packing algorithms:**

- Basic idea:
  - The problem concerns packing objects of varying sizes in boxes ("bins") with the objective of minimizing number of used boxes.
- Application to multiprocessor systems:
  - Bins are represented by processors and objects by tasks.
  - The decision whether a processor is "full" or not is derived from a utilization-based feasibility test.
- Assumptions:
  - Independent, periodic tasks
  - Preemptive, uniprocessor scheduling (RM)



## Partitioned scheduling

### Bin-packing algorithms:

Rate-Monotonic-First-Fit (RMFF): (Dhall and Liu, 1978)

- Let the processors be indexed as  $\mu_1, \mu_2, \dots$
- Assign the tasks in the order of increasing periods (that is, RM order).
- For each task  $\tau_i$ , choose the lowest previously-used  $j$  such that  $\tau_i$ , together with all tasks that have already been assigned to processor  $\mu_j$ , can be feasibly scheduled according to the utilization-based RM-feasibility test.
- Processors are added if needed for RM-schedulability.

## Partitioned scheduling

### Guarantee bound for RMFF:

The utilization guarantee bound  $U_{RMFF}$  for a system with  $m$  processors using the RMFF scheduling policy (with arbitrary task-assignment order) is

$$m(2^{1/2} - 1) \leq U_{RMFF} \leq (m + 1) / \left(1 + 2^{1/(m+1)}\right) \quad (\text{Oh \& Baker, 1998})$$

Note:  $(2^{1/2} - 1) \approx 0.41$

Thus: task sets whose utilization do not exceed  $\approx 41\%$  of the total processor capacity is always RMFF-schedulable.

## Guided search

### Branch-and-bound algorithms:

- Basic idea:
  - A set of solutions to a given problem is organized in a search tree.
  - A vertex in the search tree corresponds to a specific solution structure.
  - A goal vertex corresponds to a complete solution to the problem and is located at the highest level of the search tree.
  - The root vertex corresponds to an initial solution at the lowest level of the search tree.
  - The search for a solution starts with only the root vertex.
  - Search objective is to find a goal vertex that optimizes a given cost (performance measure).

## Guided search

### Branch-and-bound algorithms:

- Basic idea (cont'd):
  - For each vertex, a set of child vertices is generated by modifying the structure of the current vertex ("branching").
  - To check if a tree branch may lead to an acceptable solution, a lower-bound function is applied to each of the child vertices.
  - If a child vertex looks promising, it will be further investigated.
  - If a child vertex will only lead to inferior solutions, that entire branch is pruned ("bounding").

**Note:** An initial solution could be used for making good bounding operations early in the search. When an acceptable goal vertex is reached the bounding operation can be made more accurate.

## Guided search

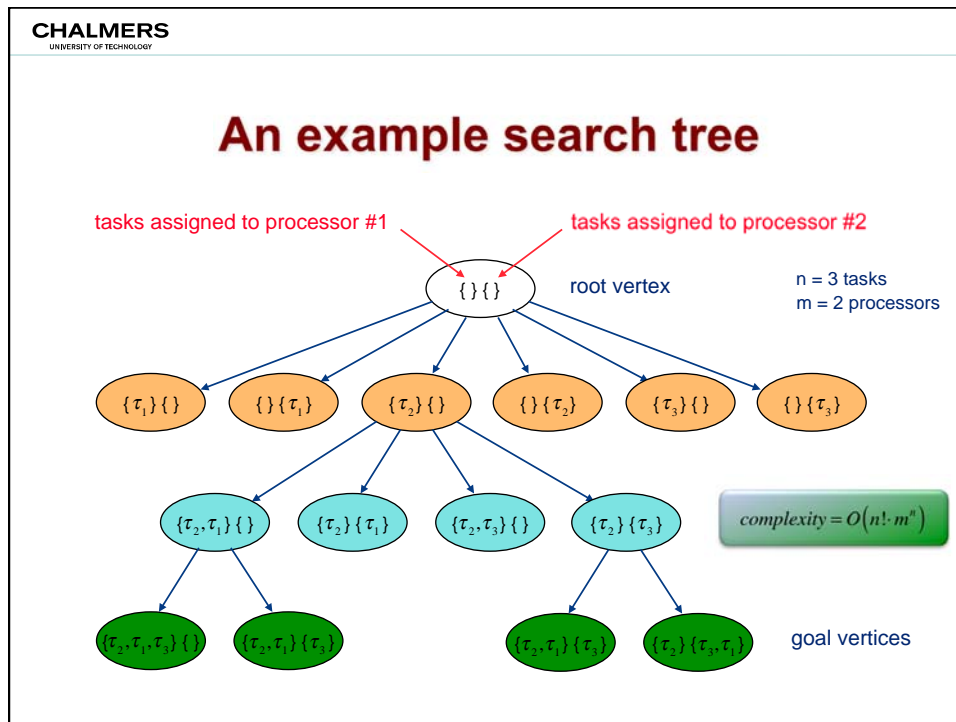
### Branch-and-bound algorithms:

- Application to multiprocessor scheduling:
  - The search tree represents the set of all task-to-processor assignments for a given set of tasks and processors.
  - A vertex in the search tree is a partial or complete assignment of tasks to processors.
  - The root vertex corresponds to an initial (empty or complete) schedule.
  - A goal vertex corresponds to a complete schedule.
  - The purpose of the lower-bound function is to assess whether a child vertex is feasible, that is, whether the corresponding branch in the search tree contains a feasible schedule.

## Guided search

### Branch-and-bound for multiprocessor scheduling:

- Initial schedule is empty:
  - At each vertex in the search tree, a set of ready tasks (candidates for execution) are available for scheduling.
  - Generation of a child vertex corresponds to adding one of the ready tasks to the schedule in the current vertex.
- Initial schedule is complete (but possibly suboptimal):
  - At each level of the search tree, a set of scheduling changes (e.g., modified constraints or assignments) are available.
  - Generation of a child vertex corresponds to applying one or more of the changes to the schedule in the current vertex.



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## Guided search

How do we avoid an exhaustive search?

- Bound pruning
  - use optimistic lower bounds
- Redundancy pruning
  - exploit symmetries in task set and processors
- Algorithm configuration
  - use suitable exploration order for promising vertices
- Performance guarantees
  - solution is within guaranteed bound from optimum
- Local optimization
  - only a subset of child vertices are retained

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## Guided search

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## Guided search

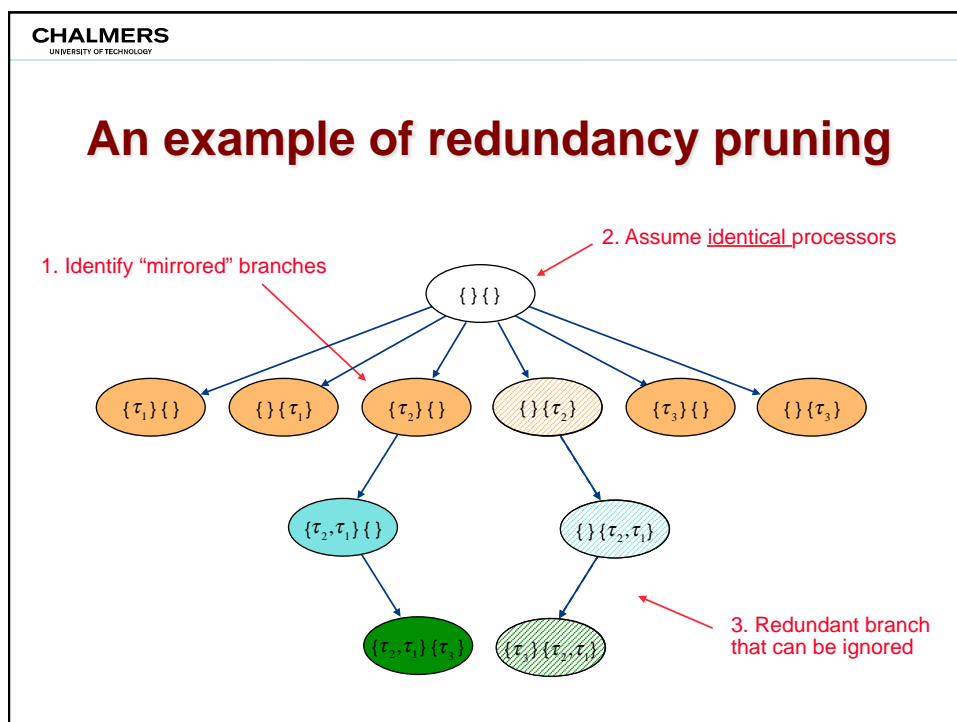
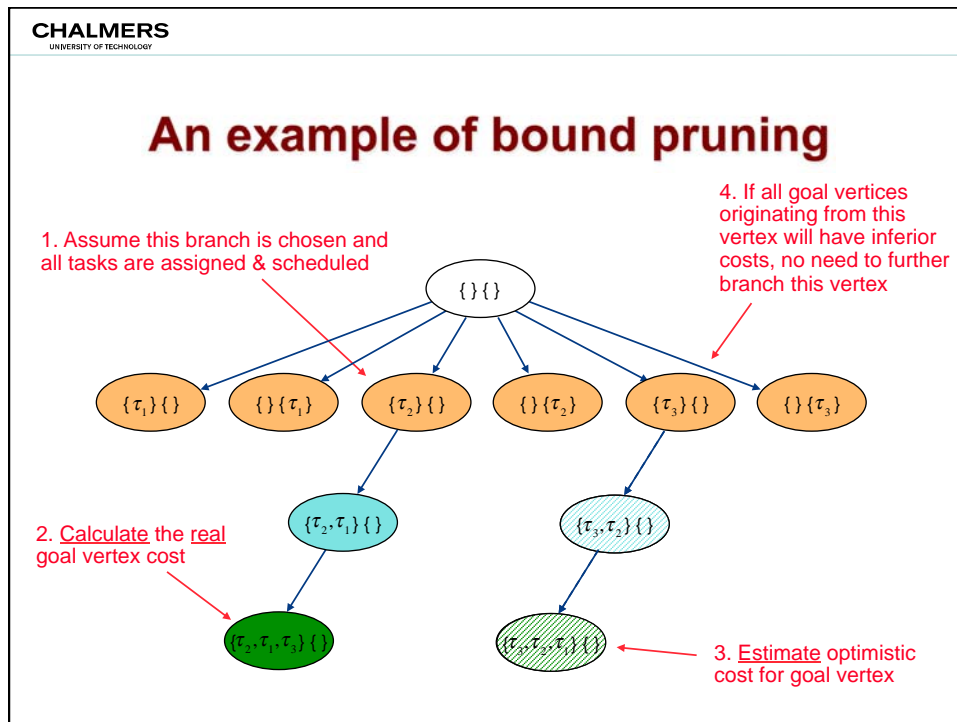
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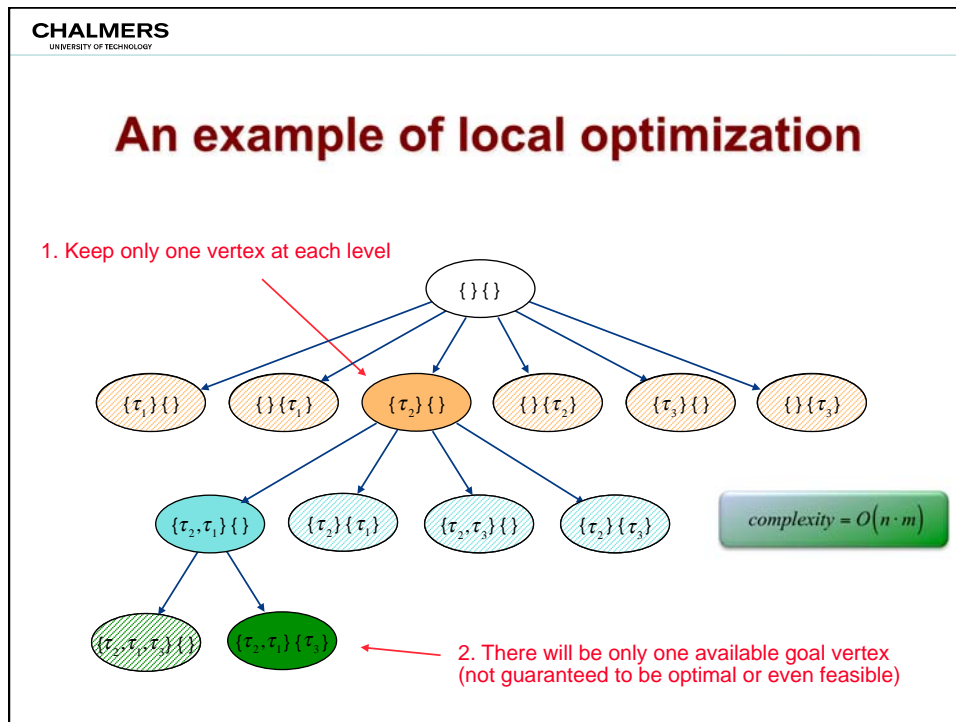
- **Bound pruning**
  - use optimistic lower bounds

**Additional reading:**  
Read the paper by Jonsson and Shin (ICPP'97)  
Study how different vertex selection rules and estimated bounds affect the performance of the search algorithm

- **Performance guarantees**
  - solution is within guaranteed bound from optimum
- **Local optimization**
  - only a subset of child vertices are retained







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## Guided search

Some (optimal) branch-and-bound algorithms:

- Distributed real-time systems: (Peng and Shin, 1989)
  - Minimizes system hazard (maximum normalized task response time)
  - Starts with an empty schedule
- Fault-tolerant real-time systems: (Hou and Shin, 1994)
  - Maximizes probability of no dynamic failure (probability that all deadlines are met in the presence of component failures)
  - Starts with an empty schedule
  - May change degree of replication and restart the algorithm

## Guided search

### Some (optimal) branch-and-bound algorithms:

- Uniprocessor real-time systems: (Xu and Parnas, 1990)
  - Minimizes maximum task lateness
  - Starts with an initial (complete) schedule
  - Modifies preemption, precedence and exclusion constraints
- Multiprocessor real-time systems: (Xu, 1993)
  - Minimizes maximum task lateness
  - Starts with an initial (complete) schedule
  - Modifies preemption, precedence and exclusion constraints

## Guided search

### Some good local-optimization algorithms:

- Myopic scheduling: (Ramamritham, Stankovic and Shiah, 1990)
  - Promising vertices are explored in the order of decreasing search-tree level; within each level, exploration order is given by a heuristic function that calculates a weighted sum of task execution time, deadline, earliest start time and laxity.
  - Lower-bound function determines for the current vertex whether it is strongly feasible, that is, whether a feasible schedule can be obtained by expanding any of its child vertices.
  - Reduces search complexity by only investigating the  $k$  child vertices with closest deadline in the check for strong feasibility.
  - Reduces search complexity by limiting the number of allowed backtracks (to vertices at lower search-tree levels)

$$complexity = O(k \cdot n \cdot m)$$

## Guided search

### Some good local-optimization algorithms:

- Pair-wise clustering: (Ramamritham, 1995)
  - Promising vertices are explored in the order of decreasing search-tree level; within each level, exploration is made in the order of increasing task LFT (latest finishing time).
  - Lower-bound function determines for the current vertex whether it is feasible using simple heuristics that keep track of latest start time and available time resources.
  - LFT is derived from task set end-to-end deadlines.
  - Pairs of communicating tasks are clustered based on the communication volume ratio. If the ratio between the task pair's execution times and communication volume is below a certain bound, the two tasks are assigned to the same processor.

## Non-guided search

### General characteristics:

- Each non-guided search is given an initial task-to-processor assignment from which the search starts.
- Within each iteration step during search, different derivable alternatives of changing the current assignment are examined.
- To check whether an alternative is feasible or not, a run-time efficient feasibility test has to be used.
- In order to help the search find better assignments, the number of deadline misses is included as a penalty into the function calculating the goodness of the assignment.

## Non-guided search

### Examples:

- Simulated annealing
- Genetic optimization
- Tabu search
- Neighbourhood search
- ...

These techniques all have in common that it is sufficient to state what makes a good solution, not how to get one!

## Non-guided search

### Simulated annealing: (Kirkpatrick, Gelatt and Vecchi, 1983)

- Basic idea:
  - Simulated annealing is a global optimization technique which borrows ideas from statistical physics. The technique is derived from observations of how slowly-cooled molten metal can result in a regular crystalline structure.
  - The salient property of the technique is the incorporation of random jumps from local minima to potential new solutions. As the algorithm progresses, this ability is lessened, by reducing a temperature factor, which makes larger jumps less likely.
  - The main objective of the technique is to find the lowest point in an energy landscape.

## Non-guided search

### Simulated annealing:

- Application to multiprocessor scheduling:
  - The set of all task-to-processor assignments for a given set of task and processors is called the problem space. A point in the problem space is an assignment of tasks to processors.
  - The neighbor space of a point is the set of points that are reachable by moving any single task to any other processor.
  - The energy of a point in problem space is a measure of the goodness of the task assignment represented by that point.
  - The energy function determines the shape of the problem space. It can be visualized as a rugged landscape, with deep valleys representing good solutions, and high peaks representing poor or infeasible ones.

## Non-guided search

### Simulated annealing:

- Algorithm:

A random starting point is chosen, and its energy  $E_s$  is evaluated.  
A random point in the neighbor space is then chosen, and its energy  $E_n$  is evaluated. This point becomes the new starting point if either  $E_n \leq E_s$ , or if  $E_n > E_s$  and

$$e^x \geq \text{random}(0,1) \quad \text{where } x = -(E_n - E_s)/C$$

The control variable  $C$  is analogous to the temperature factor in a thermodynamic system. During the annealing process,  $C$  is slowly reduced (cooling the system), making higher energy jumps less likely. Eventually, the system freezes into a low energy state.

## Non-guided search

### Simulated annealing:

- Implementation: (Tindell, Burns & Wellings, 1992)

**Neighbor function:** Choose a random task and move it to a randomly-chosen processor.

**Energy function:** The weighted sum of the following characteristics of the assignment:

- Number of tasks assigned to the wrong processor
- Number of replicas assigned to the same processor
- Number of processors with too high a memory utilization
- Number of tasks which do not meet their deadlines
- Total communication bus utilization

## Non-guided search

### Genetic optimization: (Goldberg, 1989)

- Basic idea:
  - Based on Darwin's evolution theory: "Survival of the Fittest"
  - Solutions to a problem is viewed as individuals forming a population.
  - Pair of individuals can create children (new individuals)
  - New individuals are created by applying a crossover operator to the genes of the parents
  - Genes of a new individual may mutate

## Non-guided search

### Genetic optimization:

- Application to multiprocessor scheduling:
  - Tasks assignments and orderings are viewed as “chromosomes”
  - Tasks represent “genes”
  - Mutation means that a task is moved to another processor